A study on an anti-lock braking system controller and rear-wheel controller to enhance vehicle lateral stability

Jeonghoon Song¹, Heungseob Kim², and Kwangsuck Boo²

¹Department of Mechatronics Engineering, Tongmyong University, Busan, Republic of Korea
²Innovation Centre for Automobile Parts, School of Mechanical-Automotive Engineering, Inje University of Technology, Malaysia

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Abstract: This paper presents a mathematical vehicle model that is designed to analyse and improve the dynamic performance of a vehicle. A wheel slip controller for anti-lock braking system (ABS) brakes is formulated using a sliding mode controller and a proportional–integral–derivative (PID) controller for rear wheel steering is also designed to enhance the stability, steerability, and driveability of the vehicle during transient manoeuvres. The braking and steering performances of controllers are evaluated for various driving conditions, such as straight and J-turn manoeuvres. The simulation results show that the proposed full car model is sufficient to predict vehicle responses accurately. The developed ABS reduces the stopping distance and increases the longitudinal and lateral stability of both two- and four-wheel steering vehicles. The results also demonstrate that the use of a rear wheel controller as a yaw motion controller can increase its lateral stability and reduce the slip angle at high speeds.

Keywords: vehicle model, anti-lock braking system (ABS), sliding mode wheel slip control, PID rear wheel control, yaw motion controller (YMC), four-wheel steering (4WS), two-wheel steering (2WS)

1 INTRODUCTION

The primary objective of anti-lock braking systems (ABSs), which were first implemented in vehicles in the late 1970s, is to prevent the wheels from locking while braking. Most ABS controllers on the market are based on tables and relay feedbacks, and make use of hydraulic actuators to deliver the braking force. One approach that is used with hydraulic brakes is to measure wheel rotational velocity and use this to compute wheel deceleration. Then, given prescribed thresholds for wheel deceleration, the braking pressure is increased, held, or decreased while trying to maintain a wheel slip value that is close to the point that gives the maximum amount of friction [1, 2].

Many researchers have developed numerous control strategies to improve the performance of ABSs, such as the sliding mode controller. Kazemi et al. designed a wheel angular acceleration feedback controller [3] and Drakunov et al. formulated ABS control methods by estimating the optimal friction force [4]. These methods do not require any prior knowledge of the optimal wheel slip ratio. Thus, several studies have obtained satisfactory results for longitudinal manoeuvres during braking. However, satisfactory performance is not obtained for lateral motion, indicating that more extensive research is required to guarantee vehicle stability. Bang et al. developed a yaw controller that controls the yawing motion by generating additional braking pressure on the outer front wheel when the vehicle is in a turning manoeuvre [5]. Ikushima and Sawase proposed a brake torque distribution controller to enhance lateral motion [6]. However, these control strategies can negatively affect the driveability and steerability of the vehicle, especially on slippery roads.

The present study proposes a new yaw motion controller (YMC) that controls the steering angle of the rear wheels to improve lateral stability of a
vehicle. Various electronic stability program (ESP) or vehicle dynamic control (VDC) systems have been proposed, but they are brake torque control systems. Recent developments and research show that four-wheel steering (4WS) systems can effectively improve the handling behaviour of vehicles [7, 8]. Such systems are easy to combine with ABSs and traction control systems (TCSs) to improve their performance. However, the YMC system proposed in this study is different from a 4WS system. A typical 4WS system is developed to improve handling performance and calculates the steer angle of the rear wheels by using the vehicle velocity and the steer angle of the front wheels. Thus the rear wheels are steered always when front wheel steering input is applied and the vehicle runs regardless of the driving conditions. In the case of YMC, this has been developed to improve the lateral stability and controls the steering angle based on the reference yaw rate. Thus YMC eliminates unnecessary steering input and the unfamiliar handling feeling of 4WS.

