LDPC CODED OFDM MODULATION FOR HIGH SPECTRAL EFFICIENCY TRANSMISSION

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***Abstract*– This paper investigates efficient low-density parity-check (LDPC) coded orthogonal frequency division multiplexing (OFDM) modulation schemes for fixed wireless application. We use partially LDPC coded with double gray code labeling technique and Reed-Solomon code with LDPC Coded Modulation (RS-LCM) to achieve better performance than the conventional LDPC bit-interleaved coded modulation (BICM) scheme. RS-LCM scheme outperforms BICM scheme by 0.4 dB at a BER of 10^{-5}.**

***Index Terms*–**Low-density parity-check (LDPC) code, orthogonal frequency division multiplexing (OFDM), coded modulation, LDPC coded modulation (LCM), RS-LCM, bit-interleaved coded modulation (BICM).

I. INTRODUCTION

Low-Density Parity-Check (LDPC) codes are a class of linear block codes. A (N, K) LDPC code is specified by a very sparse parity-check matrix having M rows, N columns and the code rate is $R=K/N$, where $K=N-M$. It was originally invented by Gallager in 1963 [3] and rediscovered by Mackay and Neal recently [4]. LDPC codes can achieve reliable transmission for coding performance that is very close to the Shannon's limit and can outperform Turbo codes at long block length but with relatively low decoding complexity. LDPC has been adopted as the DVB-S2 standard.

Trellis Coded Modulation (TCM) proposed by Ungerbock [1], Multilevel Coding (MLC) proposed by Imai/Hirakawa [2] and Bit-Interleaved Coded Modulation (BICM) introduced by Zehavi [3] are famous coded modulation schemes that can achieve both power and bandwidth efficiency. Most papers investigating the capacity and threshold of LDPC coded modulation focus on 8PSK constellation. This paper investigates the performance of LDPC coded OFDM modulation schemes on higher order QAM modulations.

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carriers modulation technique that has multi-path delay spread tolerance and immunity to frequency selective fading channel. High spectral efficiency and efficient modulation and demodulation by IFFT/FFT are also advantages of OFDM. Therefore, OFDM has emerged as a widely used modulation scheme for next generation high speed wireless transmission applications. For example, DAB, DVB-T, UWB, IEEE 802.11 a/g/n, and 802.16a are all OFDM based communication systems. LDPC code has recently achieved performance approaching Shannon's limit by using iterative decoding. The combination of high spectral efficiency OFDM modulation technique and LDPC code will be a good candidate for high speed broadband wireless applications. For example, IEEE 802.11n proposal takes rate-compatible LDPC code as an advanced coding option, and IEEE 802.16e proposal is based on OFDM modulation and adopt LDPC code as one of its FECs.

Almost all the applications mentioned above adopt LDPC code by directly adding it in front of modulator by using Bit-Interleaved Coded Modulation (BICM) scheme. In this scheme, a bit-interleaver is added after the encoder with Gray labeled constellation to improve code diversity, and hence the reliability of coded modulation. The coded bits are interleaved bit-wise and grouped into blocks of m labeling bits $c_0, c_1, \cdots, c_{m-1}$. The signal point labeled by $c_0, c_1, \cdots, c_{m-1}$ is transmitted through the channel. When LDPC is used as component code in this scheme, the bit-interleaver is not needed. The permutation property of the columns in the parity-check matrix is equivalent to an interleaver. [7]

At the receiver, the encoded bits are decoded using the reliability information generated by the soft demapper from the noisy received signal y_k. The a posteriori probability (APP) of code bit c_l is equal to $x = 0, 1$ as

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$$\Pr(c_l = x \mid y_k) = \frac{\sum_{a_m \in A_{l,x}} e^{-\frac{(y_{k,I}-a_{I,m})^2+(y_{k,Q}-a_{Q,m})^2}{2\sigma_n^2}}}{\sum_{a_m \in A} e^{-\frac{(y_{k,I}-a_{I,m})^2+(y_{k,Q}-a_{Q,m})^2}{2\sigma_n^2}}}, l = 0,1,...,m-1$$

,where **A** = {a_0, a_1, ⋯, a_{M-1}} is a signal set (constellation) of an M = 2^m-ary modulation scheme. After the demaping, these bit probabilities are fed into the LDPC decoder.

