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Abstract—This paper presents a novel battery-less wireless sensor that can be embedded in the road and used for measurement of traffic flow rate. Compared to existing inductive loop based traffic sensors, the new sensor is expected to provide increased reliability, easy installation and low cost. The sensor requires no external power source and has zero idle power loss. Energy to power this sensor is harvested from the short duration vibrations that results when an automobile passes over the sensor. Since much of the earlier work in literature on vibration energy harvesting has focused on continuous sources of vibration, this paper focuses on short duration vibrations and on developing low power control algorithms that can be implemented on the sensor using an analog circuit. To effect this paper develops and compares three control algorithms “Fixed threshold switching”, “Maximum voltage switching” and “Switched inductor.” The “Switched inductor” algorithm is shown to be the most effective at maximizing harvested energy. Experimental results are presented and validate the fact that adequate energy can be harvested from each passing vehicle to enable successful wireless transmission of traffic data.

Index Terms—Traffic sensor, Batteryless, Wireless, Energy harvesting.

I. REVIEW OF CURRENT TRAFFIC SENSORS

Transportation agencies all around the country monitor traffic flow rates on major highways using inductive loop detectors (ILDs). The Minnesota Department of Transportation (MnDOT), for example, monitors the flow rates at over 6000 points in the Minneapolis/St. Paul metro area using such ILDs. An ILD consists of a big loop of metallic coil buried in the lane. This loop is connected to a road-side station which provides power to the loop and processes the information obtained from it to determine if a vehicle is passing over the sensor. The traffic flow rate information from such sensors is used to control ramp meters, identify congestion points, detect incidents and for a number of other applications.

Inductive loop detectors exhibit high accuracy in detecting vehicles ([29]). Hence, despite various new non-intrusive technologies for detecting vehicles such as image processing based detectors ([6],[9],[15],[17]) and systems based on audio processing ([2],[3]), inductive loop detectors remain the most widely used technology.

Despite their popularity, ILDs are not perfect. They are prone to breakdowns [1], and can only measure traffic flow rate, not vehicle speed or vehicle classification. There have been attempts to use estimation algorithms to do vehicle classification using ILDs and identification techniques such as fuzzy logic and artificial neural networks (ANNs) ([13],[27]).

Despite many improvements, the installation of the ILD involves cutting a large section of the roadway in each lane and therefore causes considerable traffic disruption. Owing to its operating principle, the ILD needs to be continuously powered resulting in considerable idle power loss. For example, an ILD needs to be continuously powered at night, even if there is very little traffic flowing on the highway.

II. NEW BATTERY-LESS WIRELESS TRAFFIC SENSORS

A. Overview

The researchers in this paper have developed a novel battery-less wireless traffic sensor which requires zero external energy. The sensor is completely autonomous and can be embedded in the lane without the need for power or data cables. In the absence of any automobile, the sensor stays turned off, consuming no power. Thus, the sensor has zero idle power loss. When an automobile passes over the sensor, the sensor turns on and a RF pulse is transmitted wirelessly to a receiving station. The receiving station can be as much as 500 feet away from the sensor. The sensor requires no external power source as it is powered by harvesting all its required energy from vibrations that result when a vehicle passes over it. Further this sensor is significantly smaller compared to an ILD and can be installed with lower traffic disruptions. This is especially true because the sensor does not need a power source and power lines do not need to be run to the sensor. This new sensor, like the ILD, does not use complex image processing or audio processing techniques and hence is expected to provide the same level of high reliability. Owing to the battery-less and wireless nature of the sensor, low maintenance costs can also be expected. Further the sensor can measure the number of axles (vehicle classification) and can be modified to also measure the weight of the passing vehicle in addition to the traffic flow rate. It is also possible to configure several sensors to transmit to a single receiving station.

B. Principle

The proposed sensor is based on the principle of vibration energy harvesting (VEH) to enable wireless transmission of signals. Reference [26] provides a good
review of many of these VEH techniques. Some of the earlier work has also focused on developing control algorithms to optimize the amount of energy harvested ([16], [22]). However, the VEH techniques in literature focus predominantly on harvesting energy from a continuous source of vibration. When a vehicle passes over the sensor, the resulting mechanical vibrations are of short duration. Hence, although the concept of VEH is not new, it has never before been used to power a traffic sensor. Further the optimal algorithms that have been proposed earlier cannot be implemented in a stand-alone sensor as they require an external control input (and possibly an external power source). Hence new algorithms have been developed and implemented in this paper.

C. Hardware

The proposed sensor, as shown in figure 1(a), consists of a structure made up of three beams. Beam #1 is 6ft long (1.8 m) and is called the “main beam”. The sensor will be embedded in the road such that automobiles pass directly over this main beam. The load acting on the main beam is transmitted to two “support beams” located at either ends. The support beams have a gross length of 10” (254 mm) (with an effective length of 8” or 200 mm), and the ends of each support beam are rigidly fixed. A total of eight piezo elements (four piezos for each of the support beams) are bonded to the support beams. The location of the piezos were chosen as close to the ends of the support beam as possible. The piezos are connected electrically in parallel. ANSYS simulations reveal that the average of the strain over the area of all the piezos depends only on the total load acting on the main beam and is independent of the lateral location of the load on the main beam. Hence this configuration was chosen in order to make the average voltage developed by the piezos independent of the location of the load. It should be further noted that the speed of the passing vehicle can be measured by measuring the time difference in the loading between two consecutive sensors placed a short longitudinal distance apart.

Regarding installation of the sensor in the road, the sensor can be placed in a slot made in the road. The slot should be made bigger than the size of the sensor and a flexible sealant used in the gap around the sensor. This will ensure that expansion and contraction of the sensor due to temperature variations can be accommodated. This accommodation is necessary in order to prevent mechanical failure and also to prevent unloaded mechanical displacement of the sensor beam in the absence of vehicle loads on it. A photograph of the sensor is shown in figure 1(b).