The current research has three aims: to develop a model that can be used to predict vehicle dynamics for steering and braking manoeuvres; to propose control strategies to obtain higher longitudinal and lateral stabilities; and to investigate the dynamic characteristics of YMC vehicles using a mathematical model and a control algorithm.

2 VEHICLE DYNAMIC MODEL

In general, modelling a vehicle requires that the effects of the powertrain, suspension, tyres, chassis, road conditions, and so on are taken into consideration (Fig. 1). In order to develop such a model, the dynamics of the vehicle must be defined by describing how the vehicle responds to given force inputs [1, 2].

2.1 Modelling assumptions

The modelling assumptions will be clearly defined before generating the equations of motion for the vehicle. One of the most important assumptions made in this study is that the vehicle is travelling on a smooth road, regardless of whether it is fully or partly covered by asphalt, ice, etc., and thus there are no vertical wheel motions. Under this assumption, the vehicle is never airborne and the four tyres remain in contact with the ground at all times.

The next assumption is that the angle of the roll axis relative to the horizontal is negligible, so that the roll and longitudinal axes are essentially the same. This is a reasonable assumption for the majority of vehicles. In the Ford Taurus, for example, the roll axis has an angle of 0.43 degrees relative to the x axis [2].

The other assumption made in this study is that the centre of gravity (CoG) is located laterally at 17.5 mm to the left of centre. According to reference [2], the CoG of the Ford Taurus with a driver plus measuring instrumentation is only 17.5 mm to the left of centre.

2.2 Equations of motion

When modelling a vehicle, it is convenient to use a reference coordinate frame attached to and moving with the vehicle. This is because the moments of inertia of the vehicle are constant with respect to such a coordinate frame. In this study, a Society of Automobile Engineers (SAE) standard coordinate system is used, in which the positive x axis points...
forward, the positive $y$ axis points to the right, and the positive $z$ axis points down [9]. Positive rotations are determined by the right-hand rule for these axes. Figure 2 defines the axis system and necessary degrees of freedom to accommodate Fig. 1, and Table 1 shows the vehicle and controller design parameters.

### 2.2.1 Longitudinal and lateral dynamics

Longitudinal dynamics involves the speed, longitudinal acceleration, longitudinal aerodynamic drag, and pitching motions of the vehicle. Lateral dynamics involves the yawing, rolling, and lateral acceleration motions. These are dominated by the tyre forces, which depend on the longitudinal slip, the lateral slip angle, camber angle, normal load, and so on [1].

The movement of a vehicle in the longitudinal and lateral directions when steering and braking manoeuvres are performed is described by

$$m_{\text{total}}(v_x - v_y) = \sum_{i=1}^{4} F_{x_i} - F_D \cos \beta_i$$

(1)

$$m_{\text{total}}(v_y + v_x) = \sum_{i=1}^{4} F_{y_i} - F_D \sin \beta_i$$

(2)

where $F_D$ is the drag force which opposes the vehicle. The values of $i = 1, 2, 3, \text{ and } 4$ in equations (1) and (2) represent the front left, front right, rear left, and rear right wheels, respectively. The forces generated at each wheel are calculated as

$$F_{x_i} = (F_{x_i} - F_{n_i}) \cos \delta_i - F_{y_i} \sin \delta_i$$

(3)

$$F_{y_i} = (F_{y_i} - F_{n_i}) \sin \delta_i - F_{x_i} \cos \delta_i$$

(4)

Here, $\delta$ is a steering angle of the wheel, $F_n$ is a rolling resistance force that is calculated from the friction coefficient $\mu$ and normal force $F_z$ of each wheel.

