Low-loss HVDC transmission system with self-commutated power converter introducing zero-current soft-switching technique

T. Senjyu  K. Kurohane  J. Miyagi  N. Urasaki

Faculty of Engineering, University of the Ryukyus, 1 Senbaru, Nishihara-cho, Nakagami, Okinawa 903-0213, Japan
E-mail: b985542@tec.u-ryukyu.ac.jp

Abstract: In recent years, application of self turn-off devices for the HVDC transmission system is very important for flexible and high efficient power transport and supply. However, for self turn-off devices that are applied to high-power system, excessive switching losses due to voltage/current surges occurred on hard switching and the current tail characteristics on turn-off lead to the considerable switching loss. Therefore switching-loss reduction by using a soft-switching technique is very effective for improvement of the considerable switching losses. A self-commutated HVDC circuit topology with soft-switching characteristics is discussed. The proposed topology is able to achieve a zero-current switching in self-commutated power converters. In order to examine the effectiveness of the proposed system, the system is analysed in terms of the characteristics of switching losses through a theoretical approach and computer simulations.

1 Introduction

Recently, HVDC transmission technology has been widely applied in some countries because of the advantage of its economical efficiency, power system stability and load flow control. The AC–DC power converters are indispensable in HVDC transmission systems and mostly consist of thyristors on line-commutated devices. In recent years, the usage of flexible and efficient power control and transfer by use of self turn-off devices for HVDC power converters have been increasing. However, self turn-off devices such as gate turn-off thyristor (GTO) and insulated gate bipolar transistor (IGBT) have some limitations. These limitations include the increase in voltage/current stress and high switching losses, rising heat of converters and the generation of electromagnetic noise.

The solutions for these problems include improvement of performances of power semiconductors and circuit topology. The former method means technical advantages for power devices using the silicon–carbide (SiC) process technology, which replaces the silicon (Si) process technology [1, 2], and the later method means an application of soft-switching circuit topology based on resonance phenomenon using L-C elements [3–9].

Soft-switching techniques are beneficial in reducing the switching losses. In the past, several soft-switching circuit topologies for DC–DC converters and AC–DC converters on the power applications have been proposed [1–8]. DC-link circuit topology with zero-voltage switching (ZVS) as DC–DC converters have been reported in [3–5]. The AC-link circuit topology with ZVS manner that envisions the charge and discharge of power energy is presented in [6]. The ZCS static var compensator circuit topologies using current-source-inverter have also been reported in [7–10]. Furthermore, studies on low-loss snubber circuit [11, 12] for improving the switching losses and voltage/current stresses have also been reported. However, a self-commutated HVDC system with soft-switching technique is skill unavailable.

The current tail characteristics greatly increase the switching loss of the power converter. However, the ZCS technique is capable of controlling the current tail that flows to self turn-off devices by adding an auxiliary circuit.
Therefore ZCS is the most successful soft-switching technique for application of high-power converters. Besides, the soft-switching technique contributes to the reduction of EMI noise. On the other hand, ZVS controls voltage that occurs in the terminal of devices, and the voltage is cancelled by resonant operation. Therefore it is difficult to decrease of the switching losses significantly caused by the current tail based on ZVS is difficult. Also the resonant circuit is an additional circuit that increases the costs and power losses of components.

This paper proposes a zero-current switched HVDC circuit topology with self-commutated converter to improve the switching loss of HVDC. The switching losses of HVDC are reduced by adding a commutation circuit. In order to verify the effectiveness of the proposed system, the simulations are carried on by using MATLAB/SIMULINK and SimPowerSystems based on instantaneous modelling.

This recent paper is organised as follows. Circuit configuration and operating principle of the proposed system are given in Section 2. Section 3 shows ZCS requirements for soft switching. To confirm the robustness of the proposed circuit, fault simulations for soft-switching HVDC are executed in Section 4. Section 5 analyses the characteristics of switching loss and conversion efficiency through a theoretical approach and computer simulations. Finally, conclusions are drawn in Section 6.

