ABS Control Using Optimum Search via Sliding Modes

Sergey Drakunov, *Member, IEEE*, Ümit Özgüner, *Member, IEEE*, Peter Dix, *Member, IEEE*, and Behrouz Ashrafi

***Abstract*—The paper presents the control design for Antilock Braking Systems via the Sliding Mode approach. In this study we formulate the problem as that of extremum searching in a highly uncertain situation. We consider the friction force as an output of the dynamic system which includes mechanical motion equations and the hydraulic circuit equations. This setting is complicated by the optimized function being *a priori* unknown, and the input (slip) not being measurable.**

I. Introduction

THE main issue of concern during braking on a slippery surface is that the wheels of the car may lock. This phenomenon is strongly undesirable. The friction force on the locked wheel is usually considerably less when sliding on the road. Furthermore, while the wheels are locked, steering becomes impossible, leading to loss of control of the vehicle.

The main difficulty arising in the design of Antilock Braking System (ABS) control is due to the strong nonlinearity and uncertainty in the problem. It is difficult and in many cases impossible to solve this problem by using classical linear, frequency domain methods.

A typical ABS measures the wheels' angular speed and possibly linear acceleration. Then the decision is made if the wheel is about to lock. If it is, the pressure in the brake cylinder is reduced until the angular velocity of the wheel exceeds some threshold value. At this time the pressure is allowed to increase. Such algorithms produce noticeable vibrations in the car.

Several results have been published coupling the ABS problem and the VSS design technique [4], [5]. In these papers the authors design sliding-mode controllers under the assumption of knowing the optimal value of the target slip. A problem of concern here is the lack of direct slip measurements. In all previous investigations *the separation* approach has been used. The problem was divided into the problem of optimal slip estimation and the problem of tracking the estimated optimal value.

Our goal is to obtain a control algorithm which allows the maximal value of the tire/road friction force to be reached during emergency braking without *a priori* knowledge of the optimal slip. The algorithm we develop below allows one to track an unknown optimal value even in the case that the value changes in real-time.

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S. Drakunov and Ü. Özgüner are with the Control Research Laboratory, Department of Electrical Engineering, Ohio State University, Columbus, OH 43210 USA.

P. Dix and B. Ashrafi are with the Electronics Division, Ford Motor Company, Dearborn, MI 48212 USA.

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II. Mathematical Model of the Slipping Wheel

The simplified four-wheel vehicle brake model without lateral motion contains equations describing the mechanical motion of the wheel and the brake hydraulic system dynamics. In this section the mechanical equations of the rotating wheel with slipping are considered. Neglecting lateral motion and yaw, the mechanical model consists of the rotational dynamics and linear vehicle dynamics.

A. Rotational Dynamics

The rotational dynamics of the ith wheel ($i = 1, \cdots, 4$) is modeled by the equation

$$J\dot{\omega}_i = -T_{bi}\,\mathrm{sgn}\,(\omega_i) - R_c F_i + T_{di} \tag{1}$$

where

ω_i is the angular velocity of the wheel

J is the moment of inertia of the wheel about the axis of rotation

T_{bi} is the brake torque at ith wheel

$R_c F_i$ is the tire/road torque produced by the friction reaction force

T_{di} is the engine torque which is assumed to be zero during braking.

B. Linear Dynamics

The linear dynamics is described by standard Newtonian equation of motion

$$M\dot{v} = \sum_{i=1}^{4} F_i - F_a \tag{2}$$

where

v is the linear velocity

F_a is the aerodynamic drag force, which is modeled as $F_a = A_{dx}\dot{x}^2$

F_i is the tire/road friction force for the ith wheel.