D. Controller constraint

In this paper, we develop a controller to extract energy harvested from short duration vibration loads similar to impact loading. The control systems that enables Vibration Energy Harvesting (VEH) itself needs to be completely powered from the energy that is harvested. The design has been restricted to controllers that can be implemented using simple onboard analog electronics.

III. SYSTEM MODEL FOR CONTROL

A. Mechanical sub-system

The VEH sensor consists of a mechanical sub-system and an electrical sub-system. When the piezo is open (the terminals are unconnected), the mechanical sub-system is simply modeled as a vibrating beam structure. For a simple beam structure in vibration, the various modes of vibration can be calculated using equation (1). More complicated structures require a FEM model solution.

\[
\rho A \frac{\partial^2 v(x,t)}{\partial t^2} + b \frac{\partial v(x,t)}{\partial t} + EI \frac{\partial^2 v(x,t)}{\partial x^2} = 0
\]

where

- \( \rho A \) is the linear density of the beam
- \( b \) is the damping in the beam
- \( EI \) is the stiffness of the beam

![Figure 1: Sensor schematic and photograph](image)

![Figure 2: Bode magnitude plot of impulse response of the sensor](image)
It is possible to construct a reduced order model of the system, using only the most dominant vibration mode. To determine this dominant frequency, the impact response of the sensor was experimentally obtained. Using the impact response data shown here in figure 2, the frequency of the first mode of vibration is determined to be 38Hz.

For purposes of developing the control system, this dominant frequency is used to model the mechanical system as a spring mass damper system

\[ \ddot{u} + 2 \zeta \omega_n \dot{u} + \omega_n^2 u = \frac{F}{m} \]  

(2)

It must be noted that the system model deviates from equation (2) when the piezo sources current to any connected electronics, and voltage difference across the piezo decreases. This change in piezo voltage will induce a proportional force, \( F_{\text{elec}} \) on the mechanical system and hence equation (2) is modified to

\[ \ddot{u} + 2 \zeta \omega_n \dot{u} + \omega_n^2 u = \frac{F}{m} + \frac{F_{\text{elec}}}{m} \]  

(3)

where \( F_{\text{elec}} \) is the force applied on the mechanical system by the piezo and is proportional the voltage across the capacitor of the piezo \( V_{\text{piezo}} \).

The strain, \( \varepsilon \), at the location of the piezo can be represented in terms of a length scale \( L \) as

\[ \varepsilon = \frac{u}{L} \]  

(4)

From equations (3) and (4), the mechanical sub-system dynamics can be rewritten in terms of strain as

\[ \ddot{\varepsilon} + 2 \zeta \omega_n \dot{\varepsilon} + \omega_n^2 \varepsilon = \eta \omega_n \frac{F - \eta_{\text{elec}} \omega_n^2 V_{\text{piezo}}}{m} \]  

(5)

where \( \eta \) is the amount of static strain produced for a unit applied force and \( \eta_{\text{elec}} \) is the amount of static strain produced for a unit change in piezo voltage.

In order to determine the value of \( \eta \), let us consider a unit static load acting at the center of the main beam of the traffic sensor. This load would results in an average load of \( 1/2 \) units being transmitted to each support beam. It can be inferred from Ketchum et. al. ([10]) that the resulting bending moment at the location of the piezo \( (x_n) \) is given by

\[ M_{\text{piezo}} = \left( \frac{L'}{8} - \frac{x_n^2}{2} \right) \left( \frac{1}{2} \right) \]  

(6)

For the support beam, \( L' = 0.2m \), \( x_n^2 = 0.01875m \), \( b' = 0.0025m \), \( h' = 0.00625m \) and \( E' = 200GPa \). Hence \( L' = 5.0863 \times 10^{-9} \) mm and

\[ \varepsilon = 2.4 \times 10^{-7} \]  

(7)

Thus

\[ \eta = 2.4 \times 10^{-7} \]  

(8)

The value of \( \eta_{\text{elec}} \) is determined in section III.C. A quick brief analysis is presented here on parameter variations and how these variations would affect beam displacement. The ratio of beam vibration amplitude \( X \) to load magnitude \( F \) is given as a function of frequency \( \omega \) by

\[ \frac{X}{F} = \frac{1}{\sqrt{\frac{1}{\omega_n^4} + \frac{2 \zeta \omega}{\omega_n^2}}} \]  

(9)

Since the load in this case is approximately that of an impulse, the vibration of the beam will largely occur at the resonant frequency \( \omega = \omega_n \). With \( \omega = \omega_n \), the ratio of vibration amplitude to force becomes

\[ \frac{X}{F} = \frac{1}{2 \zeta} = \frac{1}{2 \zeta \omega_n} \]  

(10)

Hence, it can be seen that if the resonant frequency \( \omega_n \) changes by 20\%, the vibration amplitude can decrease to 0.7 times its nominal value or increase to 1.56 times the nominal value. Likewise, if the damping ratio changes by 20\%, the vibration amplitude can decrease to 0.85 times the nominal amplitude or increase to 1.25 times its nominal value. In the design of the energy harvesting system, it is important to ensure that the power needed by the wireless transceiver and associated electronics is a factor of 2 less than the power provided by the nominal system design. This would ensure that the system would continue to function reliably even if there were changes in mechanical parameters in the sensor.

