Optimal design of a composite leaf spring using genetic algorithms

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Abstract

A formulation and solution technique using genetic algorithms (GA) for design optimization of composite leaf springs is presented here. The suspension system in an automobile significantly affects the behaviour of vehicle, i.e. vibrational characteristics including ride comfort, directional stability, etc. Leaf springs are commonly used in the suspension system of automobiles and are subjected to millions of varying stress cycles leading to fatigue failure. If the unsprung weight (the weight, which is not supported by the suspension system) is reduced, then the fatigue stress induced in the leaf spring is also reduced. Leaf spring contributes for about 10–20% of unsprung weight. Hence, even a small amount of weight reduction in the leaf spring will lead to improvements in passenger comfort as well as reduction in vehicle cost. In this context, the replacement of steel by composite material along with an optimum design will be a good contribution in the process of weight reduction of leaf springs. Different methods are in use for design optimization, most of which use mathematical programming techniques. This paper presents an artificial genetics approach for the design optimization of composite leaf spring. On applying the GA, the optimum dimensions of a composite leaf spring have been obtained, which contributes towards achieving the minimum weight with adequate strength and stiffness. A reduction of 75.6% weight is achieved when a seven-leaf steel spring is replaced with a mono-leaf composite spring under identical conditions of design parameters and optimization. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Optimization; Genetic algorithms; Composite materials; Leaf spring; Automobiles; Weight reduction; Unsprung weight

1. Introduction

To meet the needs of natural resources conservation, automobile manufacturers are attempting to reduce the weight of vehicles in recent years. The interest in reducing the weight of automobile parts has necessitated the use of better material, design and manufacturing processes. The suspension leaf spring is one of the potential elements for weight reduction in automobiles as it leads to the reduction of unsprung weight of automobile. The elements whose weight is not transmitted to the suspension spring are called the unsprung elements of the automobile. These include wheel assembly, axles, and part of the weight of suspension spring and shock absorbers [1]. The leaf spring accounts for 10–20% of the unsprung weight [2]. The reduction of unsprung weight helps in achieving improved ride characteristics and increased fuel efficiency. The cost of materials constitutes nearly 60–70% of the vehicle cost and contributes to the better quality and performance of the vehicle. The introduction of fibre reinforced plastics (FRP) made it possible to reduce the weight of a machine element without any reduction of the load carrying capacity [5]. Because of FRP material’s high elastic strain energy storage capacity, and high strength-to-weight ratio compared with those of steel, multi-leaf steel springs are being replaced by mono-leaf FRP springs [2,3]. FRP springs also have excellent fatigue resistance and durability. Here, the weight reduction of the leaf spring is achieved not only by material replacement but also by design optimization.
Traditional optimization methods operate on a mathematical abstraction of the real design problem. These methods usually give only a local optimum. While traditional techniques are of little value here, new non-traditional techniques such as genetic algorithm (GA) give a global optimum. Genetic optimization is a process, which emulates the evolution of natural processes and as such, maintains a close link to the physical nature of the design rather than simply providing a mathematical representation. This paper presents an artificial genetic approach for design optimization leaf spring problems, including formulation of a fitness function applicable to the present study.

A GA proposed by Goldberg [8] based on natural genetics has been used in this work. In a previous study by the authors [6], a GA was applied for the design optimization of steel leaf springs. Although design optimization of steel leaf springs has been the subject for quite a few investigators [6,7], no paper has been reported (to the best of the knowledge of the authors) on composite leaf springs using the GA approach. However, there is some literature, which discusses the optimum design of structures using GAs [11–14]. A constrained optimization problem is transformed into an unconstrained problem using a different approach than the penalty function approach [9]. GAs are fairly new and are described in greater detail in the literature [8–10]. However, a brief description is included here for ease of understanding.