C. Tire Friction Force Models

The crucial point is the model of the friction forces F_i. These forces depend on the road surface, tire, weather and many other conditions. The Pacejka model [3] has been used in many studies. In this model it is assumed that friction force at

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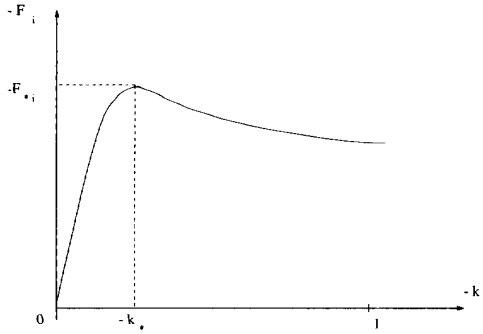


Fig. 1. Pacejka tire model.

each wheel during braking is a nonlinear function $F_i = F_i(k_i)$ of the relative slip k_i, defined as

$$k_i = \frac{R_c\omega_i - v}{v} \tag{3}$$

for the case of braking, when

$$R_c\omega_i \le v \tag{4}$$

and as

$$k_i = \frac{R_c\omega_i - v}{R_c\omega_i} \tag{5}$$

for the acceleration case when

$$R_c\omega_i > v. \tag{6}$$

The values of the function $F_i(k_i)$ were obtained experimentally for different types of surface conditions. Experiments showed that in the region $k_i > 0$ the function has a single global extremum-maximum and in the region $k_i < 0$ a minimum. The form of the function F_i for $k_i < 0$ is shown in Fig. 1.

We consider a more general tire model which does not contradict the Pacejka model described above, but includes it as a particular case.

It will be assumed that each friction force F_i is a nonstationary function of the slip k_i

$$F_i = F_i(t, k_i) \tag{7}$$

with bounded partial derivatives

$$\left|\frac{\partial F_i}{\partial k_i}\right| + \left|\frac{\partial F_i}{\partial t}\right| \le C_0 \tag{8}$$

and such that, for every t an inequality

$$k_i F_i(t, k_i) \ge 0 \tag{9}$$

is maintained, and the function F_i has a unique global maximum at

$$k_i^*(t) \ge \delta > 0 \tag{10}$$

$$y_i^* = F_i(t, k_i^*) \tag{11}$$

and a unique global minimum at

$$k_{*i}(t) \le -\delta < 0, \tag{12}$$

$$y_{*i} = F_i(t, k_{*i}). \tag{13}$$

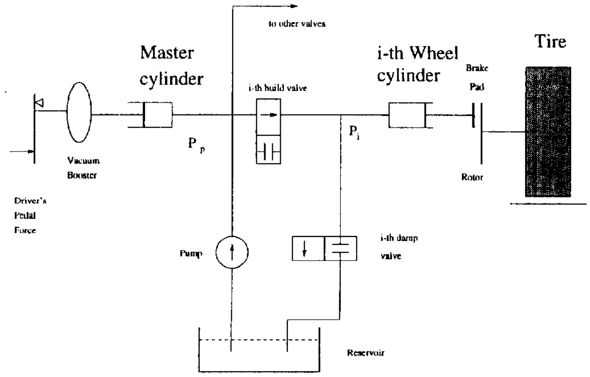


Fig. 2. ABS structure.

By assumption F_i is a sufficiently smooth function of k_i in the regions $k_i > 0$ and $k_i < 0$ and in ε-vicinity ($\varepsilon > 0$) the extremal points k_i^* and k_{*i} it satisfies the conditions

$$\left|\frac{\partial F_i(t, k_i)}{\partial k_i}\right| \ge K_0|k_i - k_{*i}| \tag{14}$$

and

$$\left|\frac{\partial F_i(t, k_i)}{\partial k_i}\right| \ge K_0|k_i^* - k_i|. \tag{15}$$

III. Model of the Brake Hydraulic System Dynamics

The hydraulic system has the standard structure shown in Fig. 2.

The common part of the system consist of a master cylinder, pump and low pressure reservoir. For each wheel there are two valves: a build valve, a dump valve and a wheel cylinder. The valves are on/off type devices, and can be only in two positions: closed/open.

The pressure created by the driver and the pump is transferred to the wheel cylinder only if the build valve is open and the dump valve is closed. If the build valve is closed and the dump valve is open the pressure in the wheel cylinder decreases due to the fluid flow in the direction of the low pressure reservoir. The case when the build valve and the dump valve are open is not allowed, but it is possible to have these valves closed at the same time. In this case, neglecting the fast transition process in the hydraulic line, the pressure in the wheel cylinder can assumed to be constant.