B. Electrical model

At low frequencies, the piezo electric material which is the critical part of the VEH system, is modeled as a voltage source in series with a capacitance using equations (11-13) ([5], [20]). A more sophisticated model can be found in Weinbergs et. al.([32]).

\[ V_{\text{piezo}} = V_{\text{strain}} - V_{\text{piezo}} \]  

(11)

\[ V_{\text{piezo}} = \frac{1}{C_{\text{piezo}}} \int i \, dt \]  

(12)

\[ V_{\text{strain}} = \varphi \varepsilon \]  

(13)

The piezo electric material consists of T1077-A4E-602 piezo sheets that were purchased from Piezo Systems, Inc. The piezo sheets were cut to rectangular strips of size 37.5mm \times 25mm. The piezos were bonded to the sensor such that they are subject to strain in the “1” direction. Hence the “31” parameters of the piezo are used in the calculation. For A4E piezo, using the modulus of elasticity at constant electric field \( E \) and thickness \( \delta \), the open circuit voltage per unit strain is

\[ \varphi = 1.4623 \times 10^7 \sqrt{V} \]  

(14)

The equivalent capacitance was estimated to be

\[ C_{\text{piezo}} = 4 \times 10^{-9} \]  

\[ \mu F \]

In order to calculate the overall electrical dynamics, other components making up the electrical subsystem need to be considered. The electrical system shown in figure 3 also contains a bridge rectifier (denoted by “Diode Bridge”) connected to the piezo element. The diode bridge is connected to a storage capacitor \( C_{r} \), which in turn is connected to transmitter via a load switch (SWL). Each diode making up the bridge is modeled by a piecewise linear model ([24])
The tire passing over in (5) that were considered, bonded to the mechanical system, this decrease in piezo voltage translates to a steady state stress

\[ \sigma_C^{\text{piezo}} = E^{\text{piezo}} d_{31} V_{\text{piezo}} / \delta \] (17)

The force developed by the piezo as a result of this stress is given by

\[ F_{\text{Cpiezo}} = -b^{\prime} \delta \sigma_C^{\text{piezo}} \] (18)

The pair of piezos that were considered, bonded to the top and bottom of the support beam are separated by the height of the support beam \( h^{\prime} \). The moment developed by them is

\[ M_{\text{Cpiezo}} = -b^{\prime} \delta h^{\prime} \varepsilon^{\text{piezo}} \] (19)

This moment acting on the mechanical system would result in a static strain

\[ \varepsilon = \frac{M_{\text{Cpiezo}}}{E I^{\prime}} \left( h^{\prime}/2 \right) \] (20)

Noting that \( I^{\prime} = (1/12) b^{\prime} \left( h^{\prime} \right)^3 \) we get

\[ \varepsilon_{\text{elec}} = \frac{6 E^{\text{piezo}}}{E^{\prime} h^{\prime} - d_{31}} V_{\text{Cpiezo}} \] (21)

From equation (5) we can deduce that for a static

\[ V_{\text{piezo}} \]

\[ \varepsilon_{\text{elec}} = \eta_{\text{elec}} V_{\text{Cpiezo}} \] (22)

Hence

\[ \eta_{\text{elec}} = \frac{6 E^{\text{piezo}}}{E^{\prime} h^{\prime} - d_{31}} \] (23)

Thus the coupling coefficient is

\[ \eta_{\text{elec}} = 6.0192 \times 10^3 \text{ V}^{-1} \] (24)

IV. PROPOSED CONTROL SYSTEMS

Of the total energy generated in the piezo, only the fraction transferred to the storage capacitor \( C_s \) is available to drive the electrical load. The peak voltage across the storage capacitor, \( V_{\text{Cs}}^{\text{max}} \), is a measure of this energy. The peak power at the load, given by \( \left( V_{\text{Cs}}^{\text{max}} \right)^2 / R_L \), is also a function of \( V_{\text{Cs}}^{\text{max}} \). Hence in the following subsections, the available storage capacitor voltage, \( V_{\text{Cs}}^{\text{max}} \), is determined for each of three different control algorithms.

For the purpose of simulation, we consider the sensor model with the parameter values calculated in section III. The input load resulting from the front and the rear tires, is modeled as two short duration pulses with a magnitude of 3937.5 N as shown in figure 4. The tire passing over the sensor would damp out the mechanical vibrations in the sensor. Hence the impulse response in figure 2 could not be directly used to find the damping ratio in section III. Instead a value of \( \zeta = 0.7 \) was found to be appropriate for the simulations. The components making up the electrical system are modeled by an equivalent 1\( \Omega \) load resistor.

![Diagram of energy harvesting circuit](70x404 to 109x408)

**Figure 3: Energy harvesting circuit**

C. Electro-mechanical coupling

The VEH system consists of piezo electric elements bonded to the support beams as shown in figure 1. The piezos are bonded to the top and bottom surfaces of the support beam. When a vehicle passes over the sensor, the piezo experiences a strain from the loading and thus generates a voltage. When the piezo sources current to the circuit, \( V_{\text{Cpiezo}} \) increases and \( V_{\text{piezo}} \) decreases. Since the piezo is bonded to the mechanical system, this decrease in piezo voltage translates to a steady state stress

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For the purpose of simulation, we consider the sensor model with the parameter values calculated in section III. The input load resulting from the front and the rear tires, is modeled as two short duration pulses with a magnitude of 3937.5 N as shown in figure 4. The tire passing over the sensor would damp out the mechanical vibrations in the sensor. Hence the impulse response in figure 2 could not be directly used to find the damping ratio in section III. Instead a value of \( \zeta = 0.7 \) was found to be appropriate for the simulations. The components making up the electrical system are modeled by an equivalent 1\( \Omega \) load resistor.