2 Circuit configuration and operating principle

2.1 Circuit configuration of the proposed system

Replacing line-commutated devices to self-turn-off devices causes an increase in switching loss. Therefore this paper presents a soft-switching HVDC circuit topology. By connecting commutation circuits for DC-link part of power converters, the proposed circuit executes the soft-switching ZCS. Fig. 1 shows the main configuration of the soft-switching HVDC transmission system. The circuit configuration contains point-symmetry except the DC-line. The power system side consists of DC smoothing reactor, commutation circuit, current source self-commutated converter, output capacitors, converter transformer, AC filter and line impedances. The function of output capacitors \( C_{ac1}, C_{ac2} \) is to keep the voltage by regulating the reactive power. A commutation circuit consists of two thyristors \( T_1 \) and \( T_2 \) (or \( T_3 \) and \( T_4 \)), resonant capacitor \( C_{cr1} \) (or \( C_{cr2} \)) and resonant inductors \( L_{cr1} \) and \( L_{cr2} \) (or \( L_{cr3} \) and \( L_{cr4} \)). By connecting the commutation circuits for the DC-link, it is possible to achieve a perfect ZCS in HVDC. That is to say, the commutation circuits aim to intermit \( i_{con1} \) and \( i_{con2} \) flowing to the power converters. The operating analysis of the commutation circuit is described in detail in the next section.

2.2 Operating principle and analysis for commutation circuit

Fig. 2 shows equivalent circuits in each mode (Mode 1–Mode 6) of commutation-circuit operation. Since the simulation results of the operated commutation circuits on each side are nearly the same, only the right-side one is analysed. The commutation circuit has six operating modes for a changing cycle of switching devices, and the resonant-modes occur at Modes 1, 3 and 5. It is assumed that the DC current \( I_{dc1} \) flowing to converter is sufficiently smoothed by the DC reactor. A thyristor model is treated as the same as GTO thyristor model that is shown in the Fig. 3. The equations needed for analysis of operation are derived on the basis of this figure.

Mode 1 \((t_0 - t_1; \text{ Fig. 2a})\): As the initial state in Mode 1, main-switches \( U_{t1} \) and \( W_{t1} \) on the right-side converter and thyristor \( T_2 \) on the commutation circuit are in the conduction state as shown in Fig. 2a. Other GTO devices and thyristor are in off-state. In addition, the voltage \( V_{cr1} \) at both ends of the resonant capacitor \( C_{cr1} \), are charged up to \( V_{cr} \).

In the beginning of this mode \( (t = t_0) \), when \( T_2 \) is on by a turn-on signal, the commutation circuit attains resonant condition by capacitor \( C_{cr1} \) and inductor \( L_{cr1} \). Then, the resonant current \( i_{t1} \) flows in the commutation circuit. The \( i_{t1} \) has the resonant frequency \( f_r \) determined by resonant capacitor and inductor. An equivalent circuit of Mode 1 is shown in Fig. 2a. The charging voltage of resonant capacitor

![Figure 1](https://www.ietdl.org)
C\textsubscript{cr1} begins to decrease by the flow of resonant current \( i_r \). The mode condition shifts to next mode, Mode 2, when all \( i_{con1} \) commutate to the commutation circuit.

The voltage equation with Mode 1 is given as

\[
V_{dc} = (R_{Lr1} + R_{on})i_r + (L_{cr1} + L_{on}) \frac{di_r}{dt} + V_f + \frac{1}{C_{cr1}} \int i_r \, dt + V_{co}
\]

where \( R_{Lr1} \) is the inner electrical resistance in resonant element \( L_{cr1} \). By solving the differential equation (1), resonant current \( i_{r1} \) is expressed as

\[
i_{con1} = I_{dc} - i_{r1} = I_{dc} - C_{cr1} V_1 \omega_1 e^{-\alpha_1 t} \sinh \gamma_1 t
\]

where \( V_1 = V_{dc} - V_f - V_{co} \)

\[
\alpha = -\alpha_1 = \sqrt{\omega_1 \omega_2}, \quad \beta = j \omega_1
\]

Mode 2 (\( t_1 \leq t < t_2 \); Fig. 2b): The converter current \( i_{con1} \) commutates to the commutation circuit side, and the equivalent circuit in this mode is shown in Fig. 2b. When \( i_{con1} \) flowing to the converter is zero, by switching \( U_{g1} \) and \( V_{p1} \), they are turned-off and turned-on at ZCS, respectively. After switching-on and switching-off the main
devices, the $i_{\text{con1}}$ again begins to flow to the primary side, and the operating mode shifts to Mode 3.