The flow in and out of the hydraulic circuit for the ith wheel cylinder can be modeled as a flow through an orifice [6]

$$Q_i = A_1 c_{d1i}\sqrt{\frac{2}{\rho}(P_p - P_i)} - A_2 c_{d2i}\sqrt{\frac{2}{\rho}(P_i - P_{\text{low}})} \tag{16}$$

where P_p is the constant pump pressure, P_{low} is the constant reservoir pressure, A_1, A_2 are constants representing the orifice area, ρ is the density of the fluid. P_i is the hydraulic pressure at the valves from the ith wheel cylinder side. Neglecting the inertial properties of the fluid and the resistance in the hydraulic line it will be assumed that the pressure in the wheel cylinder is also equal to P_i.

The coefficients c_{d1i}, c_{d2i} are in fact the control inputs which can take the values 0 or 1 depending on the corresponding valve being open or closed.

If the nonlinearities and temperature dependence are neglected, the brake torque $T_b i$ is a linear function of the brake pressure P_i

$$T_i = (P_i - P_{\text{out}})A_{wc}\eta B_F r_r \tag{17}$$

where P_{out} is a push out pressure; A_{wc}, η, B_F, r_r are constants (wheel cylinder area, mechanical efficiency, brake factor, effective rotor radius).

IV. Problem Statement

Assuming that the engine torque is equal to zero during braking and neglecting the aerodynamic drag force, the full model of the plant described above is the ninth order nonlinear system

$$J\dot{\omega}_i = -T_{bi}\,\mathrm{sgn}\,(\omega_i) - R_c F_i, \tag{18}$$

$$M\dot{v} = \sum_{i=1}^{4} F_i \tag{19}$$

$$F_i = F_i(t, k_i), \tag{20}$$

$$k_i = \frac{R_c\omega_i - v}{v} \tag{21}$$

$$T_{bi} = (P_i - P_{\text{out}})A_{wc}\eta B_F r_r \tag{22}$$

$$c_f\frac{dP_i}{dt} = A_1 c_{d1i}\sqrt{\frac{2}{\rho}(P_p - P_i)} - A_2 c_{d2i} \cdot \sqrt{\frac{2}{\rho}(P_i - P_{\text{low}})} \tag{23}$$

where $i = 1, \cdots, 4$.

There are eight control inputs $c_{d1i}, c_{d2i}, i = 1, \cdots, 4$, which can take values 0 or 1 with constraints $c_{d1i}c_{d2i} = 0$.

In this study it was assumed that only the angular velocities ω_i and the pressures P_i are accessible for measurement.

The simplified problem of designing antilock brake control can now be formulated as follows:

For the system (18)–(23) to design a control algorithm which steers the slip at each wheel k_i to its extremal value $k_{*i}(t)$ and tracks this value during braking.

The objective considered is, of course, just a simplified version of the problem, since it does not take into account the lateral motion of the car (it is, actually, a car on rails) but the method can be easily modified for the general problem including brake torque proportioning in order to avoid the skid.

V. Algorithm of the Optimum Search for ABS

Two versions of the control algorithm with different levels of complexity were developed. These were based on different approximations of the hydraulic system model. In the first version the static model of the hydraulic system is considered. In this case it is assumed that T_{bi} are control variables, which can be switched by closing and opening the valves from its lowest value $T_{bi}^{\min} = 0$ when $c_{d1i} = 0, c_{d2i} = 1$, to the maximal value

$$T_{bi}^{\max} = (P_p - P_{\text{out}})A_{wc}\eta B_F r_r \tag{24}$$

when $c_{d1i} = 1, c_{d2i} = 0$.

The second version was based on the first order model of the hydraulic system dynamics (23) for each wheel.

The optimal braking problem is solved for two models and with the assumptions that the current values of the friction forces

$$y_i(t) = F_i(t, k_i(t)) \tag{25}$$

and the hydraulic pressures, can be directly measured. In the following section we will demonstrate the design of the sliding-mode observer for estimating F_i by using the measurements of the angular velocity ω_i and the pressure P_i.

