Chapter 8

ELECTRONIC STABILITY CONTROL

8.1 INTRODUCTION

8.1.1 The functioning of a stability control system

Vehicle stability control systems that prevent vehicles from spinning and drifting out have been developed and recently commercialized by several automotive manufacturers. Such stability control systems are also often referred to as yaw stability control systems or electronic stability control systems.

Figure 8-1 schematically shows the function of a yaw stability control system. In this figure, the lower curve shows the trajectory that the vehicle would follow in response to a steering input from the driver if the road were dry and had a high tire-road friction coefficient. In this case the high friction coefficient is able to provide the lateral force required by the vehicle to negotiate the curved road. If the coefficient of friction were small or if the vehicle speed were too high, then the vehicle would not follow the nominal motion expected by the driver – it would instead travel on a trajectory of larger radius (smaller curvature), as shown in the upper curve of Figure 8-1. The function of the yaw control system is to restore the yaw velocity of the vehicle as much as possible to the nominal motion expected by the driver. If the friction coefficient is very small, it might not be possible to entirely achieve the nominal yaw rate motion that would be achieved by the driver on a high friction coefficient road surface. In this case, the yaw control system might only partially succeed by making the
vehicle’s yaw rate closer to the expected nominal yaw rate, as shown by the middle curve in Figure 8-1.

![Diagram of vehicle yaw control system](image.png)

*Figure 8-1. The functioning of a yaw control system*

The motivation for the development of yaw control systems comes from the fact that the behavior of the vehicle at the limits of adhesion is quite different from its nominal behavior. At the limits of adhesion, the slip angle is high and the sensitivity of yaw moment to changes in steering angle becomes highly reduced. At large slip angles, changing the steering angle produces very little change in the yaw rate of the vehicle. This is very different from the yaw rate behavior at low frequencies. On dry roads, vehicle maneuverability is lost at vehicle slip angles greater than ten degrees, while on packed snow, vehicle maneuverability is lost at slip angles as low as 4 degrees (Van Zanten, et. al., 1996).

Due to the above change of vehicle behavior, drivers find it difficult to drive at the limits of physical adhesion between the tires and the road (Forster, 1991, Van Zanten, et. al., 1996). First, the driver is often not able to recognize the friction coefficient change and has no idea of the vehicle’s stability margin. Further, if the limit of adhesion is reached and the vehicle skids, the driver is caught by surprise and very often reacts in a wrong way and usually steers too much. Third, due to other traffic on the road, it is
8. Electronic stability control

important to minimize the need for the driver to act thoughtfully. The yaw control system addresses these issues by reducing the deviation of the vehicle behavior from its normal behavior on dry roads and by preventing the vehicle slip angle from becoming large.

8.1.2 Systems developed by automotive manufacturers

Many companies have investigated and developed yaw control systems during the last ten years through simulations and on prototype experimental vehicles. Some of these yaw control systems have also been commercialized on production vehicles. Examples include the BMW DSC3 (Leffler, et. al., 1998) and the Mercedes ESP, which were introduced in 1995, the Cadillac Stabilitrak system (Jost, 1996) introduced in 1996 and the Chevrolet C5 Corvette Active Handling system in 1997 (Hoffman, et. al., 1998).

Automotive manufacturers have used a variety of different names for yaw stability control systems. These names include VSA (vehicle stability assist), VDC (vehicle dynamics control), VSC (vehicle stability control), ESP (electronic stability program), ESC (electronic stability control) and DYC (direct yaw control).

8.1.3 Types of stability control systems

Three types of stability control systems have been proposed and developed for yaw control:
1) **Differential Braking** systems which utilize the ABS brake system on the vehicle to apply differential braking between the right and left wheels to control yaw moment.
2) **Steer-by-Wire** systems which modify the driver’s steering angle input and add a correction steering angle to the wheels
3) **Active Torque Distribution** systems which utilize active differentials and all wheel drive technology to independently control the drive torque distributed to each wheel and thus provide active control of both traction and yaw moment.

By large, the differential braking systems have received the most attention from researchers and have been implemented on several production vehicles. Steer-by-wire systems have received attention from academic researchers (Ackermann, 1994, Ackermann, 1997). Active torque distribution systems have received attention in the recent past and are likely to become available on production cars in the future.

Differential braking systems are the major focus of coverage in this book. They are discussed in section 8.2. Steer-by-wire systems are discussed in
section 8.3 and active torque distribution systems are discussed in section 8.4.

8.2 DIFFERENTIAL BRAKING SYSTEMS

Differential braking systems typically utilize solenoid based hydraulic modulators to change the brake pressures at the four wheels. Creating differential braking by increasing the brake pressure at the left wheels compared to the right wheels, a counter-clockwise yaw moment is generated. Likewise, increasing the brake pressure at the right wheels compared to the left wheels creates a clockwise yaw moment. The sensor set used by a differential braking system typically consists of four wheel speeds, a yaw rate sensor, a steering angle sensor, a lateral accelerometer and brake pressure sensors.

8.2.1 Vehicle model

The vehicle model used to study a differential braking based yaw stability control system will typically have seven degrees of freedom. The lateral and longitudinal velocities of the vehicle ($\dot{x}$ and $\dot{y}$ respectively) and the yaw rate $\dot{\psi}$ constitute three degrees of freedom related to the vehicle body. The wheel velocities of the four wheels ($\omega_{fl}$, $\omega_{fr}$, $\omega_{rl}$ and $\omega_{rr}$) constitute the other four degrees of freedom. Note that the first subscript in the symbols for the wheel velocities is used to denote front or rear wheel and the second subscript is used to denote left or right wheel. Figure 8-2 shows the seven degrees of freedom of the vehicle model.
Vehicle Body Equations

Let the front wheel steering angle be denoted by $\delta$. Let the longitudinal tire forces at the front left, front right, rear left and rear right tires be given by $F_{xf\ell}$, $F_{xfr}$, $F_{xrl}$ and $F_{xrr}$ respectively. Let the lateral forces at the front left, front right, rear left and rear right tires be denoted by $F_{yf\ell}$, $F_{yfr}$, $F_{yr\ell}$ and $F_{yrr}$ respectively.

Then the equations of motion of the vehicle body are

$$m\ddot{x} = (F_{xf\ell} + F_{xfr}) \cos(\delta) + F_{xrl} + F_{xrr} - (F_{yf\ell} + F_{yfr}) \sin(\delta) + m \dot{\psi} \dot{\psi}$$  \hspace{1cm} (8.1)

$$m\ddot{y} = F_{yr\ell} + F_{yrr} + (F_{xf\ell} + F_{xfr}) \sin(\delta) + (F_{yf\ell} + F_{yfr}) \cos(\delta) - m \dot{\psi} \dot{\psi}$$  \hspace{1cm} (8.2)
Chapter 8

Here the lengths \( \ell_f \), \( \ell_r \) and \( \ell_w \) refer to the longitudinal distance from the c.g. to the front wheels, longitudinal distance from the c.g. to the rear wheels and the lateral distance between left and right wheels (track width) respectively.