### 2.2.2 Yawing movement

If $I_z$ is the moment of inertia of the entire vehicle referenced about the $z$ axis then, the total torque $\Gamma$ acting on the vehicle in the yawing plane about the $z$ axis is calculated as (see Fig. 2)

$$\Gamma = I_z \frac{d^2 \gamma}{dt^2} = a F Y_1 + \frac{t_r}{2} F X_1 + a F Y_2 - \frac{t_r}{2} F X_2 - b F Y_3$$

$$+ \frac{t_r}{2} F X_3 - b F Y_4 = \frac{t_r}{2} F X_4 + M_{str} + M_{str} + M_{str}$$

(5)

where $a$ and $b$ are the distance from CoG to the front wheel and rear wheel, and $t_l$ and $t_r$ are the front and rear wheel distances. The first eight terms are the influence of the tyre forces, the second four terms represent the tyre aligning moments, and the final term is the aerodynamic yawing moment.

### 2.2.3 Rolling movement

$I_{roll}$ denotes the moment of inertia of the vehicle about its roll axis, and $B_{roll}$ and $K_{roll}$ are the roll damping and roll stiffness constants, respectively.

### Table 1  Vehicle and controller design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1.203 m</td>
</tr>
<tr>
<td>$b$</td>
<td>1.217 m</td>
</tr>
<tr>
<td>$t_l$</td>
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<tr>
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<td>$R_u$</td>
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<tr>
<td>$K_f$</td>
<td>19980.0 N/rad</td>
</tr>
<tr>
<td>$K_r$</td>
<td>25020.0 N/rad</td>
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<tr>
<td>$I_z$</td>
<td>1627.0 kg m$^2$</td>
</tr>
<tr>
<td>$h_f$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$h_r$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$h_s$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$h_d$</td>
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<tr>
<td>$m_{sd}$</td>
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<tr>
<td>$m_{sr}$</td>
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<td>$C_{sd}$</td>
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<td>$C_{sr}$</td>
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</tr>
<tr>
<td>$A_d$</td>
<td>0.0013 m$^2$</td>
</tr>
<tr>
<td>$K_d$</td>
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</tr>
<tr>
<td>$K_s$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Fig. 2** Vehicle model
For a small roll angle, \( \phi \)
\[
B_{\text{roll}} = \frac{1}{2} (B_1 t_1^2 + B_2 t_2^2) \tag{6}
\]
\[
K_{\text{roll}} = \frac{1}{2} (K_1 t_1^2 + K_2 t_2^2) \tag{7}
\]
where \( B_1 \) and \( B_2 \) are the damping of the vehicle front and rear, respectively, while \( K_1 \) and \( K_2 \) are the vehicle front and rear spring rates, respectively. Then, using Fig. 3
\[
I_{\text{roll}} \ddot{\phi} + B_{\text{roll}} \dot{\phi} + K_{\text{roll}} \phi = m_s g h_s \sin \phi - m_s (\ddot{y} + \dot{x} \dot{y}) h_s \cos \phi \tag{8}
\]
where \( m_s \) is the sprung mass and \( h_s \) is the distance from sprung mass CoG to roll axis.

2.2.4 Vertical dynamics

The normal load present at each tyre is dependent on the stiffness of the tyre and the amount of tyre deflection when weight is applied. The normal forces acting on both the front and rear tyres have two components: the longitudinal or lateral weight transfer owing to braking or acceleration or steering, and a component owing to the static distribution of the masses [10].

The equations that model the normal forces acting at the front and rear tyres are developed in a quasi-static manner in this study. The lateral weight transfers are modelled using three components: the lateral weight transfer owing to roll (\( F_{\text{rr}}, F_{\text{fr}} \)), owing to the height of the roll centre (\( F_{\text{rf}}, F_{\text{fr}} \)), and owing to the unsprung weight (\( F_{\text{ut}}, F_{\text{ur}} \)). The longitudinal weight transfer owing to braking at the front and rear tyres is considered through the \( F_{\text{uf}}, F_{\text{ur}} \) terms. The model also includes the normal force acting on each tyre owing to the static distribution of the masses (\( F_{\text{mf}}, F_{\text{mr}} \)).