![Diagram of force input used for simulation](71x395 to 108x401)

**Figure 4: Force Input used for simulation**

A. Fixed Threshold Switching

This algorithm is an adaptation of traditional algorithms used for harvesting energy from sustained oscillations. In this algorithm, the load is connected to \( C_s \), by setting the control input to logic high (1) when the
value of $V_{Cs}$ crosses a predetermined high threshold $V_{\text{high}}$. The control is turned off (0), if $V_{Cs}$ falls below a low threshold $V_{\text{low}}$. The control signal to SW$_L$ can be given by the control law state transition diagram shown in figure 5.

$$\begin{align*}
V_{control} &= 0, & V_{Cs} &> V_{\text{high}} \\
V_{control} &= 1, & V_{Cs} &\leq V_{\text{high}} \\
\end{align*}$$

Figure 5: State transition diagram for “Fixed Threshold Switching”

Once the load switch SW$_L$ is closed, the voltage across the storage capacitor $V_{Cs}$ does not rise any further. The maximum value $V_{\text{max}}^{Cs}$ is equal to the on-threshold $V_{\text{high}}$. Thus

$$V_{\text{max}}^{Cs} = \begin{cases} \min \left( \frac{C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \times \left[ \left| V_{\text{max}}^{\text{strain}} \right| - 2V_d \right], V_{\text{high}} \right) \\ 0 \quad \text{otherwise} \end{cases}$$

For a sufficiently large $V_{\text{max}}^{\text{strain}}$,

$$V_{\text{max}}^{Cs} = V_{\text{high}}$$

The fixed threshold switching is the simplest algorithm, and would serve as a baseline for evaluating the performance of other control algorithms. Simulation results obtained using this baseline control law are shown below. The voltage in the storage capacitor for $V_{\text{high}} = 2.75V$ and $V_{\text{low}} = 1.5V$ is seen in figure 6. Figure 7 shows the current through the 1K (1 Kohm) load resistance and figure 8 shows the cumulative energy transferred to the resistor. The peak power is 7.56 mW and the total energy transferred to the load is 30 µJ.

Figure 6: Voltage Output for “Fixed Threshold switching” algorithm

Figure 7: Load current for 1K load with “Fixed Threshold switching” algorithm

Figure 8: Cumulative energy transferred to a 1K Load with “Fixed Threshold switching algorithm”

For benchmark numbers on power required for wireless transmission, the Linx transceiver TXM-418-LC requires 2.5 mW while the CC2500 transmitter from Texas Instruments requires 300 micro-watts for its operation. Hence both of these transceivers can be reliably powered from the energy harvested in this traffic sensor.

**B. Max Voltage Switching**

In this algorithm, the load is connected to the storage capacitor $C_s$, when the voltage $V_{Cs}$ reaches a maximum value. The control SW$_L$ is turned off, if $V_{Cs}$ falls below the off-threshold $V_{\text{low}}$. The control law can be given by the state transition diagram shown in figure 9.

$$\begin{align*}
V_{Cs} &= \text{max} \quad \text{im}um \\
|V_{Cs}| &< V_{\text{low}} \\
V_{Cs} &< V_{\text{low}} \\
V_{Cs} &\geq V_{\text{low}} \\
\end{align*}$$

Figure 9: State transition diagram for “Max Voltage switching”

The occurrence of maximum can be determined using analog electronics. For instance, the max-detector can be
realized using a high pass RC filter given by equation (27). A maximum is declared when the output of this filter falls below a threshold. The value of this threshold is small and determines how close to zero the derivative must become for the voltage to be recognized as maximum.

\[
V_{\text{max}} = \frac{RC_s}{RC_s + 1} V_{Cs}
\]  

(27)

As shown in Appendix A, if the displacement of the beam has only one extremum value \( u_{\text{max}} \) (with a corresponding \( V_{\text{strain}} = V_{\text{strain}}^{\text{max}} \)), then \( V_{\text{CS}}^{\text{max}} \) can be calculated using

\[
V_{\text{CS}}^{\text{max}} = \begin{cases} \left( \frac{C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \right) \times \left( V_{\text{strain}}^{\text{max}} - 2V_d \right) & \text{if } |V_{\text{strain}}^{\text{max}}| > 2V_d \\ 0 & \text{otherwise} \end{cases}
\]  

(28a)

\[
V_{\text{Cpiezo}}^{\text{max}} = \begin{cases} \left( \frac{C_s}{C_s + C_{\text{piezo}}} \right) \times \left( V_{\text{strain}}^{\text{max}} - 2V_d \right) & \text{if } |V_{\text{strain}}^{\text{max}}| > 2V_d \\ 0 & \text{otherwise} \end{cases}
\]  

(28b)

Thus for a sufficiently large strain voltage, the difference between \( V_{\text{CS}}^{\text{max}} \) and \( 2V_d \) is distributed between \( C_s \) and \( C_{\text{piezo}} \) in the inverse ratio of their capacitance and

\[
V_{\text{CS}}^{\text{max}} = \frac{C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \times \left( V_{\text{strain}}^{\text{max}} - 2V_d \right)
\]  

(29)

Figure 10 shows simulation results for the max voltage algorithm.