**Slip Angle and Slip Ratio**

Define the slip angles at the front and rear tires as follows

\[
\alpha_f = \delta - \frac{\dot{y} + \ell_f \dot{\psi}}{\dot{x}} \quad (8.4)
\]

\[
\alpha_r = \frac{\dot{y} - \ell_r \dot{\psi}}{\dot{x}} \quad (8.5)
\]

Define the longitudinal slip ratios at each of the 4 wheels using the following equations

\[
\sigma_x = \frac{\omega_w - \dot{x}}{\dot{x}} \quad \text{during braking} \quad (8.6)
\]

\[
\sigma_y = \frac{\omega_y}{\omega_w} \quad \text{during acceleration} \quad (8.7)
\]

Let the slip ratios at the front left, front right, rear left and rear right be denoted by \( \sigma_{fl} \), \( \sigma_{fr} \), \( \sigma_{rl} \) and \( \sigma_{rr} \) respectively.
Combined Lateral-Longitudinal Tire Model Equations

The Dugoff tire model discussed in section 13.10 of this book can be utilized for calculation of tire forces. Let the cornering stiffness of each tire be given by $C_\alpha$ and the longitudinal tire stiffness by $C_\sigma$. Then the longitudinal tire force of each tire is given by (Dugoff, et. al., 1969)

$$F_x = C_\sigma \frac{\sigma}{1 + \sigma} f(\lambda)$$  \hspace{1cm} (8.8)

and the lateral tire force is given by

$$F_y = C_\alpha \frac{\tan(\alpha)}{1 + \sigma} f(\lambda)$$  \hspace{1cm} (8.9)

where $\lambda$ is given by

$$\lambda = \frac{\mu F_z (1 + \sigma)}{2 \left[ (C_\sigma \sigma)^2 + (C_\alpha \tan(\alpha))^2 \right]^{1/2}}$$  \hspace{1cm} (8.10)

and

$$f(\lambda) = (2 - \lambda)\lambda \text{ if } \lambda < 1$$  \hspace{1cm} (8.11)

$$f(\lambda) = 1 \text{ if } \lambda \geq 1$$  \hspace{1cm} (8.12)

$F_z$ is the vertical force on the tire while $\mu$ is the tire-road friction coefficient.
Using equations (8.8), (8.9), (8.10), (8.11) and (8.12), the longitudinal tire forces $F_{x f \ell}$, $F_{x f r}$, $F_{x r l}$ and $F_{x r r}$ and the lateral tire forces $F_{y f l}$, $F_{y f r}$, $F_{y r l}$ and $F_{y r r}$ can be calculated. Note that the slip angle and slip ratio of each corresponding wheel must be used in the calculation of the lateral and longitudinal tire forces for that wheel.

**Wheel dynamics**

The rotational dynamics of the 4 wheels are given by the following torque balance equations:

\[
J_w \dot{\omega}_f = T_{df \ell} - T_{bf \ell} - r_{eff} F_{x f \ell} \tag{8.13}\\
J_w \dot{\omega}_{fr} = T_{df r} - T_{bf r} - r_{eff} F_{x f r} \tag{8.14}\\
J_w \dot{\omega}_{rl} = T_{dr l} - T_{br l} - r_{eff} F_{x r l} \tag{8.15}\\
J_w \dot{\omega}_{rr} = T_{drr} - T_{brr} - r_{eff} F_{x r r} \tag{8.16}
\]

Here $T_{df \ell}$, $T_{df r}$, $T_{dr l}$ and $T_{drr}$ refer to the drive torque transmitted to the front left, front right, rear left and rear right wheels respectively and $T_{bf \ell}$, $T_{bf r}$, $T_{br l}$ and $T_{brr}$ refer to the brake torque on the front left, front right, rear left and rear right wheels respectively.

In general, the brake torque at each wheel is a function of the brake pressure at that wheel, the brake area of the wheel $A_w$, the brake friction coefficient $\mu_b$ and the brake radius $R_b$. For instance, the brake torque at the front left wheel $T_{bf \ell}$ is related to the brake pressure at the front left wheel $P_{f \ell}$ through the equation


\[ T_{bf\ell} = A_w \mu_b R_b P_{bf\ell} \]  

(8.17)

Similar equations can be written for the brake pressures \( P_{bfr} \), \( P_{br\ell} \) and \( P_{brr} \) at the front right, rear left and rear right wheels respectively.

### 8.2.2 Control architecture

The control architecture for the yaw stability control system is hierarchical and is shown in Figure 8-3. The upper controller has the objective of ensuring yaw stability control and assumes that it can command any desired value of yaw torque. It uses measurements from wheel speed sensors, a yaw rate sensor, a lateral accelerometer and a steering angle sensor. Using these measurements and a control law to be discussed in the following sub-sections, it computes the desired value of yaw torque. The lower controller ensures that the desired value of yaw torque commanded by
the upper controller is indeed obtained from the differential braking system. The lower controller utilizes the wheel rotational dynamics and controls the braking pressure at each of the 4 wheels to provide the desired yaw torque for the vehicle. The inherent assumption is that the rotational wheel dynamics are faster than the vehicle dynamics.

### 8.2.3 Desired yaw rate

In Chapter 3 (section 3.3), we saw that the steady state steering angle for negotiating a circular road of radius $R$ is given by

$$\delta_{ss} = \frac{\ell_f + \ell_r}{R} + K_V a_y$$

(8.18)

where $K_V$ is the understeer gradient and is given by

$$K_V = \frac{\ell_f m}{2C_{cf} (\ell_f + \ell_r)} - \frac{\ell_f m}{2C_{cr} (\ell_f + \ell_r)}$$

where $C_{cf}$ and $C_{cr}$ are the cornering stiffness for each front and rear tire respectively.

Hence, the steady state relation between steering angle and the radius of the vehicle's trajectory is

$$\delta_{ss} = \frac{\ell_f + \ell_r}{R} + \left(\frac{m\ell_r C_{cr} - m\ell_f C_{cf}}{2C_{cf} C_{cr} (\ell_f + \ell_r)}\right) \frac{V^2}{R}$$

(8.19)

and the radius can be expressed in terms of steering angle as
Here $L = \ell_f + \ell_r$ is used to denote the wheelbase of the vehicle.