The total normal forces acting on the front and left tyres are
\[
F_{xf} = F_{frr} + F_{sfr} + F_{utf} + 0.5F_{bfr} + 0.5F_{mf} \tag{10}
\]
\[
F_{xf} = (a_x \cos \phi + g \sin \phi) \frac{K_{h} m_s}{K_{r} + K_{f}} + \frac{m_b h_f a_y}{I_{f}(a + b)} + \frac{1}{2} \frac{m_{\text{total}} g}{a + b} \tag{9}
\]
where \( a_x \) and \( a_y \) are acceleration in the \( x \) and \( y \) directions, \( K_{h} = (K_1 t_1^2/2) \) and \( K_{f} = (K_2 t_2^2/2) \) are the front and rear roll stiffness coefficients, \( m_{\text{uf}} \) and \( m_{\text{ur}} \) are front and rear unsprung masses, \( m_{\text{total}} \) is the total vehicle mass, and \( h_f \) and \( h_r \) are the height of front and rear unsprung mass CoGs. Similarly, the total forces acting on the other tyres are
\[
F_{xz} = -F_{fr} - F_{sfr} - F_{utf} - 0.5F_{bfr} + 0.5F_{mf} \tag{11}
\]
\[
F_{xz} = F_{fr} + F_{sfr} + F_{utf} - 0.5F_{bfr} + 0.5F_{mf} \tag{12}
\]
These equations neglect contributions from the aerodynamic lift and drag. The lift and drag terms contribute to a negligible load shift that is less than 5 per cent of either \( F_{\text{mf}} \) or \( F_{\text{mr}} \) [11].

2.2.5 Tyre model

The forces acting on each tyre are analysed to develop equations for modelling the tyre forces. The lateral and longitudinal forces acting on each tyre are determined from the normal forces acting on each wheel and the tyre slip angles. The tyre slip angle is the angle between the intended direction of motion of the tyre and the direction of travel at the centre of the tyre contact [12]. From this point of view, the slip angle \( \sigma \) is defined as
\[
\sigma_i = \delta_i - \zeta_i \tag{13}
\]
The direction angle of each tyre \( \zeta \) is represented in reference [11]. The longitudinal and lateral forces are calculated using the Dugoff model [13], which is analysed in detail by Song et al. [10] and Maalej et al. [14].

2.2.6 Wheel dynamics model

From Newton’s Second Law, rotational equations of motion for the wheels that consider the rolling resistance can be written as
\[
I_{w} \ddot{\omega}_i = -KP_{bs}A_w R_b - F_{si} R_w - T_{rolli} \tag{14}
\]
where \( I_w \) is the rotating inertia of a wheel, \( \omega \) is the wheel speed, \( A_w \) is the area of the master cylinder, \( R_w \) is the distance from the centre of the wheel to the brake path, \( R_a \) is the wheel radius, and \( T_{rolli} \) is the wheel torque owing to resistance.

The last term is attributed to the rolling resistance. The rolling resistance force exists during rotation, and its direction is against the rotation of the tyre [3]. The effect of this force is considered as a rolling resistance torque if the wheel slides [3]. Therefore, a controller that maintains a suitable braking torque is required to ensure that the stopping distance is as short as possible. In the present study, a sliding mode controller is introduced to control the minimum stopping distance.

From equation (14)

\[
\omega_i = -\frac{1}{I_{wi}}(KP_{bi}A_w R_b + F_{xi} R_w + T_{rolli})
\]

\[
= -(K_i u + \tau_x + \tau_i)
\]

(17)

where \( K_i = KA_w R_b / I_{wi} \), \( \tau_x = F_{xi} R_w / I_{wi} \), \( \tau_i = T_{rolli} / I_{wi} \), and the control input \( u_i = P_{bi} \). The dynamics of \( \tau_x \) and \( \tau_i \) are not known exactly, but they can be estimated as \( \dot{\tau}_x \) and \( \dot{\tau}_i \). The estimation error for \( \tau_x \) and \( \tau_i \) is assumed to be bounded by known values of \( \dot{\tau}_x^* \) and \( \dot{\tau}_i^* \).