2. Details of genetic algorithms

GAs solve optimization problems, imitating the nature in the way it has been working for millions of years on the evolution of life. These are search procedures based on the mechanics of natural genetics and natural selection that can be used to obtain global and robust solutions for optimization problems. GA combines “survival of the fittest” among string structures with a structured, randomized information exchange. These together, form a search algorithm with the innovative flair of human search. A new set of artificial string is created using bits and pieces of the fittest of the old to form a new generation. Occasionally, a new bit is tried by replacing an existing one for good measure. The present study concentrates on a simple GA consisting of reproduction, cross-over and mutation operators. Although these operators look very simple at first sight, their combined action is responsible for the power of the GA. However, computer implementation involves only random number generation, string copying, partial string swapping and bit conversion. GAs differ from conventional optimization and search procedures in several fundamental ways. Goldberg [8] has summarized this as follows:

1. GAs work with a coding of solution set, not the solution itself.
2. GAs search from a population of solutions, not from a single solution.
3. GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge.
4. GAs use probabilistic transition rules, not deterministic rules.

In this study, the minimization of weight of the leaf spring is considered as the objective function. GA uses the objective function for its selection, cross-over and mutation operations, i.e. the solution with less weight will have higher probability of survival during each generation. GA does not require any other additional information such as derivatives of the objective function, etc. Fittest solutions are retained and unfit solutions are removed based on the probability of survival. In conventional optimization approach, all solutions are considered for the improvement of the objective function. Whereas, in GA, the fittest solutions are only considered for optimization.

3. Design of composite leaf spring

Flexural rigidity is an important parameter in the leaf spring design, and it should increase from two ends of the spring to its centre. This idea gives different types of design possibilities, namely, constant cross-section design, constant width with varying thickness design, and constant thickness with varying width design. The constant cross-section design is selected due to its capability for mass production, and to accommodate continuous reinforcement of fibres. The design of composite leaf spring aims at the replacement of seven-leaf steel spring of an automobile with a mono-leaf composite spring. The design requirements are taken to be identical to that of the steel leaf spring:

- design load, \( W = 4500 \text{ N} \),
- maximum allowable vertical deflection, \( \delta_{\text{max}} = 160 \text{ mm} \),
- distance between eyes in straight condition, \( L = 1220 \text{ mm} \),
- spring rate, \( K = 28–32 \text{ N/mm} \).

The composite leaf spring is designed to cater the above requirements based on the design procedures given in the literature [3].

3.1. Benefits of design optimization theory over conventional design theory

Whatever may be the geometric variation of the leaf spring, it is desirable that the leaf spring is designed to
have minimum weight. This should be compatible with
the other requirements of a particular suspension to
keep the vehicle weight to a minimum [4]. It is desirable
for a suspension to provide the required deflection to
enhance cushioning ability together with adequate ri-
gidity. Therefore, the common goal in designing a leaf
spring is to obtain the lightest spring under the given
functional and geometrical constraints (load, spring rate
and desired length). The conventional design method for
leaf spring is by “trial and error”. This process depends
on the designer’s intuition, experience and skill. He se-
lects the design parameters and checks whether these
satisfy the design constraints. If not, he changes these
parameters till the desired result is achieved, which is a
tedious exercise.

It is therefore a challenge for the designers to design
efficient and cost-effective systems. The ability to bring
the highest quality products to the market within the
shortest lead-time is becoming highly necessary. Scarcity
and need for efficiency in today’s competitive world has
forced designers to evince greater interest in economical
and better designs. The design optimization of the leaf
spring aims at minimizing the weight of leaf spring
subjected to certain constraints. In general, there will be
more than one acceptable design and the purpose of
design optimization is to choose the best one out of the
many alternatives available. An attempt has been made
to develop a powerful and efficient computer program
using c language to design a minimum weight leaf
spring. The application is restricted to the computer
aided optimum design of single stage seven-leaf steel
spring and mono-leaf composite spring. Here, the multi-
leaf steel spring has constant leaf thickness and leaf
width along the length. The mono-leaf composite spring
has varying thickness and width along the length, but
maintains a constant cross-sectional area.

4. Optimal problem formulation

The purpose of the formulation is to create a mathe-
matical model of the optimal design problem, which can
be solved using an optimization algorithm. Optimization
problem must be formulated as per the format of the
algorithm. The problem formulation of steel leaf spring
is reported in the article [6]. Hence, here, the problem
formulation is limited only to composite leaf spring.

4.1. Objective function

The objective is to minimize the weight of the leaf
spring with the prescribed strength and stiffness. The
objective function identified for the leaf spring problem
is given below:

\[ f(w) = \rho Lbt, \]  \hspace{1cm} (1)

where \( \rho \) is the material density, \( t \), the thickness at centre,
\( b \), the width at centre and \( L \), the length of the leaf spring.

