Fuzzy Logic and Fuzzy Control Systems

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ABSTRACT

A. Fuzzy sets
Fuzzy sets are class of connected series of objects, characterized by grade of membership ranging between zero and one.

B. Fuzzy logic
As an extension of the case of multi-valued logic, valuations $\mu : V \to W$ of propositional variables $V$ into a set of membership degrees $W$ can be thought of as membership functions mapping predicates into fuzzy sets (or more formally, into an ordered set of fuzzy pairs, called a fuzzy relation). With these valuations, many-valued logic can be extended to allow for fuzzy premises from which graded conclusions may be drawn.

C. Fuzzy control system

I. Inference Mechanisms
Inference mechanisms can be either based on fuzzy logic or non-fuzzy (crisp) logic; Fuzzy inferencesystems will be discussed in this report.

II. Defuzzification and Fuzzification
In reality systems, when we want to control system with fuzzy logic controller (FLC) we should changed crisp numbers into fuzzy data, following operations is being done by fuzzifiers. Moreover, the requirement of the systems is crisp data, however, the output data is fuzzy, in this case, defuzzifiers will be needed.
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INTRODUCTION
Fuzzy logic was first proposed by Lotfi A. Zadeh of the University of California at Berkeley in a 1965 paper. He elaborated on his ideas in a 1973 paper that introduced the concept of "linguistic variables", which in this article equates to a variable defined as a fuzzy set. Other research followed, with the first industrial application, a cement kiln built in Denmark, coming on line in 1975.[4]

During the past several years, fuzzy logic and fuzzy control has appeared as one the most active areas for research in the applications of fuzzy set theory, especially in the realm of industrial processes, which do not lend themselves to control by conventional methods because of lack of quantitative data regarding the input-output relations.

For preparing this report, some essays and articles studied and scanned about fuzzy logic, fuzzy sets and fuzzy logic in control systems. There are also thorough and complete researches arranged about this issue. This report has discussed fuzzy sets and fuzzy logic at the beginning, and then it debates fuzzy control followed by fuzzification and defuzzification.
A fuzzy set is a class of objects with a connected series of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one. For the convenience of the reader, this report shall introduce some brief summarization of the basic concepts of fuzzy set theory.\[1\]

**Fuzzy Sets and Terminology:** Let $U$ be the collection of objects denoted generically by $\{u\}$, which could be separate or continuous. $U$ is called the universe of the discourse, and $u$ represents the generic element of $U$. A fuzzy set $F$ in a universe of discourse $U$ is characterized by a membership function $\mu$ which takes values in the interval $[0,1]$ namely,

$$\mu: U \rightarrow [0,1].$$

A fuzzy set may be viewed as a generalization of the concept of an ordinary set whose membership function only takes two values $\{0,1\}$. Thus, a fuzzy set $F$ in $U$ may be represented as a set of ordered pairs of a generic element $u$ and its grade of membership function:

$$F = \{ (u, \mu(u)) \mid u \in U \}$$

The support of a fuzzy set $F$ is the crisp set of all points $u$ in $U$ such that $\mu(u) > 0$. In particular, the element $u$ in $U$ at which $\mu = 0.5$, is called the crossover point and the fuzzy set whose support is a single point in $U$ with $\mu = 1.0$ is referred to as fuzzy singleton.

**Set Theoretic Operations:** Let $A$ and $B$ be two fuzzy sets in $U$ with membership function $\mu_A$ and $\mu_B$, respectively. The set theoretic operations of union, intersection and complement for fuzzy sets are defined via their membership functions:

**Union:** The membership function $\mu_{A \cup B}$ of the union $A \cup B$ is point wise defined for all $u \in U$ by:

$$\mu_{A \cup B}(u) = \max \{ \mu_A(u), \mu_B(u) \}$$

**Intersection:** The membership function $\mu_{A \cap B}$ of the intersection $A \cap B$ is point wise defined for all $u \in U$ by:

$$\mu_{A \cap B}(u) = \min \{ \mu_A(u), \mu_B(u) \}$$

**Complement:** The membership function $\mu_C$ of the complement of a fuzzy set $A$ is point wise defined for all $u \in U$ by:

$$\mu_C(\mu) = 1 - \mu_A(\mu)$$

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**Fuzzy Logic**
Fuzzy logic is derived from fuzzy set theory dealing with reasoning that is approximate rather than precisely inferred from classical predicate logic. It can be thought of as the application side of fuzzy set theory dealing with well thought out real world expert values for a complex problem.[2]

The traditional way of the computer is binary, either true, 1, or false, 0, and nothing in between. There are some great advantages to digital representation, but ultimately we live in an analogue world and when describing it we are used to analogue terms defined through language.