In order that the slip of the braking system, \( \lambda_{ai} \), tracks the desired slip ratio, \( \dot{\lambda}_{ai} \), the sliding surface is defined as

\[
S = \left( \frac{d}{dt} + \lambda \right)^{n-1} \lambda
\]

(18)

where \( \lambda = \dot{\lambda}_{ai} - \dot{\lambda}_{ai} \). The derivative of the sliding surface is calculated from equation (17)

\[
\dot{S} = -\dot{\lambda}_{ai} = \frac{R_x}{v_x^2} \left[ -(K_i u + \tau_x + \tau_i) v_x - \omega a_x \right]
\]

(19)

The best approximation \( \dot{u} \) of a continuous control law is

\[
\dot{u} = \frac{1}{v_x K_i} [- (\dot{\tau}_x + \dot{\tau}_i) v_x - \omega a_x]
\]

(20)

In order to satisfy a sliding condition that maintains a zero value for the scalar \( S \),

\[
\frac{1}{2} \frac{d}{dt} S^2 = S \times \dot{S} \leq -\eta |S| \quad (\eta \geq 0)
\]

(21)

If it is defined that

\[
\ddot{u} = \dot{\tau}_x^* + \dot{\tau}_i^* + \eta \frac{s \mathrm{gn}(S)}{K_i}
\]

(22)

then since \( u = \ddot{u} + \ddot{u} \), equation (21) can be rewritten as follows

\[
S \times \dot{S} = S \times \frac{R_x}{v_x^2} \left[ (\dot{\tau}_x + \dot{\tau}_i - \tau_x - \tau_i) v_x 
\right.

\]

\[
- v_x (\dot{\tau}_x^* + \dot{\tau}_i^* + \eta \mathrm{sgn}(S))
\]

(23)

3 CONTROLLER DESIGN

3.1 Sliding mode ABS controller

The stopping distance is supposed to decrease as the braking torque increases. However, the stopping distance can also increase with increased braking torque if the wheel slides [3]. Therefore, a controller that maintains a suitable braking torque is required to ensure that the stopping distance is as short as possible. In the present study, a sliding mode controller is introduced to control the minimum stopping distance.

### 2.2.7 Four-wheel steering input

Steering using both the front and rear wheels can significantly improve the maneuverability of a vehicle. Recent developments and research show that 4WS systems can effectively improve the handling behaviour of vehicles [7, 8]. At low speeds, the vehicle manoeuvres better and passengers tend to feel more relaxed when the steering angles of all of the wheels are in the same direction. Therefore, the most common rear wheel steering law involves turning the front and rear wheels in opposite directions at low speeds and in the same direction at high speeds.

Sano and Furukawa determined the steering angle of the rear wheels from [7]

\[
\frac{\delta_i}{\delta_r} = \frac{-b + [m_{total} a (c_{af} (a + b))] v^2}{a + [m_{total} b (c_{af} (a + b))] v^2}
\]

(15)

where \( \delta_i \) and \( \delta_r \) are the steering angles of the front and rear wheels, \( c_{af} \) and \( c_{ar} \) are the front and rear tyre cornering sti

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From equation (14)

\[
\omega_i = -\frac{1}{I_{wi}} (KP_{bi} A_w R_b + F_{xi} R_w + T_{rolli})
\]

\[
= -(K_i u + \tau_x + \tau_i)
\]

(17)

where \( K_i = KA_w R_b / I_{wi} \), \( \tau_x = F_{xi} R_w / I_{wi} \), \( \tau_i = T_{rolli} / I_{wi} \), and the control input \( u_i = P_{bi} \). The dynamics of \( \tau_x \) and \( \tau_i \) are not known exactly, but they can be estimated as \( \dot{\tau}_x \) and \( \dot{\tau}_i \). The estimation error for \( \tau_x \) and \( \tau_i \) is assumed to be bounded by known values of \( \dot{\tau}_x^* \) and \( \dot{\tau}_i^* \).