Mode 3 ($t_2 - t_5$: Fig. 2c): Equivalent circuit of Mode 3 is shown in Fig. 2c. The commutation circuit once again is in resonant condition. As the voltage equation in Mode 3 is similar to (1) of Mode 1, a equation of resonant current $i_{r3}$ and (2) are equivalent. In this mode, a resonant capacitor changes in reverse polarity by the flow of $i_{r3}$. When $i_{r3}$ is zero, then thyristor T2 is turned-off with ZCS, and the condition also shifts to Mode 4.

Mode 4 ($t_3 - t_6$: Fig. 2d): Resonant current that is flowing to commutation circuit is zero, and $v_{c1}$ has reverse polarity. In this mode, the DC (current) $I_{dc1}$ flows to the primary inverter. An equivalent circuit of Mode 4 is shown in Fig. 2d.

Mode 5 ($t_4 - t_6$: Fig. 2e): To reverse the polarity of the resonant capacitor voltage $v_{c1}$, a turn-on signal that has time delay $T_{\text{delay}}$ later than the turn-on signal of thyristor T2, is given to the thyristor T1. An equivalent circuit of this mode is shown in Fig. 2e. By turning on T1, a resonant current $i_{r5}$ that has reverse direction flows in the commutation circuit, and $v_{c1}$ is charged in straight polarity.

The voltage equation of Fig. 2e is written as

$$V_{co} = (R_{L2} + R_{on})v_{c5} + (L_{con} + L_{c1} + L_{c2}) \frac{dv_{c5}}{dt} + \frac{1}{C_{c1}} \int i_{r5} \, dt$$

where $R_{L2}$ is inner electrical resistance in resonant element $L_{c2}$. By solving the differential equation (3), the current equation of Mode 5 is given as

$$i_{r5} = C_{c1} V_{co} \frac{\alpha_5^2}{\gamma_5} e^{-\alpha_5 t} \sinh \gamma_5 t \quad \text{(4)}$$

where $\alpha = j\beta = -\alpha_5 \pm j\sqrt{\alpha_5^2 - \omega_5^2}$

$$= R_{L2} + R_{on} \frac{1}{2(L_{c1} + L_{c2} + L_{con})} \pm j \sqrt{\left( \frac{R_{L2} + R_{on}}{2(L_{c1} + L_{c2} + L_{con})} \right)^2 - \frac{1}{C_{c1}(L_{c1} + L_{c2} + L_{con})}}$$

where $\alpha_5$ is a constant value and $j\beta$ means the same angular frequency as $\alpha_5$ in resonant current $i_{r5}$. When $i_{r5}$ reaches zero, the operating mode shifts to Mode 6.

Mode 6 ($t_5 - t_6$: Fig. 2f): In this mode, the resonant current is zero, the equivalent circuit of Mode 6 is shown in Fig. 2f. When $T2$ has a turn-on signal, a commutation circuit repeats the similar operating modes from Mode 1.