The desired yaw rate for the vehicle can therefore be obtained from steering angle, vehicle speed and vehicle parameters as follows

$$\dot{\psi}_{des} = \frac{\dot{x}}{R} = \frac{\dot{x}}{\ell_f + \ell_r + \frac{mV^2(\ell_r C_{ar} - \ell_f C_{af})}{2C_{af} C_{ar} L}} \delta$$

Note that in the above equation, $C_{af}$ and $C_{ar}$ stand for the cornering stiffness of each front and rear tire and it is assumed that there are two tires in the front and two tires in the rear. If the cornering stiffness of the front and rear tires are equal, then $C_{af} = C_{ar} = C_{a}$.

### 8.2.4 Desired side-slip angle

In Chapter 3, we found that the steady state yaw angle error during cornering is

$$e_{2,ss} = -\frac{\ell_r}{R} + \frac{\ell_f}{2C_{ar}(\ell_f + \ell_r)} \frac{mV^2}{R} = -\frac{\ell_r}{R} + \alpha_r$$

and the steady state slip angle of the vehicle is

$$\beta = -e_{2,ss}$$

or

$$\beta = -\frac{\ell_r}{R} + \frac{\ell_f}{2C_{ar}(\ell_f + \ell_r)} \frac{mV^2}{R}$$
The above expression for steady state slip angle is in terms of velocity and road radius. This expression can be rewritten so that the steady state slip angle is expressed in terms of the steady state steering angle.

The steady-state steering angle, from equation (8.19) is

$$
\delta_{ss} = \frac{\ell_f + \ell_r}{R} + \left(\frac{m \ell_r C_{\alpha} - m \ell_f C_f}{2 C_{\alpha} C_{\alpha} (\ell_f + \ell_r)}\right) \frac{V^2}{R}
$$

Hence, the curvature of the road can be expressed as

$$
\frac{1}{R} = \frac{\delta_{ss}}{\ell_f + \ell_r + \frac{mV^2 (\ell_r C_{\alpha} - \ell_f C_{\alpha})}{2 C_{\alpha} C_{\alpha} L}}
$$

Combining equations (8.23) and (8.20), the steady state slip angle is

$$
\beta = \frac{1}{R} \left( \ell_r - \frac{\ell_f}{2 C_{\alpha} (\ell_f + \ell_r)} mV^2 \right)
$$

or

$$
\beta = \frac{\delta_{ss}}{\ell_f + \ell_r + \frac{mV^2 (\ell_r C_{\alpha} - \ell_f C_{\alpha})}{2 C_{\alpha} C_{\alpha} L}} \left( \ell_r - \frac{\ell_f}{2 C_{\alpha} (\ell_f + \ell_r)} mV^2 \right)
$$

which after simplification turns out to be

$$
\beta_{des} = \frac{\ell_r - \frac{\ell_f mV^2}{2 C_{\alpha} (\ell_f + \ell_r)}}{\left(\ell_f + \ell_r\right) + \frac{mV^2 (\ell_r C_{\alpha} - \ell_f C_{\alpha})}{2 C_{\alpha} C_{\alpha} (\ell_f + \ell_r)}} \delta_{ss} \quad (8.24)
$$
Note: The above expression assumed that the cornering stiffness of each front tire is $C_{ef}$ and of each rear tire is $C_{cr}$.

Equation (8.24) describes the desired slip angle as a function of the driver’s steering angle input, the vehicle’s longitudinal velocity and vehicle parameters.

### 8.2.5 Upper bounded values of target yaw rate and slip angle

The desired yaw rate and the desired slip angle described in sections 8.2.3 and 8.2.4 cannot always be obtained. It is not safe, for example, to try and obtain the above desired yaw rate if the friction coefficient of the road is unable to provide tire forces to support a high yaw rate. Hence the desired yaw rate must be bounded by a function of the tire-road friction coefficient.

The lateral acceleration at the center of gravity (c.g.) of the vehicle is given by

$$a_{y_{-}cg} = \dot{x} \psi + \dot{y}$$ (8.25)

Since $\dot{y} = \dot{x} \tan(\beta)$, the lateral acceleration can be related to the yaw rate and the vehicle slip angle by the equation

$$a_{y_{-}cg} = \dot{x} \psi + \tan(\beta) \dot{x} + \frac{\dot{x} \dot{\beta}}{\sqrt{1 + \tan^2 \beta}}$$ (8.26)

The lateral acceleration must be bounded by the tire-road friction coefficient $\mu$ as follows

$$a_{y_{-}cg} \leq \mu g$$ (8.27)

The first term in the calculation of the lateral acceleration in equation (8.26) dominates. If the slip angle of the vehicle and its derivative are both assumed to be small, the second and third terms contribute only a small
fraction of the total lateral acceleration. Hence, combining equations (8.26) and (8.27), the following upper bound can be used for the yaw rate

\[ \psi_{\text{upper\_bound}} = 0.85 \frac{\mu g}{\dot{x}} \]  

(8.28)

The factor 0.85 allows the second and third terms of equation (8.26) to contribute 15% to the total lateral acceleration.

The target yaw rate of the vehicle is therefore taken to be the nominal desired yaw rate defined by equation (8.21) as long as it does not exceed the upper bound defined by equation (8.28):

\[ \psi_{\text{target}} = \psi_{\text{des}} \text{ if } |\dot{\psi}_{\text{des}}| \leq \psi_{\text{upper\_bound}} \]  

(8.29)

\[ \psi_{\text{target}} = \psi_{\text{upper\_bound}} \text{ sgn}(\dot{\psi}_{\text{des}}) \text{ if } |\dot{\psi}_{\text{des}}| > \psi_{\text{upper\_bound}} \]  

(8.30)

The desired slip angle, for a given steering angle and vehicle speed, can be obtained from equation (8.24). The target slip angle must again be upper bounded so as to ensure that the slip angle does not become too large. At high slip angles, the tires lose their linear behavior and approach the limit of adhesion. Hence, it is important to limit the slip angle.

The following empirical relation on an upper bound for the slip angle is suggested

\[ \beta_{\text{upper\_bound}} = \tan^{-1}(0.02 \mu g) \]  

(8.31)

This relation yields an upper bound of 10 degrees at a friction coefficient of \( \mu = 0.9 \) and an upper bound of 4 degrees at a friction coefficient of \( \mu = 0.35 \). This roughly corresponds to the desirable limits on slip angle on dry road and on packed snow respectively.