Suppose we say *Bob* is tall. In the digital world, it would look something like this:

![Figure 1. Degree of membership-Height diagram in crisp criterion][6]

However, this would mean that while *Bob*, who is 180.1 cm in height, is tall, while Frank who is 179.9 cm is not. That doesn’t really reflect the way we would normally think. Instead, we would probably say that *Bob* and Frank are both rather tall. A fuzzy representation of the tall variable would look perhaps something like this:

![Figure 2. Degree of membership-Height diagram in fuzzy criterion][6]

As you can see, no sharp boundary at 180 cm, but a smooth transition. On the vertical axis
we have Membership Degree, indicating to what degree the value of a variable is in a set. In the digital world, membership degree can be 1 or 0, but in the fuzzy world it can be any value between 0 and 1.

Suppose we wish to divide height into three classes: short, average and tall. A Boolean representation would perhaps look something like this:

![Crisp membership function](image1)

Again we see those sharp boundaries. A fuzzy representation on the other hand might look something like this:

![Fuzzy membership function](image2)

Note how the regions are overlapping, this is one fundamental feature of fuzzy sets; membership in a set is not exclusive. A person that is 165 cm in height is a member of both the Short and the Average sets. Also note that there is nothing preventing you from creating Boolean sets by using fuzzy logic. The regular Boolean logic is in fact a special case of the more general fuzzy logic. Degrees of truth are often confused with probabilities. However, they are conceptually distinct; fuzzy truth represents membership in vaguely defined sets, not likelihood of some event or condition.

Fuzzy logic is not any less precise than any other form of logic; it is an organized and mathematical method of handling inherently imprecise concepts. The concept of
coldness cannot be expressed in an equation, because although temperature is a quantity, coldness is not. However, people have an idea of what cold is, and agree that there is no sharp cutoff between cold and not cold, where something is cold at N degrees but not cold at N+1 degrees, a concept classical logic cannot easily handle due to the principle of bivalence. The result has no set answer so it is believed to be a fuzzy answer.

Fuzzy set theory defines fuzzy operators on fuzzy sets. The problem in applying this is that the appropriate fuzzy operator may not be known. For this reason, fuzzy logic usually uses If / Then rules, or constructs that are equivalent, such as fuzzy associativematrices. Rules are usually expressed in the form: If variable is set Then action. For instance, an extremely simple temperature regulator that uses a fan might look like this: [6]

If temperature is very cold Then stop fan
If temperature is cold Then turn down fan
If temperature is normal Then maintain level
If temperature is hot Then speed up fan

Notice there is no Else. All of the rules are evaluated, because the temperature might be cold and normal at the same time to differing degrees. However, this criticism is mainly due to the fact that there exist problems with conditional possibility, the fuzzy set theory equivalent of conditional probability. This makes it difficult to perform inference. Nevertheless, there have not been many studies comparing fuzzy-based systems with probabilistic ones.
Fuzzification is related to the vagueness and imprecision in a natural language. It is a subjective valuation, which transforms a measurement into valuation of a subjective value, and hence it could be defined as a mapping from an observed input space to fuzzy sets in certain input universes of discourse. Fuzzification plays an important role in dealing with uncertain information that might be objective or subjective in nature. In fuzzy control applications, the observed data are usually crisp. Since the data manipulation in a fuzzy logic controller is based on fuzzy set theory, fuzzification is necessary during an earlier stage. Experience with the design of a fuzzy logic controller suggests the following principle ways of dealing with fuzzification.[4]

1. A fuzzification operator conceptually converts a crisp value into a fuzzy singleton within a certain universe of discourse.

2. Observed data are disturbed by random noise. In this case, a fuzzification operator should convert the probabilistic data into fuzzy numbers.

3. In large-scale systems and other applications, some observations relating to the behavior of such systems are precise; while others are measurable only in a statistical sense, and some referred to as hybrids, require both probabilistic modes of characterization.