### 2.3 Simulation key waveform on soft and hard switchings

To confirm the operating characteristics of the proposed circuit shown in Fig. 1, the proposed system is simulated using MATLAB/SIMULINK and SimPowerSystems. The instantaneous simulation waveforms of a model with DC-lines are shown in Figs. 4 and 5. Fig. 4 shows simulation results with soft-switching ZCS. To confirm effectiveness of the method, simulation results by hard switching are shown in Fig. 5. From Figs. 4 and 5, it is observed that $i_{up1}$ and $i_{on1}$ flowing to each device by soft switching become nearly zero by operating the commutation circuit before the main devices $U_{p1}$ and $V_{p1}$ turn-off and turn-on, respectively. However, $i_{up1}$ and $i_{on1}$ by hard switching is not zero. As a result, excessive switching losses (losses more than 100 MW on turn-off and losses of about 1.7 MW on turn-on), occur instantaneously in the main devices on hard switching. On the contrary, switching loss on soft switching is nearly zero. Therefore the voltage and current stresses can be reduced (more than 100 MW on turn-off and about 1.7 MW on turn-on), occur instantaneously in 15 devices (the nominal voltage and current in a GTO device are 6000 V and 6000 A, respectively).

**Figure 4 Key waveforms with soft switching**
3 ZCS requirement for soft switching

In this section, the following description gives some conditional equations for a soft switching technique ZCS on the right-side power converter shown in Fig. 1, since ZCS requirements on either side are the same.

3.1 ZCS requirement for resonant frequency

The quality factor $Q$ that refers to the sharpness of the resonant curve in LCR circuit (not the quality factor of a coil) is shown as

$$Q = \frac{1}{R_L} \sqrt{\frac{L_{ro}}{C_{cr}}}$$  (5)

In case of Modes 2 and 4

$$R_L = R_{L1} + R_{on}$$

$$L_{ro} = L_{cr1} + L_{on}$$

In case of Mode 6

$$R_L = R_{L2} + R_{on}$$

$$L_{ro} = L_{cr1} + L_{cr2} + L_{on}$$

where the quality factor $Q$ means sharpness of resonant current. The resonance phenomenon can be classified as follows cases in terms of $Q$

- In case of $(1)$ $Q > 1/2$
- In case of $(2)$ $Q < 1/2$
- In case of $(3)$ $Q = 1/2$

a resonant current has

- (1) oscillatory behaviour
- (2) non-oscillatory behaviour
- (3) critical behaviour

With the above classification, resonant current must have oscillatory behaviour for the soft switching. Therefore the conditional equation is then shown as

$$\frac{1}{R_L} \sqrt{\frac{L_{ro}}{C_{cr}}} > \frac{1}{2}$$  (6)

3.2 Requirement of shunt-current $i_{con}$

Here, the maximum value of resonant current is derived. In case of $Q > 1/2$, the variables need to be changed since $\gamma_1$ in the resonant current (2) is imaginary. As $\gamma_1 = i\beta_1 = i\sqrt{\omega_1^2 - \alpha_1^2}$, then from (2), the resonant current equation is expressed as

$$i_1 = C_{cr} V_1 \frac{\alpha_1}{\beta_1} e^{-\alpha_1 t} \sin \beta_1 t$$  (7)

Therefore the extreme solution (maximum value) of (7) becomes

$$I_{2peak} = \frac{V_1}{\beta_1 (P_{cr1} + L_{on})} e^{-\alpha_1/\beta_1} \sin \Phi_1$$

$$\therefore \Phi_1 = \tan^{-1} \frac{\alpha_1}{\beta_1}$$  (8)

A soft-switching technique ZCS is executed by turning-on and turning-off the IGBT main devices on the power converters when the converter current $i_{con1}$ is zero. For this reason, the maximum resonant current $I_{2peak}$ must be

Table 1 Simulation parameters (right-side only)

<table>
<thead>
<tr>
<th>Table 1 Simulation parameters (right-side only)</th>
<th>$R_{con} = 15 , m\Omega$, $L_{con} = 15 , m\text{H}$, $V_{if} = 90 , V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GTO 1 arm</td>
<td>$R_{con1} = 28 , m\Omega$, $L_{con1} = 28 , m\text{H}$, $V_{if} = 70 , V$</td>
</tr>
<tr>
<td>thyristor 1 arm</td>
<td>$R_{con1} = 2 , \mu\Omega$, $L_{con1} = 2 , m\text{H}$, $L_{cr1} = 3 , \mu\text{H}$, $R_{L1} = 2 , m\Omega$, $L_{cr2} = 3 , m\text{H}$, $R_{L2} = 3 , m\Omega$, $C_{cr1} = 40 , \mu\text{F}$, $f_{r1} = 4.59 , kHz$, $f_{r2} = 4.38 , kHz$</td>
</tr>
<tr>
<td>resonant components</td>
<td>$I_{dc1} = 0.8 , H$, $R_{dc1} = 1.5 , \Omega$, $C_{sc1} = 110 , \mu\text{F}$, $X_1 = 82.94 , \Omega$</td>
</tr>
</tbody>
</table>