The target slip angle of the vehicle is therefore taken to be the nominal desired slip angle defined by equation (8.24) as long as it does not exceed the upper bound defined by equation (8.31):

\[ \beta_{\text{target}} = \beta_{\text{des}} \text{ if } |\beta_{\text{des}}| \leq \beta_{\text{upper\_bound}} \]  

(8.32)
8. Electronic stability control

\[ \beta_{\text{target}} = \beta_{\text{upperbound}} \text{sgn}(\beta_{\text{des}}) \text{ if } |\beta_{\text{des}}| > \beta_{\text{upper bound}} \] (8.33)

Several researchers in literature have simply assumed the desired slip angle to be zero and assumed that the upper bound on the yaw rate is given by \( \dot{\psi}_{\text{upper bound}} = \frac{\mu_g}{\dot{x}} \). However, the equations in (8.28) – (8.33) yield a better approximation to the driver-desired target values for both yaw rate and slip angle.

8.2.6 Upper controller design

The objective of the upper controller is to determine the desired yaw torque for the vehicle so as to track the target yaw rate and target slip angle discussed in section 8.2.5.

The sliding mode control design methodology has been used by several researchers to achieve the objectives of tracking yaw rate and slip angle (Drakunov, et. al., 2000, Uematsu and Gerdes, 2002, Yi, et. al., 2003 and Yoshioka, et. al., 1998). A good introduction to the general theory of sliding surface control can be found in the text by Slotine and Li (1991).

The sliding surface is chosen so as to achieve either yaw rate tracking or slip angle tracking or a combination of both. Examples of sliding surfaces that have been used by researchers include the following three

\[ s = \dot{\beta} + \xi \beta \] (8.34)

\[ s = \psi - \psi_{\text{target}} \] (8.35)

\[ s = \psi - \psi_{\text{target}} + \xi \beta \] (8.36)

By ensuring that the vehicle response converges to the surface \( s = 0 \), one ensures that the desired yaw rate and/or slip angle are obtained. A good comparison of the performance obtained with the 3 types of sliding surfaces described above can be found in Uematsu and Gerdes (2002).
This book suggests that the following sliding surface be used for control design:

\[ s = \dot{\psi} - \dot{\psi}_{\text{target}} + \xi (\beta - \beta_{\text{target}}) \]  

(8.37)

This surface is defined as a weighted combination of yaw rate and slip angle errors and takes the target values for yaw rate and slip angle discussed in sections 8.2.3 – 8.2.5 into consideration.

Differentiating equation (8.37)

\[ \dot{s} = \ddot{\psi} - \ddot{\psi}_{\text{target}} + \xi (\ddot{\beta} - \ddot{\beta}_{\text{target}}) \]  

(8.38)

The equation for \( \ddot{\psi} \) can be obtained by rewriting equation (8.3) as

\[ I_c \ddot{\psi} = \ell_f (F_{xfl} + F_{xfr}) \sin(\delta) + \ell_f (F_{yfl} + F_{yfr}) \cos(\delta) - \ell_r (F_{yrfr} + F_{yrr}) \]

\[ + \frac{\ell_w}{2} (F_{xfr} - F_{xfl}) \cos(\delta) + \frac{\ell_w}{2} (F_{xrr} - F_{xrl}) + \frac{\ell_w}{2} (F_{yfr} - F_{yfl}) \sin(\delta) \]  

(8.39)

Ignore the terms \( \ell_f (F_{xfl} + F_{xfr}) \sin(\delta) \) and \( \frac{\ell_w}{2} (F_{yfr} - F_{yfl}) \sin(\delta) \) in equation (8.39), assuming that the steering angle is small. Next, assume that the ratio of front-to-back distribution of brake torques is fixed. Set

\[ F_{xrl} = \rho F_{xfl} \]  

(8.40)

and

\[ F_{xrr} = \rho F_{xfr} \]  

(8.41)

where \( \rho \) is determined by the front-to-back brake proportioning. The front-to-back brake proportioning is determined by a pressure proportioning valve in the hydraulic system. Many pressure proportioning valves provide equal
pressure to both front and rear brakes up to a certain pressure level, and then subsequently reduce the rate of pressure increase to the rear brakes (see Gillespie, 1992).

\[ I_z \ddot{\phi} = \ell_f (F_{yf_l} + F_{yfr}) \cos(\delta) - \ell_r (F_{yr_l} + F_{yrr}) + \frac{\ell_w}{2} (F_{xfr} - F_{xfr}) \cos(\delta) + \rho \frac{\ell_w}{2} (F_{xfr} - F_{xfr}) \]

\[ (8.42) \]

Denote

\[ M_{\psi b} = \frac{\ell_w}{2} (F_{xfr} - F_{xfr}) \]

\[ (8.43) \]

\( M_{\psi b} \) is the yaw torque from differential braking and constitutes the control input for the upper controller.

Then

\[ \ddot{\phi} = \frac{1}{I_z} [\ell_f (F_{yf_l} + F_{yfr}) \cos(\delta) - \ell_r (F_{yr_l} + F_{yrr}) + (\cos(\delta) + \rho) M_{\psi b}] \]

\[ (8.44) \]

Substituting for \( \ddot{\phi} \) in equation (8.38)

\[ \dot{s} = \frac{1}{I_z} [\ell_f (F_{yf_l} + F_{yfr}) \cos(\delta) - \ell_r (F_{yr_l} + F_{yrr}) + (\cos(\delta) + \rho) M_{\psi b}] \]

\[ - \dot{\psi}_{target} + \xi (\dot{\beta} - \dot{\psi}_{target}) \]

\[ (8.45) \]

Setting \( \dot{s} = -\eta s \) yields the control law
\[ \frac{\rho + \cos(\delta)}{I_z} M_{yb} = \left[ \begin{array}{c} - \frac{\ell_f}{I_z} (F_{yf\ell} + F_{yfr}) \cos(\delta) + \frac{\ell_r}{I_z} (F_{yr\ell} + F_{yrr}) \\ - \eta_s + \dddot{\psi}_{i_{\text{target}}} - \xi (\dot{\beta} - \dot{\beta}_{i_{\text{target}}}) \end{array} \right] \]  

(8.46)

The control law described in equation (8.46) above requires feedback of slip angle, slip angle derivative, and front and rear lateral tire forces. These variables cannot be directly measured but must be estimated and used for feedback. Estimation methods in literature use a combination of algorithms based on integration of inertial sensors and dynamic model based observers (Tseng, et. al., 1999, Van Zanten, et. al., 1996, Fukada, 1999 and Ghoeneim, 2000). The use of GPS for estimation of slip ratio and slip angle has also been investigated (Daily and Bevly, 2004, Bevly, et. al., 2001).