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\[
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\]

(18)

where \( \lambda = \dot{\lambda}_{ai} - \dot{\lambda}_{ai} \). The derivative of the sliding surface is calculated from equation (17)

\[
\dot{S} = -\dot{\lambda}_{ai} = \frac{R_x}{v_x^2} \left[ -(K_i u + \tau_x + \tau_i) v_x - \omega a_x \right]
\]

(19)

The best approximation \( \dot{u} \) of a continuous control law is

\[
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In order to satisfy a sliding condition that maintains a zero value for the scalar \( S \),

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If it is defined that

\[
\ddot{u} = \dot{\tau}_x^* + \dot{\tau}_i^* + \eta \frac{s \mathrm{gn}(S)}{K_i}
\]

(22)

then since \( u = \ddot{u} + \ddot{u} \), equation (21) can be rewritten as follows

\[
S \times \dot{S} = S \times \frac{R_x}{v_x^2} \left[ (\dot{\tau}_x + \dot{\tau}_i - \tau_x - \tau_i) v_x 
\right.

\]

\[
- v_x (\dot{\tau}_x^* + \dot{\tau}_i^* + \eta \mathrm{sgn}(S))
\]

(23)
which satisfies the sliding condition of equation (21). Thus, the control input $u$ is obtained as follows

$$u = \frac{1}{v_xK_i} \left\{ -(\hat{\gamma}_x + \hat{\gamma}_r)v_x + \omega \left[ \frac{1}{m_{\text{total}}} \left( \sum_{i=1}^{4} FX_i - F \cos \beta \right) + v_y \right] + \frac{\tau^s + \tau^r + \eta}{K_i} \text{sgn}(S) \right\}$$

(24)

### 3.2 Proportional–integral–derivative (PID) controller for rear wheel steering

Wheel slip controllers were originally developed to prevent the vehicle from losing longitudinal stability while braking. However, they have several problems, which include the fact that they do not produce the yaw rate required for turning, thereby causing the vehicle to spin. These problems make it difficult for the driver to maintain safe control of the vehicle [5]. Thus, in order to enhance the lateral stability, many researchers have proposed yaw motion controllers. There is one control theory that explores the use of a yaw rate feedback control, which calculates a reference yaw rate for improved stability. This technique has been used in torque and braking force distribution control. However, the present study proposes a new yawing motion controller that controls the steering angle of the rear wheels. It can be implemented like a steer-by-wire system. It does not use a shaft from the front steering gear to the rear steering gear or any hydraulic system. An electronic control unit (ECU) system sends control signals to a motor which is connected to the rear steering gear.

The reference yaw rate is [6]

$$\dot{\gamma}_\text{ref} = \frac{v}{1 + Kv^2} \frac{\delta_1}{a + b}$$

(25)

where $K$ is

$$K = \frac{a c_{\text{a}} \mu - b c_{\text{a}} \mu}{(c_{\text{a}} \mu)(c_{\text{a}} \mu)} \frac{m_{\text{total}}}{a + b}$$

(26)

and $\mu$ and $\mu$ are the friction coefficients of the front and rear wheels, respectively. The rear wheel steering angle, $\delta_r$, that minimizes the yaw rate error ($e = \gamma_\text{ref} - \gamma$) between the desired and measured yaw rates is calculated by a digital PID controller, for which

$$\delta_r(k) = \delta_r(k-1) + (K_p + K_i + K_d)e(k)$$

$$- (K_p + 2K_d)e(k-1) + K_d e(k-2)$$

(27)

where $K_p$, $K_i$, and $K_d$ are the proportional, integral, and derivative gains, respectively.