Figure 5 Key waveforms with hard switching
Figure 6: HVDC model with fault

Figure 7: Three-phase voltage and current with terminal-fault
- a Wave form for left converter-side
- b Wave form for right converter-side
- c Wave form for left source-side
- d Wave form for right source-side
- e Wave form for left DC line-side
- f Wave form for right DC line-side
bigger than the direct current $I_{dc1}$ and can be expressed as follows

$$I_{\text{peak}} = \frac{V_1}{\beta_1(L_{cr1} + 2L_{on})} e^{-\alpha_1/\beta_1} \phi_1 \sin \phi_1 \geq I_{dc1} \quad (9)$$

### 3.3 Setting of time-delay $T_{\text{delay}}$ for commutation-circuit operation

The time-delay $T_{\text{delay}}$, that is the interval time for turn-on signals of thyristor elements in the commutation circuit, must be longer than the half-cycle of resonant frequency $\beta_1$ in Modes 1 and 3. Therefore $T_{\text{delay}}$ can be expressed as

$$T_{\text{delay}} \geq \frac{T_{r1}}{2} = \frac{1}{2f_{r1}} = \frac{\pi}{\beta_1} = \frac{\pi}{\sqrt{\omega_{r1}^2 - \alpha_1^2}} \quad (10)$$

where $T_{r1}$ mean time that $T_1$ is in resonant condition.

### 4 Fault simulation of HVDC circuit with ZCS

To confirm the robustness of the proposed circuit, fault simulations for soft-switching HVDC are executed. The

---

**Figure 8 Three-phase voltage and current with ground fault**

- **a**: Wave form for left converter-side
- **b**: Wave form for right converter-side
- **c**: Wave form for left source-side
- **d**: Wave form for right source-side
- **e**: Wave form for left DC line-side
- **f**: Wave form for right DC line-side
parameters used in this simulation are shown in Table 1, where $L_{\text{cr}}$ and $C_{\text{cr}}$ was derived from (6). Each arm of the three-phase bridge is composed of 6000 V, 6000 A gate turn-off thyristor and 4500 V, 800 A anti-parallel diode. While, the resonant circuit is composed of 3000 V inductances, 4000 V, 800 A thyristor, 2500 V capacitor. As for the other components, we employ a 6600 V, 87.5 A, high-voltage phase advance capacitor and a 420 kVA, 6600 V, 60 Hz AC filter. Regarding the voltage rating of the devices, it is possible to reduce the voltage rating of the devices with the increment of the number of L-C elements and series connection of switching devices. The simulations assume terminal and three phase to ground faults. Fig. 6 shows HVDC model with faults. The simulation timestep is variable step (0–3 ms), duration of simulation is 0.5 s and time of execution is about 17 min. The simulation sequence is as follows: fault on the right-side occurred at $t = 0.15$ s, and each power converter stops. In addition, the fault in the power system recovers at $t = 0.3$ s for considering the time when the system recovers after an accident occurred, and each converter operates. In the fault simulations, only the left-side converter is soft switching, while the right-side is conventional switching.

Fig. 7 shows three-phase voltages and currents with terminal fault. From Figs. 7a, 7b and 7c, we can confirm that the output current of stopped converters is equal to zero because of the terminal fault. Besides, after the fault in the right-side recovers at $t = 0.3$ s for considering the time when the system recovers after an accident occurred, the power converter of the right-side operates again with soft switching.