### 8.2.7 Lower controller design

The lower controller determines the brake pressure at each wheel, so as to provide a net yaw torque that tracks the desired value for yaw torque determined by the upper controller.

By definition, \( M_{yb} = \frac{\ell_w}{2} (F_{xf\ell} - F_{xf\ell}) \). Hence, the extra differential longitudinal tire force needed to produce the desired yaw torque can be obtained as

\[ \Delta F_{xf} = \frac{2M_{yb}}{\ell_w} \]  

(8.47)

Consider the dynamics of the front left and front right wheels

\[ J_w \dot{\omega}_{f\ell} = T_{dff\ell} - A_w \mu_b R_b P_{b\ell f} - r_{eff} F_{xf\ell} \]  

(8.48)

\[ J_w \dot{\omega}_{fr} = T_{dfr} - A_w \mu_b R_b P_{bf r} - r_{eff} F_{xf r} \]  

(8.49)
The drive torque variables $T_{dfe}$ and $T_{dfr}$ are determined by the driver throttle input or by a combination of the driver throttle input and a traction control system. The brake pressures $P_{bfe}$ and $P_{bfr}$ are determined from the braking input of the driver and the additional brake required to provide the differential braking torque for vehicle yaw control.

By inspection of equations (8.48) and (8.49), it can be seen that the desired differential longitudinal tire force $\Delta F_{xf}$ at the front tires can be obtained by choosing the brake pressures at the front left and right tires as follows:

\[ P_{bfe} = P_0 - a \frac{\Delta F_{xf} r_{eff}}{A_w \mu_b R_b} \]  
(8.50)

\[ P_{bfr} = P_0 + (1 - a) \frac{\Delta F_{xf} r_{eff}}{A_w \mu_b R_b} \]  
(8.51)

where $P_0$ is the measured brake pressure at the wheel at the time that differential braking is first initiated and the constant $a$ has to be chosen such that $0 \leq a \leq 1$ and $P_{bfe}$ and $P_{bfr}$ are both positive. The brake pressure at each wheel should be zero or positive. Hence, in the case where the driver is not braking, $\Delta F_{xf}$ is positive, and $P_0 = 0$, then $a$ has to be chosen to be zero. On the other hand, if the driver is braking and $P_0$ is adequately large, then $a$ could be chosen to be 0.5. This would mean that the differential braking torque is obtained by increasing the brake pressure at one wheel and decreasing the brake pressure at the other wheel compared to the driver applied values. Thus $a$ must be chosen in real-time based on the measured value of $P_0$. 
8.3 STEER-BY-WIRE SYSTEMS

8.3.1 Introduction

In the use of a steer-by-wire system for yaw stability control, the front wheel steering angle is determined as a sum of two components. One component is determined directly by the driver from his/her steering wheel angle input. The other component is decided by the steer-by-wire controller, as shown in Figure 8-4. In other words, the steer-by-wire controller modifies the driver’s steering command so as to ensure “skid prevention” or “skid control”. This must be done in such a way that it does not interfere with the vehicle’s response in following the path desired by the driver.

Significant work on the design of steer-by-wire systems for vehicle stability control has been documented by Ackermann and co-workers (Ackermann, 1997, Ackermann, 1994). The following sub-sections summarize the steer-by-wire control system for front-wheel steered vehicles designed by Ackermann (1997).

![Figure 8-4. Structure of steer-by-wire stability control system](image-url)
8. Electronic stability control

8.3.2 Choice of output for decoupling

As described in Ackermann (1997), the driver's primary task is "path following". In path following the driver keeps the car — considered as a single point mass $m$ — on her desired path, as shown in Figure 8-5. She does this by applying a desired lateral acceleration $a_{yp}$ to the mass $m$ in order to re-orient the velocity vector of the vehicle so that it remains tangential to her desired path.

![Figure 8-5. The path following task of the driver](image)

The driver has a secondary task of "disturbance attenuation." This task results from the fact that the vehicle is not really a point mass but has a second degree of freedom which is the yaw motion of the vehicle. Let the yaw moment of inertia of the vehicle be $I_z$. The yaw rate of the car is excited not only by the driver desired lateral acceleration $a_{yp}$ but also by a disturbance torque $M_{zd}$. The yaw rate excited by the lateral acceleration $a_{yp}$ is expected by the driver and she is used to this yaw rate. However, disturbances such as a flat tire and asymmetric friction coefficients at the left and right wheels induce a disturbance torque $M_{zd}$ which excite a yaw motion that the driver does not expect.

Usually, the driver has to compensate for the disturbance torque by using the steering wheel. This is a difficult task for the driver due to the fact that she is not used to counteracting for such disturbances and also due to the fact that she does not have a measure of the disturbances that cause the
unexpected yaw and therefore her reaction is likely to be delayed. It often takes time for the driver to recognize the situation and the need for her special intervention.

In Ackermann (1997), the steer-by-wire electronic stability control (ESC) system is designed to perform this task of disturbance attenuation so that the driver can concentrate on her primary task of path following. For this it is necessary to decouple the secondary disturbance attenuation dynamics such that they do not influence the primary path following dynamics. The automatic control system for the yaw rate \( \dot{\psi} \) should not interfere with the path following task of the driver. In control system terms, this means the yaw rate \( \dot{\psi} \) should be unobservable from the lateral acceleration \( a_{yp} \). The yaw rate dynamics will continue to depend on the lateral acceleration \( a_{yp} \). Only then can the driver control the car to follow a path, since the vehicle must have a yaw rate to follow a path. However, the yaw rate is commanded only indirectly by the driver via \( a_{yp} \). Nominally the driver is concerned directly only with \( a_{yp} \). But any yaw rate induced by the disturbance attenuation automatic steering control system should be such that it does not affect the lateral acceleration \( a_{yp} \). This decoupling has to be done in a robust manner. In particular, it must be robust with respect to vehicle velocity and road surface conditions.

From the above discussion, the motivation for removing the influence of yaw rate on lateral acceleration is clear. The next question to be answered is “At which point of the vehicle should the lateral acceleration be used as the output?” The lateral acceleration at any point \( P \) on the vehicle is given by

\[
a_{yp} = a_{y\_cg} + \ell_p \psi
\]

(8.52)

where \( a_{y\_cg} \) is the lateral acceleration at the c.g. of the vehicle and \( \ell_p \) is the longitudinal distance of the point \( P \) ahead of the c.g. of the vehicle. Since \( a_{y\_cg} = \frac{F_{yf} + F_{yr}}{m} \), we have
Choose the output position as

\[
\ell_p = \frac{I_z}{m\ell_r} \quad (8.55)
\]

This choice of the lateral acceleration output position ensures that the acceleration is independent of the rear lateral tire force \(F_{yr}\). Thus the uncertainties associated with some of the tire forces on decoupling are removed and more robust decoupling can be achieved.