The double tapered composite leaf spring is de-
signed based on the constant cross-section area. Hence,
centre thickness and centre width is considered for
optimization. The end thickness and end width can be
determined based on the taper ratio. The constant
cross-section area ensures that the fibres pass continu-
ously without any interruption along the length. This is
advantageous to the FRP structures. Moreover, higher
efficiency with low level of shear stress can be obtained
using this shape.

4.2. Design variables

A design problem usually involves many design par-
ameters, of which, some are highly sensitive. These
parameters are called design variables in the optimiza-
tion procedure. In the present problem, the following
variables are considered: (1) centre width, \( b \) and (2) centre thickness, \( t \).

The upper and lower bound values of design vari-
ables are given as follows:

\[ b_{\text{max}} = 50 \text{ mm} \quad \text{and} \quad b_{\text{min}} = 20 \text{ mm}, \]

\[ t_{\text{max}} = 50 \text{ mm} \quad \text{and} \quad t_{\text{min}} = 10 \text{ mm}. \]

4.3. Design parameters

Design parameters usually remain fixed in relation to
design variables. Here, the design parameters are length
of leaf spring, \( L \), design load, \( W \), material properties – (i)
density, \( \rho \), (ii) modulus of elasticity, \( E \) and (iii) maxi-
mum allowable stress, \( S_{\text{max}} \).

4.4. Design constraints

Constraints represent some functional relationships
between design variables and other design parameters,
which satisfy certain physical phenomenon and resource
limitations. In this problem, the constraints are the
bending stress, \( S_b \) and vertical deflection, \( d \).

\[ S_b = \frac{1.5WL}{bt^2}, \] \hspace{1cm} (2)

\[ d = \frac{WL^3}{4Ebt^3}, \] \hspace{1cm} (3)

\[ \text{FOS} = \frac{S_{\text{max}}}{S_b}. \] \hspace{1cm} (4)

When considering both static and fatigue behaviour of
composite leaf spring, the factor of safety (FOS) is taken
as 2.5. The upper and lower bound values of constraints are given as follows:

\[ S_{\text{max}} = 550 \text{ MPa}, \quad S_{\text{min}} = 400 \text{ MPa}, \]
\[ d_{\text{max}} = 160 \text{ mm}, \quad d_{\text{min}} = 120 \text{ mm}. \]

4.5. Fitness function

GAs mimic the “survival of the fittest” principle. So, naturally they are suitable to solve maximization problems. Minimization problems are usually transformed to maximization problems by some suitable transformation. A fitness function \( F(x) \) is derived from the objective function and is used in successive genetic operations. For maximization problems, fitness function can be considered the same as the objective function. The minimization problem is an equivalent maximization problem such that the optimum point remains unchanged. A number of such transformations are possible. The fitness function often used is

\[ F(x) = \frac{1}{1 + f(x)}. \]  

(5)

GAs are ideally suited for unconstrained optimization problems. As the present problem is a constrained optimization one, it is necessary to transform it into an unconstrained problem to solve it using GAs. To handle constraints, penalty function method has been mostly used [9]. Traditional transformations using penalty functions are not appropriate for GAs. A formulation based on the violation of normalized constraints is proposed in this paper.

4.6. Violation parameter

Here, an attempt has been made to formulate a fitness function, which includes the FOS that will become low in the process of optimization of weight of leaf spring. So it is apt to add the FOS term along with the objective function as against the use of penalty function:

\[ \text{obj}(x) = f(w) + a(FOS), \]  

(6)

where \( a \) is termed as a violation parameter.

Before evaluating the fitness function, the values are checked for constraint violation. If the design variable set violates the constraint, then a higher value will be assigned for the violation parameter. If not, a lower value will be assigned. By doing so, the design variable set, which violates the constraints will give a very high objective function value. When evaluated, this results in a very low fitness function (Eq. (7)), which reduces the probability of selection of this particular set in the next generation and so on.

\[ F(x) = \frac{1}{1 + \text{obj}(x)}. \]  

(7)

5. Parameters of genetic algorithms

Establishing the GA parameters is very crucial in an optimization problem because there are no guidelines. One has to fix the GA parameters for a particular problem based on the convergence of the problem as well as the solution time.