Coded modulation schemes when using LDPC codes also gain better performance and achieve more spectral efficiency transmission than the conventional convolutional code with TCM scheme. Therefore, we are interested in LDPC coded modulation schemes and investigated a variety of efficient modulation schemes of that. In this paper, LDPC coded modulation (LCM) and RS-LCM schemes are presented over AWGN channel and multi-path fading channel. We compare our result with a well known coded modulation scheme, BICM at various transmission spectral efficiencies for 256QAM.

The remainder of this paper is organized as follows: In section II, we describe the system model and the coded modulation schemes under study. Section III presents the simulation results and discussion, and shows BER performance improvements achieved by the LCM scheme and RS-LCM scheme as compared to the conventional Gray mapped BICM scheme. Section IV is the conclusion.

II. SYSTEM MODEL

The LDPC coded OFDM system under study is shown in Fig. 1. The OFDM modulation parameters of these coded modulation schemes are based on the IEEE 802.16a-2003 OFDM-256 PHY layer configurations.

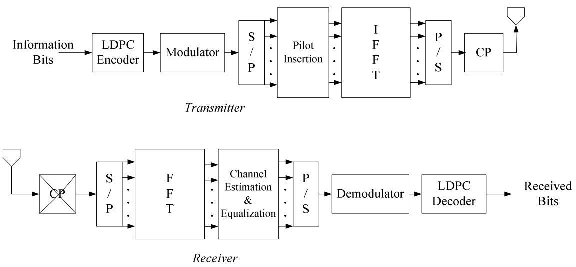

Fig. 1 Block diagram of WiMAX OFDM-256 PHYs

A. LDPC Coded Modulation (LCM) Scheme

Next, we describe a LDPC coded multilevel modulation technique based on set partitioning and double Gray code labeling [6], called LCM. The LCM scheme is used only when square-shape constellations (2^m-QAM, where m=1 or m>1 an even) are employed. The soft demapping at Rx is greatly simplified, because the a posteriori probability (APP) of each bit can be derived independently from the real or imaginary part of the noisy complex signals. The block diagram of partially encoding and symbol mapping is shown in Fig. 2. A transmitted symbol chosen from a 2^m-QAM symbol set can be divided to two binary m/2-tuples ($r_{m/2-1}$, $r_{m/2-2}$,⋯, r_1, r_0) and ($i_{m/2-1}$, $i_{m/2-2}$,⋯, i_1, i_0). The two tuples independently select two L-ary ($L=2^{m/2}$) real symbols, representing the real and imaginary parts of the complex QAM symbol to be transmitted respectively. The L-ary symbols belong to the set **A** = {$A_l = 2l-(L-1)$, l = 0, 1,⋯, L-1}. Each 2^m-QAM symbol conveys C LDPC code bits on its real and imaginary part, respectively; the remaining U bits are uncoded. So there are U uncoded bits and C coded bits in each dimension, where U+C = m/2.

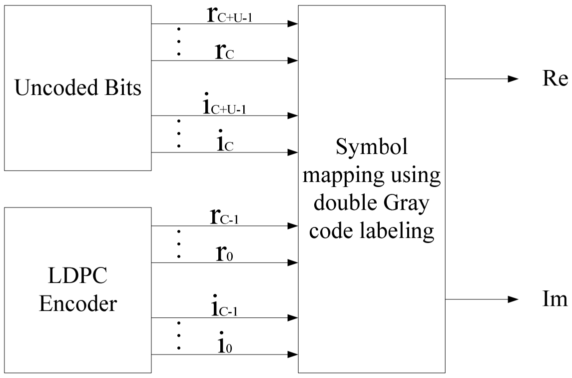

Fig. 2 LDPC coded modulation encoding and symbol mapping
(for 2^m-QAM, where U+C = m/2.)

We adopt 256-QAM constellation with this double Gray-code labeling technique to evaluate the performance of this scheme. Symbol labeling of one of the combinations of C and U for a 2^8-QAM constellation is shown in Table 1. The overall code rate R using LDPC code rate r is $2/m$(U+Cr). For example, $R = 1/4(1+3r)$ for 256-QAM scheme (1 uncoded bit, 3 coded bits for each dimension).

In 2^m-QAM constellation with U uncoded bits and C coded bits, the symbol mapping set **A** is partitioned into 2^C subsets. By this partitioning, the minimum Euclidean between the symbols within the same subset is maximized. The C least-significant bits (LSBs) of each dimension label the subsets of **A** following a Gray-coding rule. The U uncoded most-significant bits (MSBs) label symbols within a subset following a separate Gray-coding rule.