A. Algorithm Development

Differentiating $y_i = F(t, k_i(t))$ with respect to time along the trajectories of the system (18)–(23) we obtain

$$\dot{y}_i = -\frac{\partial F_f}{\partial k_i}\frac{1}{v}\left[R_c J^{-1}T_{bi}\,\mathrm{sgn}\,(\omega_i) + R_c^2 J^{-1}y_i + M^{-1}(1 + k_i)\sum_{j=1}^{4} y_j\right] + \frac{\partial F_i}{\partial t}. \tag{26}$$

This equation shows that even if the brake torque T_{bi} is considered as a control variable, which means that the hydraulic system dynamics is neglected, the main difficulty to control the friction force is in the fact that the sign of the coefficient $\partial F_i/\partial k_i$ is unknown. Due to the different road conditions the current value of the slip $k_i(t)$ can be more than the the optimum slip during braking k_{*i} or less than k_{*i} and, therefore, $(\partial F_i/\partial k_i) > 0$ or $(\partial F_i/\partial k_i) < 0$, respectively. And the value of k_{*i} is not known *a priori*.

B. First Version of the Algorithm

In the first version of the algorithm there is no pressure controller since we consider the static model of the hydraulic system assuming that the braking torque is switched from minimal $T_{b\min} = 0$ to maximal $T_{b\max}$ value by the following logic

$$c_{d1i} = \psi(\sin(Cy_i + \sigma(t))), \tag{27}$$

$$c_{d2i} = 1 - \psi(\sin(Cy_i + \sigma(t))) \tag{28}$$

where $C > 0, \sigma(t)$ is an increasing function of time and

$$\psi(\xi) = \begin{cases} 1, & \text{if } \xi > 0 \\ 0, & \text{if } \xi < 0 \end{cases}. \tag{29}$$

This corresponds in the static case to

$$T_{bi} = T_{b\max}\psi(\sin(Cy_i + \sigma(t))) \tag{30}$$

for example, if $\sigma(t) = \beta t$ we have

$$T_{bi} = T_{b\max}\psi(\sin(Cy_i + \beta t)).$$

It can be demonstrated that (30) results in the convergence of y_i to a desirably small vicinity of the optimal value $y_{*i} = F(t, k_{*i}(t))$.

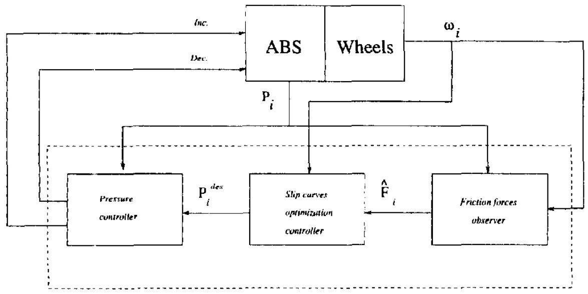


Fig. 3. Periodic switching function.

Substituting (30) into (26) we obtain

$$\dot{y}_i = A_i(t) + B_i(t)\psi(\sin(\sigma(t) + Cy_i)) \tag{31}$$

where

$$A_i(t) = -\frac{\partial F_i}{\partial k_i}\frac{1}{v}\left[R_c^2 J^{-1} y_i + M^{-1}(1+k_i)\sum_{j=1}^{4} y_j\right] + \frac{\partial F_i}{\partial t} \tag{32}$$

$$B_i(t) = -T_{b\max}\frac{\partial F_i}{\partial k_i}\frac{1}{v}R_c J^{-1}. \tag{33}$$

We assume that $\omega_i > 0$ during braking and hence $\operatorname{sgn}(\omega_i) = 1$.

By introducing new variables

$$z_i = \sigma(t) + Cy_i \tag{34}$$

(31) can be rewritten as

$$\dot{z}_i = \dot{\sigma} + CA_i + CB_i\psi(\sin(z_i)). \tag{35}$$

Under the conditions

$$|CB_i| > |CA_i + \dot{\sigma}|, \tag{36}$$

$$\operatorname{sign}(\dot{\sigma} + CA_i) = -\operatorname{sign}(CB_i) \tag{37}$$

the function in the right hand side of (35) alternates positive and negative values on the intervals $\cdots, (-\pi, 0), (0, \pi), (\pi, 2\pi), \cdots$ as illustrated in Fig. 3.

Case a) corresponds to $(\partial F_i/\partial k_i) > 0$, case b) corresponds to $(\partial F_i/\partial k_i) < 0$. The direction of motion of z on each interval is shown by arrows.