Figure 10: Voltage Output for “Max Voltage switching algorithm”

Since we have apriori knowledge about the nature of the loading, it is possible to modify the switch \( \text{SW}_1 \) to turn on only at the end of the second pulse, and not turn on after the first pulse. One way of achieving this would involve checking for a maximum larger than a low threshold. If \( V_{Cs}(0) = V_{\text{max}}^{i} \) and \( V_{\text{piezo}}(0) = V_{\text{piezo}}^{i} \), equation (28) is modified to

\[
V_{\text{effective}} = \begin{cases} \left( \frac{C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \right) \times V_{\text{effective}} + V_{Cs}^{i} & \text{if } V_{\text{effective}} > 0 \\ \left( \frac{C_s}{C_s + C_{\text{piezo}}} \right) \times V_{\text{effective}} + V_{\text{piezo}}^{i} & \text{otherwise} \end{cases}
\]  

(30a)

\[
V_{Cs}^{\text{max}} = \begin{cases} \left( \frac{C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \right) \times V_{\text{Effective}} + V_{Cs}^{i} & \text{if } V_{\text{Effective}} > 0 \\ V_{\text{piezo}}^{i} & \text{otherwise} \end{cases}
\]  

(30b)

\[
V_{\text{Cpiezo}}^{\text{max}} = \begin{cases} \left( \frac{C_s}{C_s + C_{\text{piezo}}} \right) \times V_{\text{Effective}} + V_{\text{piezo}}^{i} & \text{otherwise} \end{cases}
\]  

(30c)

For the particular load acting on the sensor (consisting of four extremums), \( V_{\text{CS}}^{\text{max}} \) is approximately four times the value of \( V_{\text{max}}^{\text{CS}} \) obtained from just the first pulse. From figure 11, \( V_{\text{CS}}^{\text{max}} \) is 3.9V. Figure 12 shows the current through the 1K (1 Kohm) load resistance and figure 13 shows the cumulative energy transferred to the resistor.

Figure 11: Voltage Output for modified “Max Voltage switching algorithm”

Figure 12: Load current for 1K load with modified “Max Voltage switching algorithm”

Figure 13: Cumulative energy transferred to the load resistance
Cumulative energy transferred to a 1K Load with modified “Max Voltage switching algorithm”

Since the storage capacitor is allowed to charge to a higher voltage, this algorithm will deliver a larger peak power in comparison to the “Fixed Threshold switching”. Hence this algorithm is always more efficient at harvesting vibration energy than the simple fixed threshold algorithm described in the previous subsection. Indeed, the value of peak power (15.4 mW) and the amount of total energy transferred to the load (70 µJ) are larger than those that were obtained for the “Fixed Threshold Switching” algorithm.

C. Switched Inductor

This section proposes a third algorithm that would further enhance $V_{max}^{CS}$. This algorithm uses the circuit shown in figure 16. The new circuit uses an inductor ($L$) and an additional switch $SW_p$ in addition to the components shown in figure 3. The voltage drop across $L$ is given by $V_L \cdot SW_p$ is turned on when $V_{piezo}^{\max}$ reaches a maximum and $SW_L$ is turned on when $V_{CS}$ reaches a maximum. The switches $SW_p$ and $SW_L$ are turned off when the respective voltages $|V_{piezo} + V_L|$ and $V_{CS}$ drops below an off-threshold $V_{low}$. As discussed in the previous subsection, the occurrence of maximum can be determined using analog electronics. The control law for $SW_p$ is given by the state transition diagram shown in figure 14 and the control law for $SW_L$ is given by the state transition diagram state transition diagram shown in figure 15.

If switch $SW_p$ is closed at first local extremum of the displacement of the beam, the voltages across capacitors $C_s$ and $C_{piezo}$ at the end of first half-LC oscillation ($V_{CS}^1$ and $V_{piezo}^1$ respectively) are given by

$$V_{CS}^1 = \begin{cases} \left( \frac{\chi C_{piezo}}{C_s + C_{piezo}} \right) \times \left| V_{max}^{\strain} - 2V_d \right| & \text{if } \left| V_{max}^{\strain} \right| > 2V_d \\ 0 & \text{otherwise} \end{cases}$$

$$V_{piezo}^1 = \begin{cases} \left( \frac{\chi C_s}{C_s + C_{piezo}} \right) \times \left| V_{max}^{\strain} - 2V_d \right| \times \text{sign}(V_{max}^{\strain}) & \text{if } \left| V_{max}^{\strain} \right| > 2V_d \\ 0 & \text{otherwise} \end{cases}$$

where

$$\chi = 1 + \exp\left( -\pi \frac{\zeta}{\sqrt{1 - \zeta^2}} \right), \quad 1 \leq \chi \leq 2$$

$$\zeta = \frac{1}{2} \frac{R_{\text{total}}}{\sqrt{L}}$$

$$C = \frac{C_s C_{piezo} \left( \frac{1}{C_s} + \frac{1}{C_{piezo}} \right)}{C_p + C_s}$$

This equation has been derived in appendix B. Thus when $\left| V_{max}^{\strain} \right| > 2V_d$

$$V_{CS}^{\max} = \left( \frac{\chi C_{piezo}}{C_s + C_{piezo}} \right) \times \left| V_{max}^{\strain} - 2V_d \right|$$

By comparing equations (33) with (29), we notice that $V_{CS}^{\max}$ is greater by a factor of at least $\chi$, this factor

Figure 13: Cumulative energy transferred to a 1K Load with modified “Max Voltage switching algorithm”

Figure 14: State transition diagram for $SW_p$ (“Switched Inductor”)
being equal to the peak overshoot of the LCR circuit. The “Switched Inductor” algorithm yields a higher \( V_{\text{max}}^{\text{Cs}} \) due to the presence of the inductor. In the absence of the diode bridge rectifier, the second order dynamics of the LCR circuit will exhibit an oscillatory behavior for an extremely small time period and the dynamics would eventually converge to its steady state value. The bridge rectifier in the circuit would however clamp \( V_{\text{Cs}} \) to the peak overshoot voltage of the transient response, resulting in higher available voltage for the storage capacitor.

Using \( R_{\text{total}} = 327.2 \) (this is the estimated total resistance in the circuit) in equations (31), we get \( V_{\text{Cs}} \approx 1.25 \) and \( V_{\text{piezo}} \approx 187.42 \). Now \( V_{\text{piezo}}^{\text{max}} - V_{\text{strain}}^{\text{max}} \) \( > V_{\text{Cs}} + 2V_{d} \). Hence the piezo would drive the circuit in the reverse direction. \( \Delta V_{\text{Cs}} = 0.27 \) and \( \Delta V_{\text{piezo}} = 40.53 \). Hence \( V_{\text{Cs}}^{\text{max}} \approx V_{\text{CS}}^{2} = 1.52 \). The estimated value for \( V_{\text{Cs}} \) is seen to be in close agreement with the simulation result from figure 17.