**Defuzzification**
Basically, defuzzification is a mapping from a space of fuzzy control actions defined over an output universe of discourse into a space of non-fuzzy (crisp) control actions. It is employed because in many practical applications a crisp control action is required. A defuzzification strategy is aimed at producing a non-fuzzy control action that best represents the possibility distribution of an inferred fuzzy control action. Unfortunately, there is no systematic procedure for choosing a defuzzification strategy. Zadeh first pointed out this problem and made tentative suggestions for dealing with it. At present, the commonly used strategies may be described as the max criterion, the mean of maximum, and the center of area.[4]

A. The max criterion method
The max criterion produces the point at which the possibility distribution of the control action reaches a maximum value.

B. The mean of maximum method
The mean of maximum strategy generates a control action, which represents the mean value of all local control actions whose membership functions reach the maximum.

C. The center of gravity method
The widely used center of area method strategy generates the center of gravity of the possibility distribution of a control action.

**Fuzzy Control: An Example**
This example devotes to control the position of a DC motor with a fuzzy control system. First the problem itself should be clarified; by giving an input the desired angle of the shaft will be recognized. Consequently, this input should be fuzzified into fuzzy sets. Then, the inference mechanism will make decision and convert fuzzy data to understandable data by defuzzification. The block set diagram below shows the control mechanism of this problem.

![Block Diagram](image)

Figure 5. A simple PD control block-set diagram [3]

Suppose we are using error function \( e(t) = r(t) - y(t) \), in which \( r \) is the desired angle and \( y \) is the sensor feedbacks of the current angle. Moreover, \( u \) indicates the voltage input to the DC motor. For simplifying the control mechanism we assume the desired angle is zero. Hence:

\[
    e(t) = -y(t), \quad \frac{d}{dt} e = \frac{d}{dt} (-y)
\]

After identifying inputs and outputs we shall choose the control method; in this very case the PD control is used, because, it is much sensible than PID, PI or P method.

The next step would be the fuzzification step. In this stage the membership functions is determined. Initially, the number of cases should be hypothesized. In this problem, poslarge, possmall, zero, negsmall, neglarge are the cases. The linguistic-numeric values are also assigned for this case in order to facilitate the notation; 2, 1, 0, -1, -2 are the assigned numbers, respectively. Hence, the membership functions will be:

![Membership Functions](image)
After fuzzifying, the rules should be defined. As said before, the rules are mainly in IF/THEN statements. For this case we have 25 rules because there are 2 inputs \( e \) and \( \dot{e} \) each containing five cases.

For instance, IF \( e \) is negsmall AND \( \dot{e} \) is possmall THEN \( u \) is possmall, i.e. the sensed position is slightly after zero and the shaft is slowly rotating counter-clockwise, hence, a small positive voltage should apply to the motor to prevent the shaft to pass the desired angle.

The next and last step is to defuzzify the outcome from the inference system. To do so, the outcome value of each rule should be interpreted in its own membership criteria. For instance sensor detects \( e = 0 \) and \( \frac{d}{dt} e = \frac{3\pi}{32} \), this condition satisfies two different membership functions; "IF \( e \) is zero AND \( \dot{e} \) is zero THEN \( u \) is zero" and "IF \( e \) is zero AND \( \dot{e} \) is possmall THEN \( u \) is negsmall". Hence, the outcomes should be inferred. In which, the degree of certainty of statements should be found.
As shown in figure 8, it is convenient to take 0.25 certainty for error change in zero region and 0.75 in possmall region. Moreover, the certainty for error is always 1 since error in this example is zero. Consequently, it is suitable to write:

$$\mu_1 = \min\{0.25, 1\} = 0.25, \quad \mu_2 = \min\{0.75, 1\} = 0.75$$

And the implied membership functions for these cases are:

![Figure 9. Implied membership functions [3]](image)

After the certainties are extracted, we use on of defuzzification methods to combine these cases and interpret a single output. In this very example, the center of gravity method will be used.

![Figure 10. Sum of answers, a step before defuzzification [3]](image)

The basic formula for calculating center of gravity or center of area is:
\[ u_{\text{equivalent}} = \frac{\sum c_i A_i}{\sum A_i} \]

Where in this case we have:

\[ u_{\text{equivalent}} = \frac{(0 \times 4.375) + (-10 \times 9.375)}{(4.375 + 9.375)} = -6.81 \]

Therefore, the voltage of -6.81 would be applied to the motor.
This process will be continued until the error and change of error with respect to time reaches to zero. In which, the position of the motor is the desired position with perfect approximation. [3]
REFERENCES