So if (36) and (37) are satisfied in both cases we obtain sliding on one of the manifolds

$$z_i - \pi l = 0 \tag{38}$$

where l is an integer number $(0, \pm 1, \pm 2, \cdots)$.

From (38) and (34) it follows that

$$Cy_i = -\sigma(t) + \pi l. \tag{39}$$

This equality means that $y_i = F_i(t, k_i(t))$ is a decreasing function of time and, therefore, it tends to the minimum with the rate $\dot{\sigma}$ no matter what the sign of the partial derivative $\partial F/\partial k$ is.

The variable y_i is decreasing until the sliding mode conditions (36) and (37) are fulfilled.

The condition (36) can be rewritten as

$$\left|\frac{\partial F_i(t, k_i(t))}{\partial k_i}\right| > \frac{|\dot{\sigma} v| + |CC_1|}{|CT_{b\max} R_c J^{-1}|}, \tag{40}$$

where C_1 is maximum of $|A_i v|$

$$C_1 = C_0\left(R_c^2 J_{-1}|y_{*i}| + 2M^{-1}\sum_{j=1}^{4}|y_{*j}| + 1\right).$$

On the other hand, condition (37) is equivalent to

$$\operatorname{sign}\left[\frac{\partial F_i(t, k_i(t))}{\partial k_i}\right](\dot{\sigma} + CA_i) > 0. \tag{41}$$

According to the assumption (12), $(\partial F_i(t, k_i(t))/\partial k_i) > 0$ in the δ-vicinity of the origin $k_i = 0$. Since for the case of braking $k_i \le 0$, all y_i are also negative and the inequality (41) can always be fulfilled by increasing $\dot{\sigma}$.

Due to the assumptions (8), (14), (15) the inequality opposite to (40) defines the vicinity of the optimal point k_{*i}. By choosing parameters $C, \dot{\sigma}$ and $T_{b\max}$ appropriately this region can be made arbitrarily small. Outside this area sliding mode exist, the variable y_i is increasing and hence k_i tends to k_{*i}.

C. Second Version of the Algorithm

Better performance can be obtained by taking the hydraulic system dynamics into account. In this case T_{bi} are not controls, but the system state variables. We design the manifold in the state space which leads to convergence of the friction force to the optimal point or, at least, to within desirably small vicinity.

It can be shown that a relation of the form

$$R_c J^{-1} T_{bi}\operatorname{sgn}(\omega_i) + R_c^2 J^{-1} y_i + M^{-1}(1+k_i)\sum_{j=1}^{4} y_j = K\sin(\sigma(t) + Cy_i) \tag{42}$$

forces y_i to converge to the ε-neighborhood of the value $y_{*i} = F_i(t, k_{*i}(t))$.

The parameters of the algorithm σ, K and C should be chosen to satisfy the accuracy and rapidity requirements on the extremum search. Then we design the pressure controller so as to maintain (42).

Substituting (42) into (26) we obtain

$$\dot{y}_i = A_i(t) + B_i(t)\sin(\sigma(t) + C_i y) \tag{43}$$

where

$$A_i(t) = \frac{\partial F_i(t, k_i(t))}{\partial t}, \tag{44}$$

$$B_i(t) = -K\frac{\partial F_i(t, k_i(t))}{\partial k_i}\frac{1}{v} \tag{45}$$

$i = 1, \cdots, 4$.

As before, by introducing a new variable

$$z_i = \sigma(t) + Cy_i \tag{46}$$

(43) can be rewritten as

$$\dot{z}_i = \dot{\sigma} + CA_i + CB_i\sin(z_i). \tag{47}$$

As can be shown by using an appropriate Lyapunov function, if the condition

$$|CB_i| > |\dot{\sigma} + CA_i| \tag{48}$$

holds, the variable z_i remains in the $\pi/2$-vicinity of one of the points πk, where l is an integer $l = \cdots, -2, -1, 0, 1, 2, \cdots$, which is closest from the left or from the right from its initial condition $z_i(0)$.

It implies that y_i is decreasing with the "mean" rate $-\dot{\sigma}$.