![Figure 17: Voltage Output for “Switched Inductor” algorithm](image)

As mentioned in the earlier section, owing to apriori knowledge about the nature of the loading, it is possible to modify the SWL to turn on only at the end of the second load pulse, instead of turning on after the first pulse. This allows greater voltage to build up across the storage capacitor. For the particular load acting on the sensor (consisting of four extremums), \( V_{\text{CS}}^{\text{max}} \) is approximately four times the \( V_{\text{CS}}^{\text{max}} \) for first extremum (6.04V). When compared to the modified “Max voltage switch” controller, it is seen from figure 11 and figure 18 that the available voltage has increased by a factor of over 1.5. Subsequently from figure 20, the peak power is obtained to be 36.3 mW (an increase of 136%) and the total energy transferred to the load equals 188 \( \mu \)J (an increase of 169%).

![Figure 18: Voltage Output for modified “Switched Inductor” algorithm](image)

![Figure 19: Load current for 1K load with modified “Switched Inductor” algorithm](image)

![Figure 20: Cumulative energy transferred to a 1K Load with modified “Switched Inductor”](image)

D. Comparison of the three algorithms for first extremum

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Approximate ( V_{\text{max}}^{\text{Cs}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Threshold Switching</td>
<td>( V_{\text{high}} )</td>
</tr>
</tbody>
</table>
modified Max Voltage Switching \[ \left( \frac{4 \times C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \right) \times \left\lfloor \frac{V_{\text{max}} - 2V_d}{\Delta V_{\text{strain}}} \right\rfloor \]
modified Switched Inductor \[ \left( \frac{4 \times C_{\text{piezo}}}{C_s + C_{\text{piezo}}} \right) \times \left\lfloor \frac{V_{\text{max}} - 2V_d}{\Delta V_{\text{strain}}} \right\rfloor \]

Table 1: Comparison of maximum storage capacitor voltage for the 3 algorithms

It must be noted that for the “Fixed Threshold Switching” algorithm to work reliably, \( V_{\text{high}} \) cannot be arbitrarily large. \( V_{\text{high}} \) must necessarily be chosen a volt or two lower than the lowest \( V_{\text{CS}}^{\text{max}} \) that can be expected corresponding to the set of all possible \( \{\Delta V_{\text{strain}}\} \). Hence \( V_{\text{high}} \) for “Fixed Threshold Switching” algorithm is necessarily smaller than \( V_{\text{CS}}^{\text{max}} \) for the modified Max Voltage Switching. From equation (32a) it is clear that \( \chi > 1 \). Thus, \( V_{\text{CS}}^{\text{max}} \) for “Fixed Threshold Switching” < \( V_{\text{CS}}^{\text{max}} \) for modified “Max Voltage Switching” < \( V_{\text{CS}}^{\text{max}} \) for modified “Switched Inductor”.

V. EXPERIMENTAL RESULTS

The three algorithms presented in section IV were implemented with a self-contained electronic circuit. The switches \( SW_L \) and \( SW_P \) were implemented using MOSFETs. The extremely small amount of power required by the control system was derived entirely from the charge energy stored in the capacitor. Sets of experiments were carried out with each of the three circuits connected to the piezo. Each experiment consisted of driving a compact car over the sensor at 20 kmph. This resulted in separate loading from the two axles, first by the front tires and then by the rear tires. Figure 21 shows a schematic of the test setup.

The speed of 20 kmph (12 mph) was chosen for the tests since it was the lowest speed with which the car could be made to move consistently over the sensor in multiple tests. Higher speeds will result in higher loads on the sensor and are likely to increase the energy harvested.

Figure 21: "Schematic of test setup" (figure not drawn to scale)

Figure 22 show the results from the testing of the “Fixed threshold algorithm”. The on-threshold for the algorithm was chosen at 2.75 V so that the sensor would detect light vehicle such as motorcycles. The MOSFET used in the switching circuit constrains the off-threshold to 1.75V. As a result, \( SW_L \) turns on when the capacitor voltage \( V_{CS} \) reaches 2.75V and turns off when \( V_{CS} \) falls below 1.75V.

The electronic circuits were modified so that \( SW_L \) turns on when \( V_{CS} \) reaches a maximum. In order to detect a global maximum, and to collect energy from both axles, \( SW_L \) was modified to turn on when \( V_{CS} \) does not increase for a period of 100ms. The off-threshold was once again chosen to be 1.75V. Figure 23 show the results from “Max Switching algorithm”. It is seen that the two axles would generate a combined voltage of 4V. This agrees well with the simulations in section IV.B. Figure 24 show the results from the “Switched Inductor Algorithm” experiments and the capacitor voltage is found to be 5.9V. This agrees well with the simulations in section IV.C.

Figure 22: “Fixed Threshold algorithm” with a high threshold of 2.75V

Figure 23: "Max Switching Algorithm"

From both the experimental data and the simulation results, it can be seen that the operating range for the capacitor voltage is expected to be between 0V and 10V and the load current is expected to be between 0 and 10 mA.
It is apparent that if SW_p is controlled as prescribed, the “Switched Inductor” offers the best energy harvesting performance.