Substituting from equation (8.55) into equation (8.54)

\[
a_{yp} = F_{sf} \left( \frac{1}{m} + \frac{\ell_f}{m\ell_r} \right)
\]

\[
a_{yp} = F_{sf} \left( \frac{\ell_r + \ell_f}{m\ell_r} \right) = \frac{L}{m\ell_r} F_{sf} \quad (8.56)
\]
8.3.3 Controller Design

The total steering angle is given by

\[ \delta = \delta_{\text{driver}} + \delta_{\text{sbw}} \]  \hspace{1cm} (8.57)

where \( \delta_{\text{driver}} \) is the steering angle input of the driver and \( \delta_{\text{sbw}} \) is the steering angle input of the disturbance attenuation control system.

First, note that the lateral force at the front tire depends on the slip angle at the front wheels. Hence

\[ a_{yp}(\alpha_f) = \frac{L}{m\ell_r} F_{yf}(\alpha_f) \]  \hspace{1cm} (8.58)

Hence the yaw rate \( \psi \) does not influence \( a_{yp} \) if and only if \( \psi \) does not influence \( \alpha_f \). Hence the controller should be designed such that the front tire slip angle does not depend on the yaw rate.

Let the vehicle velocity angle at the front tires be \( \theta_{vf} \). This is the angle between the longitudinal axis of the vehicle and the velocity vector at the front wheels. Then

\[ \alpha_f = \delta_{\text{driver}} + \delta_{\text{sbw}} - \theta_{vf} \]  \hspace{1cm} (8.59)

There is no easy way to measure \( \theta_{vf} \). Otherwise the control law could be chosen as \( \delta_{\text{sbw}} = \theta_{vf} \). That would ensure that the slip angle did not depend on the yaw rate. It would depend only on the driver commanded front wheel steering angle and would not depend on any other state variables.
8. Electronic stability control

The state equation for $\theta_{vf}$ is (Ackermann, 1994)

$$\dot{\theta}_{vf} = -\dot{\psi} + \frac{\cos^2(\theta_{vf})}{V_x} a_{yp}(\alpha_f) + g(\psi)$$

(8.60)

where

$$g(\psi) = \frac{\cos(\theta_{vf})}{V_x} \left[ (\ell_f - \ell_p) \dot{\psi} \cos(\theta_{vf}) + \left( \ell_f \psi^2 - a_x \right) \sin(\theta_{vf}) \right]$$

(8.61)

where $a_x$ is longitudinal acceleration and could be measured by an accelerometer.

Differentiating equation (8.59)

$$\alpha_f = \dot{\delta}_{driver} + \dot{\delta}_{sbw} - \dot{\theta}_{vf}$$

(8.62)

Substituting from equation (8.60) into equation (8.62), it is clear that if the control law is chosen as

$$\dot{\delta}_{sbw} = -\dot{\psi} + g(\psi) + F(\delta_{driver})$$

(8.63)

then the slip angle dynamics at the front tires would be

$$\dot{\alpha}_f = -\frac{\cos^2(\theta_{vf})}{V_x} a_{yp}(\alpha_f) + \dot{\delta}_{driver} + F(\delta_{driver})$$

(8.64)

Here $F(\delta_{driver})$ is chosen as a function of the driver input only and can be interpreted as the desired yaw rate corresponding to the driver's steering angle input $\delta_{driver}$. Thus the error in yaw rate $F(\delta_{driver}) - \dot{\psi}$ is used as a
feedback term in the calculation of the steer-by-wire correction $\delta_{sbw}$ in equation (8.63).

The assumption of a small velocity angle at the front tire leads to

$$\dot{\alpha}_f = -\frac{L}{m\ell_r V_x} F_{yf}(\alpha_f) + \dot{\delta}_{\text{driver}} + F(\delta_{\text{driver}})$$  \hspace{1cm} (8.65)

Thus the front wheel slip angle dynamics depend only on the external driver commanded steering input $\delta_{\text{driver}}$ and do not depend on the yaw rate $\dot{\psi}$. As we have seen, this also implies that the lateral acceleration $a_{yp}$ does not depend on the yaw rate $\dot{\psi}$.

One question that remains to be addressed is stability of the overall system. Decoupling does not automatically ensure stability. However, using the Lyapunov function $V = \alpha_f^2$ and the fact that

$$\alpha_f F_{yf}(\alpha_f) > 0$$  \hspace{1cm} (8.66)

it can be shown that the $\alpha_f$ sub-system is stable when $\delta_{\text{driver}} = 0$. It also turns out that the decoupled yaw sub-system is stable (Ackermann, 1994).

Further practical implementation issues and simplifications of the controller are discussed in Ackermann (1997). Experimental results are presented in Ackermann (1994) and Ackermann (1997).
8. Electronic stability control

8.4 INDEPENDENT ALL WHEEL DRIVE TORQUE DISTRIBUTION

8.4.1 Traditional four wheel drive systems

If the differential braking based yaw stability control system is used during vehicle acceleration, it reduces the acceleration of the vehicle and therefore may not provide the longitudinal response the driver needs. A solution to this problem that is being actively investigated and developed in the automotive industry is the use of independent drive torque control with all wheel drive technology to enhance both traction and handling (Sawase and Sano, 1999, Osborn and Shim, 2004).

The terms “four wheel drive” and “all wheel drive” will be quickly summarized here for the reader’s benefit. In a 4-wheel drive system the drive torque is transmitted to all four wheels (as opposed, for example, to a front wheel drive vehicle where the torque is transmitted only to the two front wheels).

The advantage of a 4-wheel drive (4WD) system is that longitudinal tire traction forces are generated at all 4 wheels to help the forward motion of the vehicle. This is very helpful in situations where loss of traction is a problem, for example in snow, off-road terrain and in climbing slippery hills. Four-wheel drive systems provide no advantage, however, in stopping on a slippery surface. This is determined entirely by the brakes and not by the type of drive system.

The major components that enable 4-wheel drive operation are the differentials at the front and rear axles and the transfer case. The differential at the front (or the rear) allows the left and right wheels to spin at different speeds. This is necessary during a turn where the outer wheel moves on a circle of larger radius and must turn faster. The transfer case routes torque from the transmission to both the front and rear axles. Depending on the design, the transfer case may provide equal amounts of torque to the front and rear axles, or it may proportion torque to the front and rear axles. The transfer case routes torque to the front and rear using a differential called the center differential.