5.1. Total string length

The lengths of the strings are usually determined according to the accuracy of the solution desired and the data available. For example, if a four-bit binary string is used to code a variable, then the substring (0000) is decoded to the lower limit of the variables, (1111) is to the upper limit and any other string to a value in the range between the lower and upper limits. There can be only \( 2^4 \) different strings possible, because each bit position can take a value of 0 or 1. So, using a four-bit binary substring one can represent 16 available sections. Here, in order to make more number of sections available for the problem, the total string length is taken as 20. Each substring will have 10 bits. A binary string of length 10 can represent any value from 0 to 1023 \((2^{10} - 1)\). It can also be a variable substring length. In the present study, the substring lengths of all variables are assumed as equal.

5.2. Maximum generation

The process of termination of the loop was carried out by fixing the maximum number of generations. This maximum number of generations is fixed after trial runs.

5.3. Cross-over and mutation probability

Generally, it is recommended to have high cross-over and low mutation probability. Here, the cross-over probability is taken as 0.9, and mutation probability as 0.001.

5.4. String length

The individual string length is taken as 10 for each of the two design variables. The strings representing individuals in the initial population are generated randomly, and the binary strings are decoded for further evaluation. Depending on the evaluation results of the first generation and the GA parameters, population for the next generation is created. Generation of population for the subsequent generation depend on the selection op-
erator as well as on the cross-over and mutation probability. The algorithm repeats the same process by generating a new population, and evaluating its fitness as well as constraint violation.

6. Computer program

A tailor made computer program using c has been developed to perform the optimization process, and to obtain the best possible design. The approach consists of minimizing the weight of the leaf spring with required strength and stiffness. The flow-chart describing the step-by-step procedure of optimizing the composite leaf spring using GA is shown in Fig. 1.

7. Results and discussion

The procedure described in the previous sections has been applied to the design of minimum weight double tapered composite leaf spring to replace the seven-leaf steel spring arranged longitudinally in the rear suspension system of a passenger car. The design parameters such as distance between spring eyes, camber and load are kept as same in both steel and composite leaf springs. The input parameters used in this work are listed in Table 1. The geometric models of steel and composite leaf springs considered for optimization are shown in Fig. 2. The input GA parameters of steel and composite leaf springs are summarized in Table 2. The number of leaves in steel spring is fixed as seven and all leaves have the same thickness and width. By applying GA procedure, optimization is performed to decide the best possible combination of thickness and width of the leaves of steel spring by satisfying the above said constraints. The same procedure is carried out to determine the optimum centre thickness and centre width of composite spring.

The variation of fitness value with the number of generations obtained during GA process of optimization for steel and composite leaf springs are shown in Figs. 3 and 4, respectively. Maximum fitness value is the best parameter among the population in each generation. Average fitness value is the average of all the fitness values of the population. In earlier generation, the value of average fitness will be less since the population consists of worst individuals also. As the generations progresses, the population gets filled by more fit individuals, with only slight deviation from the fitness of the best individual so far found. Hence, the average fitness comes very close to the maximum fitness value. This has been clearly observed from Figs. 3 and 4.

During the process of search for optimum, the variation of design variables (thickness and width) in each generation for steel and composite springs are shown in Figs. 5 and 6, respectively. It has been earlier stated that the GA does global search in a random fashion. Hence, the variables are highly fluctuating during initial generations (up to generation number 45). After that, the fluctuation of the design variables is reduced and then it converges to optimum values, due to the population being filled by more fit individuals. It has been illustrated in Figs. 5 and 6 that different values are chosen in a random manner from the specified solution space for evaluation and this process continues till the solution is converged. Figs. 7 and 8 show the variation of constraints (deflection and stress) of steel and composite
Table 1
Input parameters of steel and composite leaf spring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Steel spring</th>
<th>Composite spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring length under straight condition (mm)</td>
<td>1220</td>
<td>1220</td>
</tr>
<tr>
<td>Arc height at axle seat (camber) (mm)</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>Modulus of elasticity of material (MPa)</td>
<td>$210 \times 10^3$</td>
<td>$32.5 \times 10^3$</td>
</tr>
<tr>
<td>Material density (kg/m$^3$)</td>
<td>7800</td>
<td>2600</td>
</tr>
<tr>
<td>Load (N)</td>
<td>4500</td>
<td>4500</td>
</tr>
<tr>
<td>Maximum allowable stress (MPa)</td>
<td>800</td>
<td>550</td>
</tr>
</tbody>
</table>

Fig. 2. Model of steel and composite spring considered for optimization.