VI. CONCLUSIONS

This paper developed a battery-less wireless traffic sensor that operates by harvesting vibration energy from the passing of a vehicle over the sensor. The sensor is significantly smaller than an inductive loop detector and has other advantages of easier installation, low cost, no wiring and ability to do vehicle classification. Three different control algorithms were developed to improve the efficiency of energy harvesting from the vibrations. Of the three algorithms, the switched inductor algorithm was shown to be the best at maximizing the harvested energy. Experimental results showed that the developed traffic sensor was able to harvest adequate energy from the passing of every vehicle over the sensor to enable the powering of electronics for wireless transmission.

Appendix (Calculation of V_{C_{piezo}}^{max})

A. CALCULATION OF V_{C_{piezo}}^{max} FOR MAX VOLTAGE SWITCHING ALGORITHM

The V_{C_{piezo}}^{max} for Max Voltage Switching algorithm (equation (28)) can be derived as follows. It is clear that $V_{strain} = V_{piezo} + V_{C_{piezo}}$ (refer figure 3). Thus if $|V_{strain} - V_{C_{piezo}}| - V_{C_{piezo}} > 2V_d$, the bridge circuit rectifies the piezo current and charges the storage capacitor. When $|V_{strain} - V_{C_{piezo}}| - V_{C_{piezo}} \leq 2V_d$, the diodes block the flow of current thus preventing storage capacitor from discharging. If $i_p$ does not change signs and $|V_{strain} - V_{C_{piezo}}| - V_{C_{piezo}} > 2V_d$, the effective voltage driving the current through the resistive element in the circuit is given by

$$V_{effective}(t) = \text{sign}[V_{strain}(t) - V_{C_{piezo}}(t)] \times \max(|V_{strain}(t) - V_{C_{piezo}}(t)| - V_{C_{piezo}}(t) - 2V_d, 0)$$

In modeling overall dynamics, the first order nonlinear electrical dynamic equations (11-16) are dominated by the much slower dynamics of the mechanical system. The system exhibits a two time scale property and the faster electrical dynamics needs to be modeled by its quasi-steady state value ((11), [14]) which corresponds to $V_{effective}(t) = 0$. Equivalently

$$|V_{strain}(t) - V_{C_{piezo}}(t)| = V_{C_{piezo}}(t) - 2V_d$$

If $V_{C_{piezo}}(t)$ does not change signs

$$\frac{|V_{C_{piezo}}(t)|}{V_{C_{piezo}}} = \begin{cases} \frac{V_{C_{piezo}}}{V_{C_{piezo}}} \times |V_{strain} - 2V_d| & \text{if } |V_{strain}| > 2V_d \\ 0 & \text{otherwise} \end{cases}$$

Thus

$$V_{C_{piezo}}(t) = \begin{cases} \frac{C_{C_{piezo}}}{C_{C_{piezo}} + C_{piezo}} \times |V_{strain} - 2V_d| & \text{if } |V_{strain}| > 2V_d \\ 0 & \text{otherwise} \end{cases}$$

(37)

B. CALCULATION OF V_{C_{piezo}}^{max} FOR SWITCHED INDUCTOR ALGORITHM

The $V_{C_{piezo}}^{max}$ for Switched Inductor algorithm (equation (31)) can be derived as follows. When SW_p is closed in the LCR circuit shown figure 16, the effective voltage driving the resistive and inductive components of the circuit is given by

$$V_{effective}(t) = \text{sign}[V_{strain}(t) - V_{C_{piezo}}(t)] \times \max(|V_{strain}(t) - V_{C_{piezo}}(t)| - V_{C_{piezo}}(t) - 2V_d, 0)$$

In the absence of SW_p, the overall dynamics is dominated by the mechanical system and $V_{C_{piezo}}^{max}$ would be given by equation (28). There would be no gain in $V_C$. If SW_p is closed at some $V_{strain} = V_{C_{piezo}}^{max}$, it would be result in a step input to the electrical circuit. If $|V_{strain}(t)| > 2V_d$, the diode bridge will begin to conduct when SW_p is closed.

$$L \frac{di}{dt} + R_{total}i_p = V_{effective}(t)$$

(39)

When $i_p$ is unidirectional, the electrical dynamics can be written in terms of $Q = \int i_p dt$ and $|Q| = \int |i_p| dt$.

Hence

$$V_{C_{piezo}}^{max} = \frac{1}{C_{piezo}} \int i_p dt = \frac{Q}{C_{piezo}}$$

(40)
\[ V_{Cs} = \frac{1}{C_{piezo}} \int |v| dt = \frac{|q|}{C_s} \]  \hfill (41)

Now \( \text{sign}(V_{strain}) = \text{sign}(Q) \). When \( C_s \) and \( C_{piezo} \) are initially not charged,

\[ V_{\text{effective}}(t) = \frac{V_{\text{max, strain}}}{2} + 2V_d \times \text{sign}(V_{\text{max, strain}}) \]  \hfill (42)

The piece-wise linear dynamics of the LCR system can be written as

\[ L \ddot{Q} + R_{\text{total}} \dot{Q} + \frac{1}{C_{piezo}} \dot{q} + \frac{1}{C_s} q = V_{\text{max, strain}} - 2V_d \times \text{sign}(V_{\text{max, strain}}) \]  \hfill (43)

Now \( i_p = \dot{Q} \) and \( i_p \) is unidirectional up to the first maximum of \( Q \) at the end of the first half-oscillation. Since equation (43) is valid when \( i_p \) is unidirectional, it can be used to determine this first maximum \( Q_{\text{max}} \). For this second order system,

\[ Q_{\text{max}} = C \times \chi \times V_{0,\text{strain}} \]  \hfill (44)