In a 4-wheel drive system, when 4-wheel drive is engaged, the front and rear drive shafts are locked together so that the two axles must spin at the same speed. Four-wheel drive systems can be full-time or part-time systems. In a part-time 4-wheel drive system, the driver can select 4-wheel or 2-wheel drive operation using a lever or a switch. The driver can "shift on the fly" (switch between 2WD and 4WD while driving). This allows the
use of 2 wheel drive on regular dry roads and 4-wheel drive on slippery surfaces where more traction is needed.

A full-time 4WD system, on the other hand, lets the vehicle operate in 2WD (either front or rear) until the system judges that 4WD is needed. It then automatically routes power to all four wheels, varying the ratio between front and rear axles as necessary. Usually the detection of the fact that one of the wheels of the vehicle is slipping is used to activate a system. However, some of the more recent and sophisticated systems use software that switches the system to 4WD during specific driving conditions, even before a wheel begins to slip. A full-time 4-wheel drive system is also called an *all-wheel drive* (AWD) system.

### 8.4.2 Torque transfer between left and right wheels using a differential

As described above in section 8.4.1, a traditional differential allows the left and right wheels of a drive axle to spin at different speeds. This is necessary in order to allow the vehicle to turn. A traditional differential is also called an “open” differential.

An open differential splits the torque evenly between each of the two wheels to which it is connected. If one of those two wheels comes off the ground, or is on a very slippery surface, very little torque is required to drive that wheel. Because the torque is split evenly, this means that the other wheel also receives very little torque. So even if the other wheel has plenty of traction, no torque is transferred to it. This is a major disadvantage of an open differential.

An improvement on the open differential is a locking differential. In a locking differential, the driver can operate a switch to lock the left and right wheels together. This ensures that both wheels together receive the total torque. If one of the two wheels is on a slippery surface, the other wheel could still receive adequate torque and provide the longitudinal traction force. Thus a locking differential provides better traction on slippery surfaces and can be used when required by the driver.

Yet another type of differential is the limited slip differential (LSD). In a limited slip differential, a clutch progressively locks the left and right wheels together but initially allows some slip between them. This allows the inner and outer wheels to spin at different speeds during a turn but automatically locks the two wheels together when the speed difference is big so as to provide traction help on slippery surfaces.

From the above discussion on differentials it is clear that the ratio of torque transmitted to the left and right wheels is determined by the type of differential. In an open differential, the torque transmitted to both wheels is
always equal. In a locked differential, the speed of both wheels is equal and both wheels receive the total torque together as one integrated system. In a limited slip differential (LSD), more torque can be transferred to the slower wheel. This increase in torque to the slower wheel is equal to the torque required to overpower the clutch used in the LSD.

8.4.3 Active Control of Torque Transfer To All Wheels

The ultimate all-wheel drive system is one in which torque transfer to each of the 4 wheels can be independently controlled. Twin clutch torque biasing differentials have recently been developed in the automotive industry in which torque can be transferred to the inner or outer wheels in a variety of different ratios as required by an active control system (Sawase and Sano, 1999). The torque transfer between front and rear wheels can be similarly controlled actively using the center differential in the transfer case. By independently controlling the drive torque transferred to each of the 4 wheels, both traction and yaw stability control can be achieved. Yaw stability control can thus be achieved during the acceleration of a vehicle without requiring differential activation of the brakes which would have resulted in a net decrease in acceleration.

**Differential Braking**

Causes vehicle to slow down and may not provide driver desired longitudinal response if used during vehicle acceleration

**Controlled Limited Slip Differential**

Capable of increasing torque to inside wheel but not outside wheel as it only transfers torque from faster running wheel to slower one

**Twin-Clutch Torque Transfer Differential**

Capable of transferring torque to either wheel without a reduction in net vehicle acceleration

*Figure 8-6. Types of yaw stability control systems and their characteristics during vehicle acceleration*
Figure 8-6 shows three different types of yaw stability control systems that can be used during vehicle acceleration and their respective characteristics.

A twin-clutch limited slip differential described in Sawase and Sano (1999) allows any ratio of drive torques between the left and right wheels. The following equations can be used to model the torque transferred to each wheel with such a twin-clutch active differential:

When the right clutch is engaged with a clutch torque $T_{\text{clutch}}$, the drive torque transmitted to the left wheel is

$$T_{d_l} = \frac{1}{2}T_d - qT_{\text{clutch}} \quad (8.67)$$

while the drive torque transmitted to the right wheel is

$$T_{d_r} = \frac{1}{2}T_d + qT_{\text{clutch}} \quad (8.68)$$

where $q$ is a ratio determined by the gearing system in the twin-clutch differential and $T_d$ is the total torque transmitted to the axle under consideration.

Similarly, when the left clutch is engaged with a clutch torque $T_{\text{clutch}}$, the drive torque transmitted to the left wheel is

$$T_{d_l} = \frac{1}{2}T_d + qT_{\text{clutch}} \quad (8.69)$$

while the drive torque transmitted to the right wheel is

$$T_{d_r} = \frac{1}{2}T_d - qT_{\text{clutch}} \quad (8.70)$$

Thus, by controlling the clutch torque, the ratio of drive torque transmitted to the left and right wheels can be controlled.
8. Electronic stability control

The best configuration for independently controlling the torque to each wheel would be a system consisting of a twin-clutch torque transfer differential each at both front and rear wheels and an all wheel drive transfer case equipped with a central differential. However, weight and price considerations could make this configuration an unattractive option. An alternative is to use a central differential and just one twin-clutch torque transfer differential. Analysis in Swase and Sano (1999) shows that a torque transfer differential at the rear wheels, in addition to a central differential, is an attractive option.

Results in Sawase and Sano (1999) show performance when a stability control system that utilizes both differential braking and torque transfer is used. The upper control system to be used for such a stability control system would be similar to the one discussed in section 8.2.6. The upper controller would determine the desired yaw moment for the vehicle. The difference would be in the lower controller. In the lower controller, the active drive torque transfer would be utilized during vehicle acceleration and differential braking would be utilized during vehicle deceleration.

8.5 CHAPTER SUMMARY

This chapter reviewed three types of yaw stability control systems: differential braking based systems, steer-by-wire systems and independent drive torque control systems.

A major portion of the chapter focused on differential braking based systems. A hierarchical control architecture in which an upper controller determines desired yaw torque and a lower controller provides the desired yaw torque was presented. The driver's steering angle input together with a measure of tire-road friction conditions was used to determine a target yaw rate and a target slip angle for the vehicle. A sliding surface based control system was designed to ensure tracking of the target yaw rate and slip angle.