### 4 MODEL VALIDATION – DOUBLE LANE CHANGE

In order to validate the vehicle model, simulation results are compared with those of Smith and Starkey, which includes sufficient data to drive the simulations and gives experimental vehicle responses [11]. The upper left plot shown in Fig. 4 gives a saw tooth steering input for a vehicle travelling at 20 m/s. This condition is similar to a vehicle that is performing a double lane change manoeuvre to avoid traffic congestion. The other plots of Fig. 4 reveal the response of the vehicle displacement, yaw rate, and lateral acceleration. The data used for the model are listed in reference [11].

The differences between the simulated and referenced results arise from the model algorithm and the amount of braking torque applied during the steering manoeuvre. However, based upon this comparison, the model appears to be sufficiently valid for the manoeuvres considered in this study.

### 5 RESULTS AND DISCUSSION

#### 5.1 Controller evaluation

The current study proposes a wheel slip controller and a yaw motion controller. The performances of these controllers are evaluated in this section.

##### 5.1.1 Wheel slip controller

Figures 5 and 6 show a braking manoeuvre during longitudinal driving with an initial speed of 30 m/s (108 km/h) on dry asphalt and on an icy road, respectively. The friction coefficient $\mu$ is obtained from a look-up table that gives a slip ratio versus friction coefficient profile.

The displacement and deceleration results with the sliding mode controller for dry asphalt and a braking torque of 400 N m are given in Fig. 5. This figure indicates that the vehicle displacement between 0 and 4.5 s is reduced by 34 per cent when the proposed ABS controller is employed. When a vehicle without an ABS controller manoeuvres on an icy road and a braking torque of 200 N m is applied, the wheel locked and the vehicle began to slip at 2.0 s, as shown in Fig. 6. Once sliding, the deceleration of the vehicle decreases and the stopping distance increases, as shown in the figure.
5.1.2 Yaw motion controller

A properly implemented 4WS system can result in a vehicle that is more manoeuvrable at low speeds, and more responsive and stable in high-speed transient manoeuvres. The yaw motion controller proposed in the present study aims to enhance the lateral stability of a vehicle; thus, only the high-speed manoeuvre is described.

In order to evaluate the yaw motion controller, the steering input given in Fig. 7 is applied to a vehicle and the responses are examined. A braking input is not applied in this simulation, and the initial speed is 30 m/s.
Figure 9 shows the yaw rates, lateral accelerations, slip angles, and trajectories of the simulated vehicle. The 2WS vehicle without an ABS has the largest turning radius, which indicates that its controllability is the worst. The trajectory of the 2WS vehicle with an ABS shows the smallest turning radius, but the trajectory and the actual direction that the vehicle is heading differed slightly, suggesting that oversteer occurred.

The large yaw rate, lateral acceleration, and slip angle provide further proof of oversteer. The YMC vehicle, which is represented as 4WS with ABS and with PID, has the smallest slip angle, which indicates the highest level of controllability, as the slip angle is defined as the angle between the direction that the wheel travels and the direction of the wheel heading.

5.3 Braking and cornering – high velocity on an icy road

Braking while steering is a very common evasive manoeuvre. Acceleration or braking during cornering can reduce the available lateral forces at the tyres. If this happens at the front wheels, the vehicle understeers. If the rear wheels have insufficient lateral force, they slide outward and the vehicle turns into the corner and oversteers. These situations are more critical when a vehicle is running on a slippery road.

Figure 10 shows the responses and trajectories of a vehicle when braking and steering inputs are applied on a slippery road. The initial conditions and inputs are the same as those used previously, except for the road conditions.

The results for the 2WS vehicle without a wheel slip controller show that it is nearly impossible to steer, as indicated by the trajectories. The small yaw rate and lateral acceleration cause these results. The largest turning radius induces the smallest slip angle, which is not a desirable result.

Fig. 7 Front wheel steering and brake torque input

Fig. 8 Performance of the PID yaw rate controller on (a) dry asphalt and (b) icy road
The performances of the vehicles with and without a yaw rate controller are also depicted in Fig. 10. The yaw rate of the 4WS vehicle follows the reference yaw very well, regardless of whether the vehicle is equipped with an ABS controller. However, the lateral acceleration and slip angle of the 4WS vehicle without an ABS are more aggravated, which increases the amount of oversteer.