Hence

\[ V_{\text{C}_{\text{piezo}}}^\text{max} = \begin{cases} \frac{2C_{piezo}}{C_s + C_{piezo}} \times \left( V_{\text{max, strain}} - 2V_d \right) & \text{if } \left| V_{\text{max, strain}} \right| > 2V_d \\ 0 & \text{otherwise} \end{cases} \]  \hfill (45)

\[ V_{\text{C}_{\text{piezo}}}^\text{max} = \begin{cases} \frac{2C_{piezo}}{C_s + C_{piezo}} \times \left( V_{\text{max, strain}} - 2V_d \right) \times \text{sign}(V_{\text{max, strain}}) & \text{if } \left| V_{\text{max, strain}} \right| > 2V_d \\ 0 & \text{otherwise} \end{cases} \]  \hfill (46)

Note:

If the piezo is charged sufficiently in the reverse direction such that \( V_{\text{C}_{\text{piezo}}}^\text{max} - V_{\text{max, strain}} > V_{\text{Cs}} \), the system would continue to oscillate. When the piezo current flows in the reverse direction, the change in \( V_{\text{Cs}} \) and \( V_{\text{C}_{\text{piezo}}} \) at the end of the second half oscillation (\( dV_{\text{Cs}}^2 \) and \( dV_{\text{C}_{\text{piezo}}}^2 \)) can be obtained by replacing \( |V_{\text{max, strain}}| \) with \( |V_{\text{C}_{\text{piezo}}}^\text{max} - V_{\text{max, strain}}| \).

C. NOMENCLATURE

- \( \chi \) ratio of peak response to steady state response of the electrical RC circuit
- \( \delta \) thickness of piezo (0.191mm)
- \( \varepsilon \) strain in the piezo, from eq. (4)
- \( \eta \) constant associated with the mechanical system equaling \( 1/\omega_c \)
- \( \eta_{\text{dec}} \) constant associated with the mechanical system model arising due to electro-mechanical coupling , \( (6E^\text{piezo} d_31)^2 / (E_s h^s) \), \( \eta_{\text{dec}} = 6.0192 \times 10^8 \)
- \( \omega_n \) open-circuit natural frequency of the mechanical system (38Hz)
- \( \zeta \) damping ratio associated with the vibrating mechanical system (0.7)
- \( h^s \) width of the support beam (25mm)
- \( d_{31} \) also the width of the piezo
- \( g_{31} \) strain produced per unit applied Electric Field (-190 \times 10^{-12} \text{ m/V} )
- \( g_{1} \) open circuit electric field produced for an unit applied stress (\( g_{31} = -11.6 \times 10^3 \text{ Vm/N} \) )
- \( h^s \) piezo constant defined as the open circuit voltage developed per unit applied strain (\( g = g_{31} \Delta E_{\text{piezo}} \))
- \( \delta \) thickness of the support beam (6.25mm)
- \( i_L \) load current through \( R_L \)
- \( i_p \) current through the piezo
- \( l \) length scale associated with the mechanical system
- \( m \) mass associated with the mechanical system
- \( u \) displacement of the mechanical system
- \( u_{\text{mid}} \) the magnitude of displacement of the mechanical system at the point where the force is applied
- \( C_{\text{piezo}} \) capacitance of the piezo
- \( C_s \) storage capacitor
- \( F \) is the force applied on the mechanical system by the passing automobile
- \( E_{\text{piezo}} \) modulus of elasticity of the piezo at constant voltage (constant electric field) \( E_{\text{piezo}} = 6.6 \times 10^{10} \text{ N/m}^2 \)
- \( E^s \) elastic modulus of the main beam (200MPa)
- \( E^m \) elastic modulus of the support beam (200MPa)
- \( I^s \) area moment of inertia main beam (1.2957 \times 10^{-3} \text{ mm}^4 )
- \( I^m \) area moment of inertia support beam (5.0863 \times 10^{-10} \text{ mm}^4 )
- \( L \) value of inductance used in “Switched Inductor” algorithm (section C) (10mH)
- \( L^e \) effective length of the main beam (1.7125m)
- \( L^p \) effective length of the support beam (0.2m)
- \( R_{\text{total}} \) total resistance in the circuit due to switches and other components (not including \( R_p \)) (327 ohms)
- \( R_L \) load resistance (1000 ohms)
- \( R_d \) diode resistance
- \( \text{SW}_L \) load switch
- \( \text{SW}_P \) piezo switch
- \( V_{\text{Cs}} \) voltage across the storage capacitor \( C_s \)
- \( V_{\text{C}_{\text{piezo}}} \) maximum voltage across the storage capacitor
- \( V_{\text{C}_{\text{piezo}}} \) voltage across the piezo capacitor \( C_{\text{piezo}} \)
- \( V_d \) forward voltage drop across each diode (1.1 V for the diode used)

\(^1\) : The constant “c” is not available for the piezo material, hence an equivalent constant is used
on (high) threshold

V_{\text{high}}

off (low) threshold

V_{\text{low}}

voltage measured across the piezo

V_{\text{piezo}}

voltage open circuit voltage generated due to the strain

V_{\text{strain}}

maximum value of \( V_{\text{strain}} \) corresponding to the maximum value of the strain

\( V_{\text{max}} \)

extremum value of \( V_{\text{strain}} \) corresponding to the \( i \)th extremum of displacement \('u'\)

\( V_{\text{strain}}^{(i)} \)

VII. REFERENCES


Krishna Vijayaraghavan obtained his M.S degree from University Of Minnesota at Minneapolis in 2005 and his B.Tech degree from the Indian Institute of Technology at Madras in 2003. He is currently working towards his PhD. at the University of Minnesota. His research interests include fault-tolerant control, battery-less wireless sensors, signal processing and real-time software. Krishna’s awards include the ITS MN Graduate Student Award (2008) in recognition of outstanding research contributions to Intelligent Transportation Systems (ITS) technology and the Sivasailam Merit Prize for Best Individual B.Tech Project in Mechanical Engineering in IT Madras, (2003).
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