We investigate different combinations of C and U with 256-QAM to evaluate their BER performance. Two combinations of C and U are investigated, which are U=2, C=2 and U=1, C=3. One constellation mapping of these two configurations is listed in Table 1.

Table 1 Symbol labeling
(for the case of 256-QAM (L=16) constellations, U = 2, C = 2)

L-ary symbol A_l	$r_3(i_3)$	$r_2(i_2)$	$r_1(i_1)$	$r_0(i_0)$	Subset no.
+15	1	0	1	0	2
+13	1	0	1	1	3
+11	1	0	0	1	1
+9	1	0	0	0	0
+7	1	1	1	0	2
+5	1	1	1	1	3
+3	1	1	0	1	1
+1	1	1	0	0	0
-1	0	0	1	0	2
-3	0	0	1	1	3
-5	0	0	0	1	1
-7	0	0	0	0	0
-9	0	1	1	0	2
-11	0	1	1	1	3
-13	0	1	0	1	1
-15	0	1	0	0	0

The APP for decoding is also separately produced from the real part and imaginary part of the received signal. Assume y = A + n is the real part of a noisy signal over AWGN channel with n of variance σ_n^2, and A is a transmit symbol in the symbol set **A**. Then the APP of code bit i_m being equal to x = 0, 1 is computed as

$$\Pr(i_m = x \mid y) = \frac{\sum_{A_l \in \mathrm{A}_{m,x}} e^{-\frac{(y-A_l)^2}{2\sigma_n^2}}}{\sum_{A_l \in \mathrm{A}} e^{-\frac{(y-A_l)^2}{2\sigma_n^2}}}, m = 0,1,..., C-1$$

B. Reed-Solomon LDPC Coded Modulation (RS-LCM) Scheme

This RS-LCM scheme is a concatenated coded modulation scheme, which takes an extra Reed-Solomon code as outer code and the LCM scheme as inner code. The LCM scheme suffers error floor at low bit-error rate when low rate codes are used. The outer RS code can eliminate the errors caused by threshold decoding of the uncoded part in the LCM scheme.

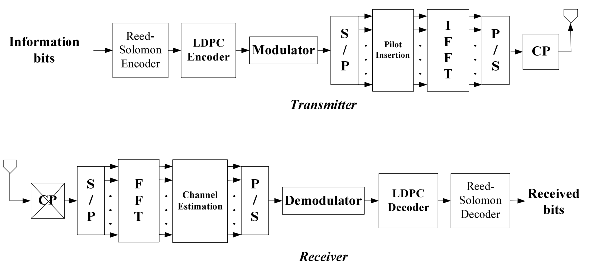

Fig. 3 Reed-Solomon with LDPC-coded modulation (RS-LCM) WiMAX OFDM-256 PHYs

In the Tx end, the information bits are encoded by the outer Reed-Solomon encoder, which is derived from a systematic RS (N=255, K=239, T=8) using symbols from GF (2^8), and the bits are then encoded using the LCM scheme. At the Rx, the inverse operations of the Tx recover the transmitted information bits.

III. Simulation Results and Analysis

In the simulation, the LDPC codes used in all schemes are code length N = 6000 regular LDPC codes. The parity check matrix is generated randomly and the maximum iteration times of decoding are set to 20. Because modern applications required high spectral efficiency transmissions, so we use 256-QAM for investigation. Since 256-QAM has more freedom choices at each dimension of constellation, we can use the LCM scheme either with 1 uncoded bit with 3 coded bits or 2 uncoded bits with 2 coded bits (abbreviated these as LCM1+3 and LCM2+2). Simulations run using various spectral efficiencies of these two configurations are listed in Table 2 and Table 3.

Table 2 – Parameters of LCM2+2 scheme
(for 256-QAM at various spectral efficiencies)

Spectral Efficiency η (bits/s/Hz)	LDPC code (N, K)	LDPC code rate r	Overall code rate R
4.5	(6000, 750)	1/8	9/16
5	(6000,1500)	1/4	5/8
5.5	(6000,2250)	3/8	11/16
6	(6000,3000)	1/2	3/4
6.5	(6000,3750)	5/8	13/16
7	(6000,4500)	3/4	7/8
7.5	(6000,5250)	7/8	15/16

Table 3 – Parameters of LCM1+3 scheme
(for 256-QAM at various spectral efficiencies)