According to the explanation by Siahkalroudi and Naraghi, the control input for a typical rear wheel controller is the steering angle of the rear wheel [8]. Thus, it is almost impossible to control both the yaw rate and lateral acceleration of a vehicle completely. When an ABS controller is added to this system, the lateral acceleration becomes stable and the slip angle is reduced because the wheel slip is controlled. Thus, although the ABS controller proposed in this study is not designed to control lateral motion, it can improve lateral motion to some degree when combined with a yaw motion controller. On a very slippery road, a YMC (4WS vehicle with wheel slip and a yaw rate controller) can achieve a good...
response, even though weak oversteering takes place, because of the load transfer from the rear to the front tyres during braking [6, 12].

In order to validate the YMC in detail, a conventional rear wheel controller, represented by equation (16), is employed in the proposed vehicle model and the vehicle response is analysed. Figure 11 shows the performance of the two controllers.

The 4WS vehicle with a conventional rear wheel controller shows a smaller yaw rate and slip angle. The YMC exhibits faster and larger lateral acceleration. The larger lateral acceleration causes larger slip angles but reduces the turning radius. Although the yaw motion controller does not allow the vehicle to achieve a perfect level of performance by reducing the slip angle, it can enhance the vehicle’s response.

The numerical model was validated by comparing the results with a known reference. The model was sufficient to predict accurately the vehicle response. The ABS controller reduces the deceleration distance by about 34 per cent between 0 and 4.5 s when the vehicle is running on dry asphalt and the initial velocity is 30 m/s. The controller also prevents the wheels from locking if the vehicle is on a slippery road when a sudden braking torque is applied.

The yaw rate of a YMC vehicle follows the reference yaw rate very well, which indicates improved stability and controllability of the lateral motion. When the vehicle performs a turning manoeuvre on dry asphalt, the YMC increases the stability and driveability; it also reduces the slip angle and turning radius when a vehicle with an ABS controller is running on an icy road.

6 CONCLUSIONS

A full car dynamic model has been developed in this study. The model has 15 degrees of freedom. A new sliding mode controller for an ABS braking system that is designed to reduce the stopping distance while maintaining the steerability of a vehicle has been presented. The applied hydraulic pressure at each wheel is controlled based on the desired slip, which is tuned for each wheel separately. A rear wheel steer system using a PID control scheme was also designed to control the yawing motion and thereby improve vehicle stability.

REFERENCES


7 Sano, S. and Furukawa, Y. Four wheel steering system with rear wheel steering angle controlled as a function of steering wheel angle. SAE paper 860625, 1986.


APPENDIX

Notation

- $a$: distance from CoG to the front wheel
- $b$: distance from CoG to the rear wheel
- $B_{roll}$: roll axis torsional damping
- $F_D$: drag force
- $F_n$: rolling resistant force
- $F_x$: longitudinal force
- $F_y$: lateral force
- $F_z$: normal force
- $h_s$: distance from sprung mass CoG to roll axis
- $I_{roll}$: vehicle moment of inertia around roll axis
- $I_{roll}$: rotating inertia of a wheel
- $I_z$: vehicle moment of inertia around z axis
- $K_{roll}$: roll axis torsional stiffness
- $P_w$: brake fluid pressure
- $R_b$: distance from centre of wheel to brake path
- $R_w$: wheel radius
- $t_f$: front wheel distance
- $t_r$: rear wheel distance
- $T_{roll}$: wheel torque owing to resistance
- $\beta$: side slip angle
- $\gamma$: yaw angle
- $\Gamma$: torque
- $\delta$: steering angle
- $\lambda_{di}$: desired slip ratio
- $\mu$: friction coefficient
- $\varphi$: roll angle