A design of a steer-by-wire system for yaw stability control was presented based on the work of Ackerman (1997). The front wheel steering angle was determined as a sum of the driver's input and an additional steer-by-wire control signal. The steer-by-wire control signal was designed so as to make the yaw rate of the vehicle unobservable from the lateral acceleration of the vehicle. This ensured that the driver could concentrate on the task of path following while the steer-by-wire controller compensated for disturbances that affected the yaw rate of the vehicle.

Finally, the design of an independent drive torque control system was discussed. A twin-clutch torque transfer differential together with a transfer case can be used to control the proportion of drive torque provided to the 4
wheels. This can be used as a control mechanism for yaw stability control. Compared to a differential braking based system, the use of a drive torque control system would ensure that the vehicle does not decelerate during yaw stability control.

**NOMENCLATURE**

- $F_y$ : lateral tire force
- $F_x$ : longitudinal tire force
- $F_{yfl}$ : lateral tire force on front left tire
- $F_{yfr}$ : lateral tire force on front right tire
- $F_{yrfl}$ : lateral tire force on rear left tire
- $F_{yrr}$ : lateral tire force on rear right tire
- $F_{xfl}$ : longitudinal tire force on front left tire
- $F_{xfr}$ : longitudinal tire force on front right tire
- $F_{xrl}$ : longitudinal tire force on rear left tire
- $F_{xrr}$ : longitudinal tire force on rear right tire
- $\dot{x}$ : longitudinal velocity at c.g. of vehicle
- $\dot{y}$ : lateral velocity at c.g. of vehicle
- $\delta$ : steering wheel angle
- $\delta_{ss}$ : steady state value of steering angle on a circular road
- $m$ : total mass of vehicle
- $I_z$ : yaw moment of inertia of vehicle
- $\ell_w$ : distance between left and right wheels (track length)
- $\ell_f$ : longitudinal distance from c.g. to front tires
- $\ell_r$ : longitudinal distance from c.g. to rear tires
- $L$ : total wheel base ($\ell_f + \ell_r$)
- $\dot{\psi}$ : yaw rate of vehicle
8. Electronic stability control

\( \alpha_f \)  
slip angle at front tires

\( \alpha_r \)  
slip angle at rear tires

\( \sigma_x \)  
slip ratio

\( \sigma_{f\ell} \)  
slip ratio at front left wheel

\( \sigma_{fr} \)  
slip ratio at front right wheel

\( \sigma_{r\ell} \)  
slip ratio at rear left wheel

\( \sigma_{rr} \)  
slip ratio at rear right wheel

\( \omega_w \)  
angular speed of a wheel

\( \omega_{f\ell} \)  
angular speed of front left wheel

\( \omega_{fr} \)  
angular speed of front right wheel

\( \omega_{r\ell} \)  
angular speed of rear left wheel

\( \omega_{rr} \)  
angular speed of rear right wheel

\( r_{\text{eff}} \)  
effective tire radius

\( C_\alpha \)  
cornering stiffness of tire

\( C_\sigma \)  
longitudinal stiffness of tire

\( F_z \)  
normal force on tire

\( \mu \)  
tire-road friction coefficient

\( J_w \)  
rotational moment of inertia of each wheel

\( T_{bf\ell} \)  
brake torque on front left wheel

\( T_{bfr} \)  
brake torque on front right wheel

\( T_{br\ell} \)  
brake torque on rear left wheel

\( T_{brr} \)  
brake torque on rear right wheel

\( P_{bf\ell} \)  
brake pressure on front left wheel

\( P_{bfr} \)  
brake pressure on front right wheel

\( P_{br\ell} \)  
brake pressure on rear left wheel
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{brr}$</td>
<td>brake pressure on rear right wheel</td>
</tr>
<tr>
<td>$P_0$</td>
<td>measured brake pressure at a wheel</td>
</tr>
<tr>
<td>$\psi_{des}$</td>
<td>desired yaw rate of driver</td>
</tr>
<tr>
<td>$\psi_{target}$</td>
<td>target yaw rate for yaw control system</td>
</tr>
<tr>
<td>$\psi_{upper_bound}$</td>
<td>upper bound on desired yaw rate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>slip angle of vehicle</td>
</tr>
<tr>
<td>$\beta_{des}$</td>
<td>desired slip angle of vehicle</td>
</tr>
<tr>
<td>$\beta_{target}$</td>
<td>target slip angle for yaw control system</td>
</tr>
<tr>
<td>$\beta_{upper_bound}$</td>
<td>upper bound on desired slip angle</td>
</tr>
<tr>
<td>$\delta_{driver}$</td>
<td>driver steering angle input in steer-by-wire system</td>
</tr>
<tr>
<td>$\delta_{sbw}$</td>
<td>steer by wire steering angle correction</td>
</tr>
<tr>
<td>$a_{yp}$</td>
<td>lateral acceleration at decoupling point P</td>
</tr>
<tr>
<td>$a_x$</td>
<td>longitudinal acceleration</td>
</tr>
<tr>
<td>$a_{y_cg}$</td>
<td>lateral acceleration at c.g. of vehicle</td>
</tr>
<tr>
<td>$\ell_P$</td>
<td>longitudinal distance of point P from vehicle c.g.</td>
</tr>
<tr>
<td>$T_{df\ell}$</td>
<td>drive torque on front left wheel</td>
</tr>
<tr>
<td>$T_{dfr}$</td>
<td>drive torque on front right wheel</td>
</tr>
<tr>
<td>$T_{drl}$</td>
<td>drive torque on rear left wheel</td>
</tr>
<tr>
<td>$T_{drr}$</td>
<td>drive torque on rear right wheel</td>
</tr>
<tr>
<td>$T_d$</td>
<td>drive torque on any axle</td>
</tr>
<tr>
<td>$T_{clutch}$</td>
<td>clutch torque in an active differential</td>
</tr>
<tr>
<td>$M_{\psi\theta}$</td>
<td>yaw torque due to differential braking</td>
</tr>
<tr>
<td>$\Delta F_{xf}$</td>
<td>extra differential longitudinal tire force required to provide desired yaw torque</td>
</tr>
<tr>
<td>$\eta$</td>
<td>constant used in sliding surface control system design</td>
</tr>
</tbody>
</table>
8. Electronic stability control

\( \xi \) constant used in definition of sliding surface for differential braking based controller
\( \rho \) front-to-back brake proportioning ratio
\( \lambda \) variable used in Dugoff tire model
\( f(\lambda) \) function used in Dugoff tire model
\( A_w \) brake area of the wheel
\( \mu_b \) the brake friction coefficient
\( R_b \) brake radius
\( q \) constant determined by gear ratios in active differential

REFERENCES