Spectral Efficiency η (bits/s/Hz)	LDPC code (N, K)	LDPC code rate r	Overall code rate R
2.5	(6000,500)	1/12	5/16
3	(6000,1000)	1/6	3/8
3.5	(6000,1500)	1/4	7/16
4	(6000,2000)	1/3	1/2
4.5	(6000,2500)	5/12	9/16
5	(6000,3000)	1/2	5/8
5.5	(6000,3500)	7/12	11/16
6	(6000,4000)	2/3	3/4
6.5	(6000,4500)	3/4	13/16
7	(6000,5000)	5/6	7/8
7.5	(6000,5500)	11/12	15/16

Fig.4 shows the BERs of LCM2+2 scheme with various spectral efficiencies over AWGN channel. We can see that there are about 0.3~0.6 dB coding gains over the BICM scheme. But it has error floor at low rates. Fig.5 also shows that LCM1+3 scheme also has 0.3 dB coding gains over BICM scheme and has error floors at low code rates. However, the error floors occurred in this scheme seems not as serious as in the LCM2+2 scheme. We can explain this phenomenon of error floors by using different LDPC code lengths.

When code length is increasing, the performance of the error floor is unchanged while the waterfall part is improved. Also, the curves of the error floor are almost parallel to the BER curve of the OFDM-256 without coding. Hence, we can conclude that the error floor is caused by the uncoded part and performance is also bounded by the Euclidean distance between the signals in the same subset. The LCM1+3 scheme has more coded bits, and the distance between signals in the same intra set is larger than that of the LCM2+2. This is why the error floors of the LCM1+3 not as serious as in the LCM2+2 scheme. In Fig. 6, we can see that the performance of LCM2+2 is better than LCM1+3 and there are error floors. The LCM2+2 has a lower component LDPC code rate than that of LCM1+3 hence more strong error-correcting capability.

The problem of error floors can be solved by adding an extra Reed-Solomon code before LDPC encoding as a concatenated code scheme i.e. RS-LDPC coded modulation scheme (RS-LCM). Table 4 lists the parameters of the RS-LCM scheme we simulated. Some BER curves are shown in Fig. 7. The RS code eliminates the error floor and has the same coding gain over BICM as LCM.

Table 4– Parameters of Reed-Solomon code with LDPC coded modulation (RS-LCM) scheme using 256-QAM

Scheme	RS Code (N, K, t)	LDPC Code (N, K)
RS-LCM1+3	(60, 48, 6)	(6000, 500)
RS-LCM1+3	(72, 64, 4)	(6000, 1000)
RS-LCM2+2	(108, 96, 6)	(6000, 750)

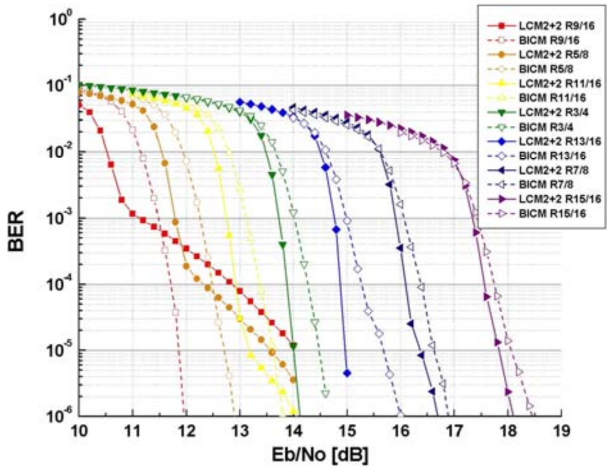

Fig. 4 BER Performance of LCM2+2 scheme versus BICM scheme (for various spectral efficiencies with 256-QAM in AWGN channel)

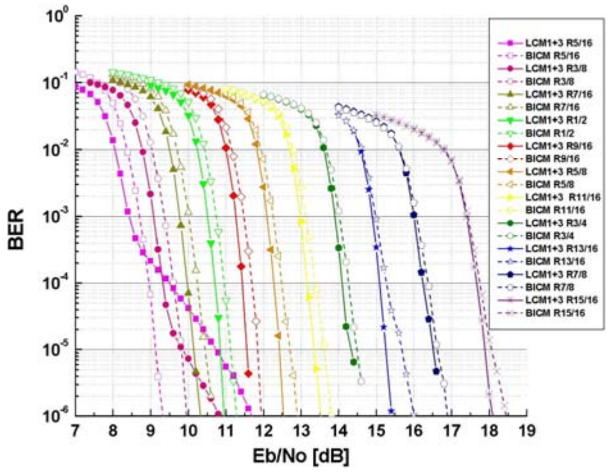

Fig. 5 BER Performance of LCM1+3 scheme versus BICM scheme (for various spectral efficiencies with 256-QAM in AWGN channel)