The condition (48) holds if

$$\left|\frac{\partial F_i(t, k_i(t))}{\partial k_i}\right| > \frac{|\dot{\sigma} v| + |CA_i v|}{|CK|}. \tag{49}$$

Due to the assumptions on the function F_i the condition (48) is violated only in the neighborhood of the extremal point k_{*i}. Therefore, y_i will decrease with the rate ω_i until it will reach this neighborhood. Correspondingly, the slip k_i will reach the vicinity of the optimal point k_{*i}.

The second version of the algorithm requires a pressure controller which supplies the necessary pressure in order to obtain the desired braking torque T_{bi}^{des}. The desired torque is obtained from the condition that (42) satisfies

$$T_{bi}^{\text{des}} = -R_c y_i - R_c^{-1} J M^{-1}(1 + k_i) \sum_{j=1}^{4} y_j + R_c^{-1} J K \sin(\sigma(t) + C y_i). \tag{50}$$

The desired pressure P_i^{des} can be obtained from (22)

$$P_i^{\text{des}} = P_{\text{out}} + (A_{wc}\eta B F r_r)^{-1} T_{bi}^{\text{des}}. \tag{51}$$

The control algorithm is the following

$$c_{d1i} = \psi(P_i^{\text{des}} - P_i), \tag{52}$$

$$c_{d2i} = \psi(P_i - P_i^{\text{des}}). \tag{53}$$

The equality

$$P_i - P_i^{\text{des}} = 0 \tag{54}$$

represents a sliding manifold in this case.

Sliding mode occurs if the condition of its existence [2]

$$(P_i - P_i^{\text{des}}) \frac{d(P_i - P_i^{\text{des}})}{dt} < 0 \tag{55}$$

holds.

Equation (55) is equivalent to the inequalities

$$A_1 \sqrt{\frac{2}{\rho}(P_p - P_i^{\text{des}})} > |c_f \dot{P}_i^{\text{des}}| \tag{56}$$

$$A_2 \sqrt{\frac{2}{\rho}(P_i^{\text{des}} - P_{\text{low}})} > |c_f \dot{P}_i^{\text{des}}| \tag{57}$$

which are restrictions on the rate of P_i^{des} and can be satisfied by appropriate choice of parameters.

In sliding mode the condition (42) is fulfilled and, therefore, the system will converge to within an arbitrary small vicinity of the optimal point.

VI. THE FRICTION FORCE OBSERVER

As mentioned in the previous section, the realization of the control algorithm requires information on the friction force. Since this quantity cannot be measured directly, an observer is developed which allows us to obtain friction force values using measurements of the angular velocity ω of the wheel and the pressure P in the hydraulic system, which defines the braking torque T_b.

The friction force observer [7] is based on the equivalent control method. The observer provides extremely accurate estimates of the friction force and is tolerant to parameter mismatches and random disturbances.

A. Observer Structure

The equations of the observer coincides with the corresponding equation (1). The model for each $i = 1, \cdots, 4$ has four inputs which are: engine torque T_{di}, angular velocities of the wheels ω_i, brake torque T_{bi} and input V_i instead of the friction force in the equation of the real wheel. The only output of the model is an estimate $\hat{\omega}_i$ of the angular velocity of the wheel

$$J\dot{\hat{\omega}}_i = -R_c V_i - T_{bi} \operatorname{sgn}(\omega_i) + T_{di}. \tag{58}$$

The function V_i is picked as

$$V_i = -M_i \operatorname{sgn}(\overline{\omega}_i) \tag{59}$$

where $\overline{\omega}_i = \omega_i - \hat{\omega}_i$ is a tracking error of the angular velocity and $M_i > 0$ is a sufficiently large constant.

Subtracting (58) from (1) we obtain

$$J\dot{\overline{\omega}}_i = -R_c M_i \operatorname{sgn}(\overline{\omega}_i) - R_c F_i. \tag{60}$$

Under the condition

$$M_i > \max\{|F_i|\} \tag{61}$$

the discontinuous feedback in the observer equation results in the tracking of the angular velocity of the wheel, when in sliding mode. During sliding $\overline{\omega}_i = 0$ and the equivalent value of the variable $V_i = -M_i \operatorname{sgn}(\overline{\omega}_i)$ is equal to F_i

$$V_{ieq} = F_i(t, k_i(t)). \tag{62}$$

As shown in [2], the equivalent value of the chattering with high frequency discontinuity can be obtained by using a lowpass filter. Note here that this chattering occurs inside the friction force observer loop and does not affect the behavior of the overall system. Since the chattering frequency of the inner loop can be assigned sufficiently high, a first order filter can be used.