Fig. 3. Variation of fitness value of steel spring with number of generations.

Fig. 4. Variation of fitness value of composite spring with number of generations.

Table 2
Input GA parameters of steel and composite leaf spring

<table>
<thead>
<tr>
<th>GA Parameters</th>
<th>Steel spring</th>
<th>Composite spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parameters</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total string length</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Population size</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Maximum generations</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Cross-over probability</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>String length for width</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Minimum and maximum bound for width (mm)</td>
<td>30–45</td>
<td>20–50</td>
</tr>
<tr>
<td>String length for thickness</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Minimum and maximum bound for thickness (mm)</td>
<td>5–8</td>
<td>10–50</td>
</tr>
</tbody>
</table>
spring with the number of generations, respectively, during GA search. It is observed that the constraints fluctuate only during the initial generations and the values are within the maximum limit.

The variations of weight of steel and composite leaf springs in each generation are shown in Figs. 9 and 10, respectively. For the first 20 generations of steel leaf spring and 30 generations of composite leaf spring, the weight is found to be highly fluctuating. The fluctuation...
is reduced to a minimum from generation nos. 21–90 in steel spring and from generation nos. 31–45 in composite spring, but later they get converged. This is because the population is filled with the best individuals, and further operations result in no change in fitness value. The optimal design values of steel and composite leaf springs are given in Table 3.

Weight reduction of leaf spring is obtained at different stages. Namely, the initial design of steel leaf spring had a weight of 9.28 kg. On optimization, using GA, it became 8.54 kg which results in a weight reduction of about 8%. Replacing steel spring with composite spring gave a weight reduction of 65.5%. Optimization of composite leaf spring yielded a further weight reduction of 23.4% giving an overall weight reduction of 75.6% (when optimized composite spring is considered in comparison with conventional steel spring). These results are shown as a flow diagram in Fig. 11. During this process of weight reduction, adequate strength and stiffness requirements are kept as constraints. The automotive suspension leaf spring contributes for about 10–20% of unsprung weight. If the unsprung weight is reduced, then the stress induced is also reduced. Hence, even a small amount of weight reduction in leaf spring will lead to improvements in passenger comfort as well as reduction in vehicle cost. In the present study, 75.6% of existing spring weight is reduced. This heavy reduction of leaf spring weight will improve the performance of the vehicle in all respects.

### Table 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Steel spring</th>
<th>Composite spring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (mm)</td>
<td>34.25 (each leaf)</td>
<td>28.475</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>6.55 (each leaf)</td>
<td>25.015</td>
</tr>
<tr>
<td>Maximum Stress (MPa)</td>
<td>799.52</td>
<td>462.17</td>
</tr>
<tr>
<td>Maximum deflection (mm)</td>
<td>144.10</td>
<td>141.03</td>
</tr>
<tr>
<td>Estimated weight (kg)</td>
<td>8.54</td>
<td>2.26</td>
</tr>
</tbody>
</table>

### 8. Conclusions

1. In the present work, the design variables (leaf thickness and width) of steel and composite leaf springs are optimized by making use of GA: a powerful non-traditional optimization method.
2. It is found that the use of violation parameter is much easier than penalty parameter for the conversion of constrained optimization problem to unconstrained optimization problem.
3. Optimization using GA has contributed to a reduction of 8% of the steel spring weight.
4. The weight of composite spring is reduced by 23.4% using GA optimization.
5. The weight of leaf spring is reduced from 9.28 to 2.26 kg, when a seven-leaf steel spring is replaced by a mono-leaf composite spring and optimized using GA. In this case around 75.6% weight saving is achieved with the same stiffness and strength.
6. It is observed that optimization using GA leads to larger weight reduction due to its search for global optimum as against the local optimum in traditional search methods.
7. These results are encouraging and suggest that GA can be used effectively and efficiently in other complex and realistic designs often encountered in engineering applications.

References