Fig. 8 shows three phase voltages and currents with a three phase to ground fault. As can be seen from Fig. 8a, 8b and 8c, that the ground fault occurs at $t = 0.15$ s, the DC current and three phase currents $i_a$, $i_b$ and $i_c$ of converters side are zero, and left-side converter is switched with ZCS.

Note that DC-current $i_{\text{dc1}}$ is zero by operating commutation circuit, and the output AC-current is zero. Thus the output current wave form of soft-switched converter (Figs. 7a and 8a) include many vertical lines than for no soft switching (Figs. 7b and 8b).

5 Evaluation of switching loss

By using the derived equations in the Appendix, the characteristics of switching loss for HVDC transmission circuit applied to self-commutated power converter is evaluated for theoretical values. The parameters used for the circuit are shown in the Appendix. In addition, the equations for power losses with hard switching (on 120°-conducting operation) are also derived in the Appendix. The power-loss characteristics with soft switching are only composed of conduction loss, because it is confirmed in Fig. 9 that switching losses with turn-on and turn-off with soft switching are nearly zero. The additional losses with commutation circuit are included in the energy losses.

Fig. 9a shows the characteristics of switching losses for hard-switching HVDC. In Fig. 9a, the solid line and the chain dashed line show the total losses and the turn-off losses, respectively. The chain double-dashed line and the chained line show the turn-on losses and the conduction losses, respectively. The dot line shows the transmission power. As the main losses that occur in the self turn-off device are generally turn-on and turn-off losses and conduction losses, the above three switching losses are evaluated. As can be seen from Fig. 9a, with the increase of DC-current $I_{\text{dc1}}$ such losses are increasing. It can be confirmed that the arising losses on hard switching in a high-power system like HVDC system are excessive, and that the switching losses in turn-off exceed conduction losses from Fig. 9a. Besides, it is confirmed that the increase in hard-switching losses for high-power HVDC circuit is high, and the switching losses with turn-off are larger than conduction loss.
Fig. 9b shows the proportions of such losses (turn-on and turn-off losses and conduction losses) to total losses (turn-on and turn-off losses + conduction losses) as a percentage. In Fig. 9b, the chain dashed line and the chain double-dashed line show the characteristics of turn-off losses, and turn-on losses respectively, whereas the chained line and the dot line show conduction losses and transmission power, respectively. From Fig. 9b, it is confirmed that the losses with turning-off indicate losses more than 50% of total losses, and the turn-off losses with switching dominate the huge percentage of total losses. From the above results, in large-scale power systems such as HVDC system it is confirmed that energy losses occurring in self turn-off devices depend a lot on switching loss.

Fig. 9c shows total loss of HVDC circuit with hard- and soft switching, respectively. To confirm effectiveness of the soft-switching technique, the total losses with hard switching are compared with soft-switching HVDC at 120° conduction operation. The total loss with soft-switching HVDC includes copper and core losses with resonant-element coil, and conduction loss with thyristors. The solid line and the dash line show the total losses (conduction loss + loss of commutation circuit ) with soft switching and total losses (conduction loss + turn-on and turn-off loss) with hard switching, respectively. From Fig. 9c, it is confirmed that switching losses with self turn-off devices are significantly improved by applying the soft-switching technique.