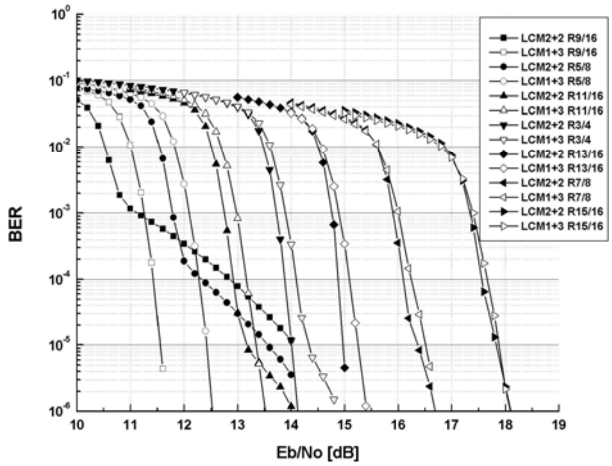

Fig. 6 Performance of LCM1+3 scheme versus LCM2+2 scheme (for various spectral efficiencies with 256-QAM in AWGN channel)

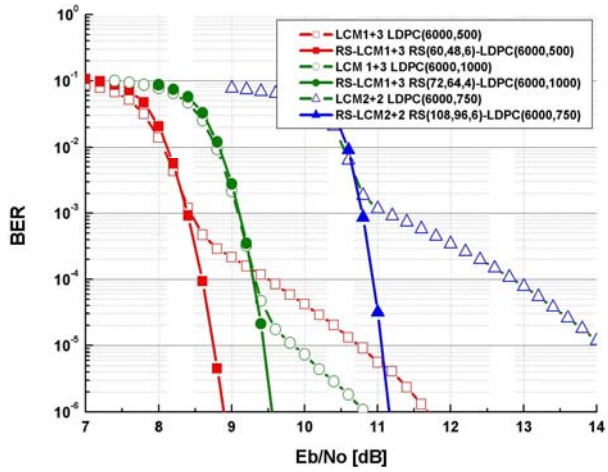

Fig. 7 Performance of RS-LCM versus LCM over AWGN channel with 256-QAM

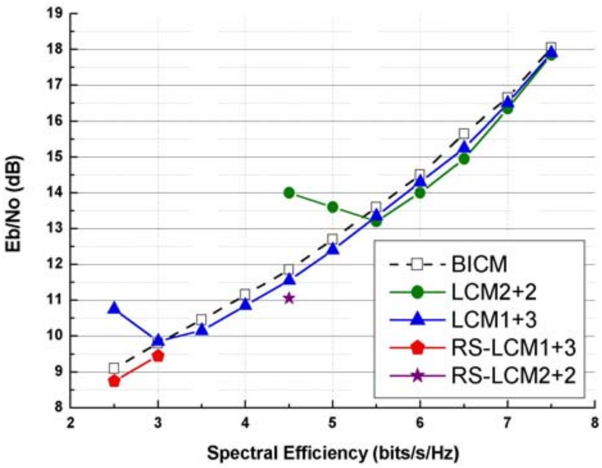

Fig. 8 Different LDPC coded modulation schemes comparison at BER 10^{-5} over AWGN channel

IV. CONCLUSION

This paper investigates the performance of efficient LDPC coded OFDM modulation schemes with 256-QAM. We investigate different coded modulation schemes, including LCM, RS-LCM, and BICM schemes. LCM scheme has 0.3~0.4 dB coding gains over conventional BICM scheme. The SNR versus spectral efficiency at a BER of 10^{-5} of these schemes is summarized in Fig. 8. The computation complexity of the likelihood ratio of LCM scheme is greatly simplified compared to that of BICM scheme. LCM2+2 scheme has better performance then LCM1+3 scheme. But there are error floors at low BER when low rates component LDPC codes are used, and the error floor of LCM1+3 scheme is not as serious as LCM2+2. RS-LCM scheme has same coding gains as LCM scheme and does not have error floors at low BERs.

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