The equation of the filter is

$$\hat{F}_i = W_{if}(s) V_i \tag{63}$$

where $W_{if}(s)$ is a transfer function of the filter

$$W_{if}(s) = \frac{1}{T_{if}s + 1}. \tag{64}$$

The time constant of the filter T_{if} is chosen to suppress the high frequency of the oscillations but not to disturb the

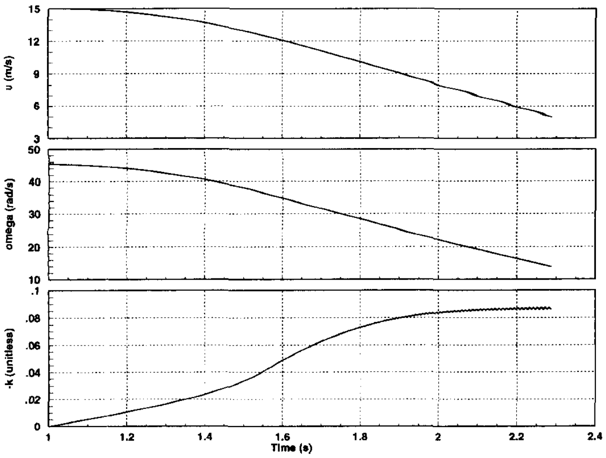


Fig. 4. Velocities and slip.

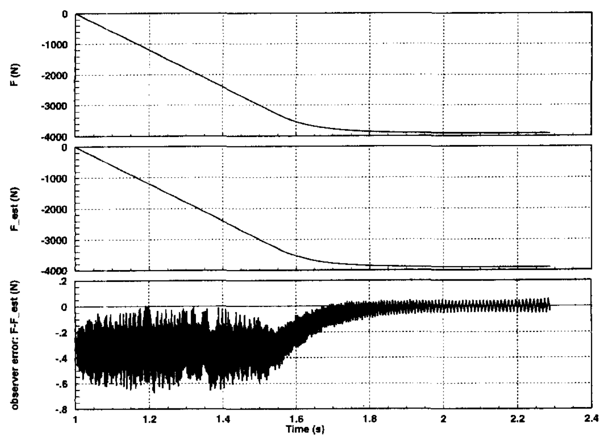


Fig. 5. Friction force observer.

relatively slow component F_i. The estimate of F_i on the output of the filter then was used in the above optimization algorithms.

Appendix

Below, we show the results of one of the simulation experiments when the optimization algorithm, together with the friction force observer was simulated with a one-wheel model for the case of Pacejka friction. The optimal value of the friction force was -3900 N at the relative slip $k_* = -0.09$.

As can be seen from Fig. 4 the absolute value of the slip k increases and then oscillates in the vicinity of the optimal value. The friction force shown in Fig. 6, correspondingly, converges to its optimum and the brake torque to the constant value.

The performance of the friction force observer is shown in Fig. 5. While the real friction follows the desired one for the optimal braking trajectory, the Friction Force Observer shows excellent tracking of the angular velocity signal and as a result, a very accurate friction force estimate. When the wheel friction force changes in the wide range from 0 N (wheel rolling without any slip) to 4000 N (with optimal slip value) the deviation of the estimate $\hat{F}$ from the real force F is less than 1 N as shows the magnified value of the error $F - \hat{F}$ in Fig. 5.

