What is an image?

We can think of an **image** as a function, $f$, from $\mathbb{R}^2$ to $\mathbb{R}$:
- $f(x, y)$ gives the **intensity** at position $(x, y)$
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$
Images as functions
What is a digital image?

We usually operate on digital (discrete) images:

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

If our samples are $\Delta$ apart, we can write this as:

$$f[i,j] = \text{Quantize}\{ f(i \Delta, j \Delta) \}$$

The image can now be represented as a matrix of integer values:

<p>| | | | | | | | |</p>
<table>
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<td>0</td>
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</tr>
</tbody>
</table>
Image Processing

An image processing operation typically defines a new image $g$ in terms of an existing image $f$. We can transform either the range of $f$:

$$g(x, y) = t(f(x, y))$$

Or the domain of $f$:

$$g(x, y) = f(t_x(x, y), t_y(x, y))$$

What kinds of operations can each perform?
Image Processing

image filtering: change **range** of image
\[ g(x) = h(f(x)) \]

\[
\begin{array}{c}
\begin{array}{c}
 f \\
 x
\end{array} \\
\xrightarrow{h} \\
\begin{array}{c}
 f \\
 x
\end{array}
\end{array}
\]

image warping: change **domain** of image
\[ g(x) = f(h(x)) \]

\[
\begin{array}{c}
\begin{array}{c}
 f \\
 x
\end{array} \\
\xrightarrow{h} \\
\begin{array}{c}
 f \\
 x
\end{array}
\end{array}
\]
Image Processing

image filtering: change **range** of image

\[ g(x) = h(f(x)) \]

image warping: change **domain** of image

\[ g(x) = f(h(x)) \]
Point Processing

The simplest kind of range transformations are these independent of position x,y:

\[ g = t(f) \]

This is called point processing.

What can they do?

What's the form of \( t \)?

**Important:** every pixel for himself – spatial information completely lost!
Basic Point Processing

**FIGURE 3.3** Some basic gray-level transformation functions used for image enhancement.
Negative

**FIGURE 3.4**
(a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)
Log

FIGURE 3.5
(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$. 
Power-law transformations

\[ s = cr^\gamma \]

**FIGURE 3.6** Plots of the equation \( s = cr^\gamma \) for various values of \( \gamma \) (\( c = 1 \) in all cases).
Image Enhancement

**FIGURE 3.9**
(a) Aerial image.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0, \text{ and } 5.0$, respectively.

*(Original image for this example courtesy of NASA.)*
Example: Gamma Correction

\[ S = r^\gamma \]

e.g. \[ 0.25 = 0.5^{2.0} \]

http://www.cs.berkeley.edu/~efros/java/gamma/gamma.html
Contrast Stretching

FIGURE 3.10
Contrast stretching.
(a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)
Image Histograms

Dark image

Bright image

Low-contrast image

High-contrast image

Cumulative Histograms

\[ s = T(r) \]

**FIGURE 3.15** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)
Histogram Equalization

**FIGURE 3.17** (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.
Neighborhood Processing (filtering)

Q: What happens if I reshuffle all pixels within the image?

A: It’s histogram won’t change. No point processing will be affected…

Need spatial information to capture this…

…switch slides