6 Conclusion

This paper proposes the HVDC circuit with soft-switching technique. The proposed circuit concludes that switching losses with all self turn-off devices are nearly zero by connecting commutation circuits for power converters. In addition, it is able to execute soft-switching ZCS only by deciding switching sequences that turn-on pulses synchronised with turn-on signals of main-devices GTO give thyristor of commutation circuit. With the operating analysis of the proposed circuit, the analysis equations for such operating modes and ZCS requirements for soft switching are derived. Besides, to confirm the robustness of the proposed circuit, fault simulations for soft-switching HVDC are executed. The simulation results show that the proposed circuit can achieve soft-switching operation, after faults in the power system recover. Finally, to confirm the effectiveness of soft-switching technique, the proposed circuit is strictly modelled using MATLAB/SIMULINK and SimPowerSystems. Switching losses with 120° conduction operation are calculated on the simulation results, and total losses for soft switching are compared with them for hard-switching. As a result, it is confirmed that the improvement of power losses are more than 50% of total losses for hard-switching HVDC. Therefore the voltage and current stresses can be reduced. There are the trade-off relationship between the increase of circuit component parts with auxiliary commutation circuit and the improvement of high power losses attributed by introducing the soft-switching technique. However, for the reduction of switching losses confirmed from simulation results, the additive losses in the commutation-circuit are generated and the voltage rating of the devise is a little high, but the total losses in power converters for the proposed system is improved. It is sufficiently possible to decrease the power losses in the total system with application of soft-switching technique. In addition to the above effectiveness, the cost reduction is led by decreasing the heat-radiation unit with the reduction of switching loss.

Developments of self turn-off device technology need no soft-switching techniques. However, there are facts that the above developments advance soft switching and control technologies. Besides, there is no ideal switching device, and the switching losses with turn-on and turn-off are not zero. For these reasons, by introducing the soft-switching technique for the high-power system, the cost reduction, energy saving and the mitigation of environmental problems is largely expected.

7 References


8 Appendix

8.1 GTO model (Figs. 3 and 10)

8.1.1 Derivations of switching losses in hard switching: Switching-loss at turn-on duration: As can be seen from Fig. 10, the current and voltage equations of GTO device at turn-on duration can be expressed as

\[ i_c = \frac{V_{off}}{L_{on}} (t - T_{on}) : (T_{on} < t < T_{on} + T_{tr}) \]  

From (11), the equation of switching loss at turn-on duration can be derived as

\[ \omega_{on} = \int_{T_{on}}^{T_{on}+T_{tr}} i_c v_{ce} \, dt = \frac{V_{rms}^2 T_{tr}^2}{2L_{on}} \]  

where \( T_{tr} = (I_{on}I_{dc})/V_{rms} \).

Switching-loss at turn-off duration: As can be seen from Fig. 10, the tail-current equations of GTO device at turn-off duration can be expressed as

\[ i_{ce} = \begin{cases} \frac{I_{dc}}{10} \left( 1 - \frac{0.9 \left( T_{off} - T_{tr} \right)}{T_{tr}} \right) & : (T_{eff} < t < T_{eff} + T_{j}) \\ \frac{I_{dc}}{10} \left( 1 - \frac{t - (T_{eff} + T_{tr})}{T_{tr}} \right) & : (T_{off} + T_{j} < t < T_{off} + T_{tr} + T_{j}) \end{cases} \]  

From (13), the equation of switching loss at turn-off duration can be derived as

\[ \omega_{off} = \int_{T_{eff}}^{T_{eff}+T_{j}} i_c v_{ce} \, dt = \frac{1}{20} V_{rms} I_{dc} (11 T_{j} + T_{f}) \]  

Conduction-loss: The conducting-period in periodical time of output-current is 120°, then AC current on 120° conduction mode is shown in (15)

\[ i_{mc} = 2 \sqrt{3} \frac{I_{dc}}{\pi} \sin \frac{\pi a t}{5} - \sin \frac{5 \pi a t}{7} + \sin \frac{7 \pi a t}{11} + \sin \frac{13 \pi a t}{13} - \frac{17 \pi a t}{19} + \cdots \]  

From (15), the conduction-loss equation in 1 s interval is shown in (16)

\[ P_{con} = \frac{2}{7} \int_{0}^{T/2} i_{mc} v_{ce} \, dt = \frac{2}{\pi^2} I_{dc} \left[ 3 R_{on} I_{dc} \left( 1 + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{11^2} + \frac{1}{13^2} + \frac{1}{17^2} + \frac{1}{19^2} + \cdots \right) + 3 V_f \left( 2 - \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{11^2} + \frac{1}{13^2} - \frac{1}{17^2} - \frac{1}{19^2} + \cdots \right) \right] \]  

Figure 10 Turn-on and off characteristics of GTO