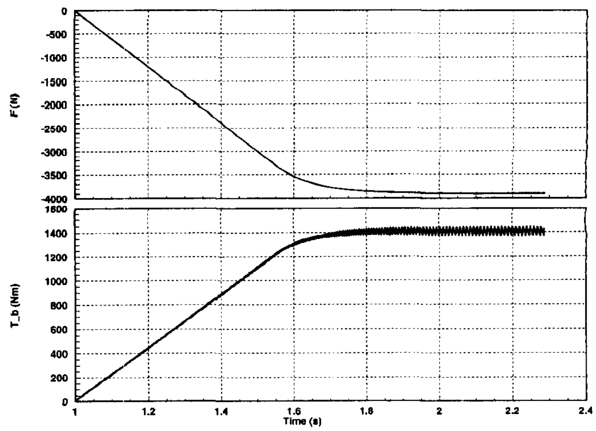


Fig. 6. Force, torque.

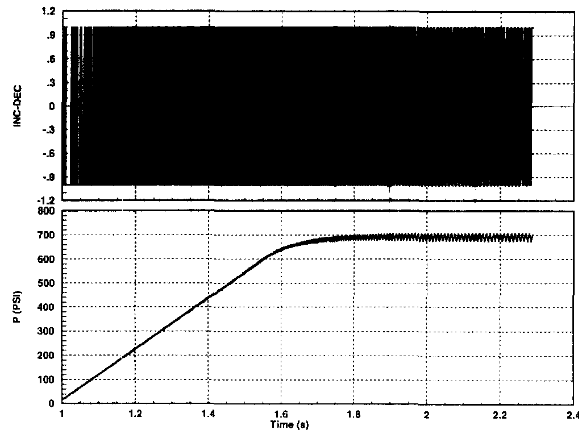


Fig. 7. Control, pressure.

In Fig. 7 the control signal is shown. The value 1 of the variable $INC - DEC$ corresponds to open position of the build valve and closed position of the dump valve, as 0 corresponds to the open position of the dump valve and closed position of the build valve. The control chatters with a very high frequency as the pressure tracks the desired value. In this simulation experiment it was assumed that the valves can be closed and opened with a desirably high frequency. In real life situation, there is of course, a restriction defined by mechanical characteristics of the valves. The delays in switchings will result in higher amplitude chattering.

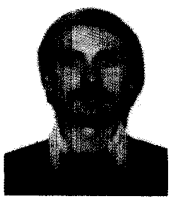
References

[1] S. Drakunov, "Sliding mode control of the systems with uncertain direction of control vector," in *Proc. 32nd IEEE Conf. Decision and Contr.*, San Antonio, TX, Dec. 15–17, 1993, pp. 2477–2478.

[2] V. I. Utkin, *Sliding Modes and Their Application In Variable Structure Systems.* Moscow: MIR, 1978.

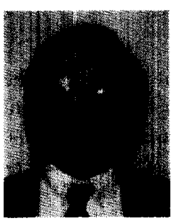
[3] E. T. Bakker, H. B. Pacejka, and L. Linder, "A new tire model with an application in vehicle dynamic studies," SAE Technical Paper No. 870421, 1982.

[4] H. S. Tan and M. Tomizuka, "An adaptive sliding mode vehicle traction controller design," in *Proc. 1989 Amer. Contr. Conf.*, 1989, pp. 1053–1058.
[5] Y. K. Chin *et al.*, "Sliding-mode ABS wheel slip control," in *Proc. 1992 ACC*, Chicago, IL, June 24–26, 1992, pp. 1–6.
[6] H. E. Merritt *Hydraulic Control Systems.* New York: Wiley, 1967.
[7] S. Drakunov, "Sliding mode observers based on equivalent control method," in *Proc. IEEE Conf. Decision and Contr.*, Tucson, AZ, Dec. 1992.



Sergey Drakunov (M'91) received the M.S. degree in applied mathematics from Moscow Institute of Electronic Engineering in 1978 and the Ph.D. degree in control theory and system analysis from the Institute of Control Sciences, Academy of Sciences, Russia, in 1985.

From 1985 until June 1991, Dr. Drakunov was with the Laboratory of Discontinuous Systems of the Institute of Control Sciences, Academy of Sciences, where he was a Senior Research Scientist, simultaneously he was an Associate Professor at the Department of Mathematics at the Moscow Institute of Electronic Engineering. Since June 1991 he has been with the Department of Electrical Engineering of the Ohio State University, where he is currently a Research Scientist. His research interests are in sliding mode control, observers and filters for finite-dimensional, discrete-time, distributed parameter and hybrid systems.



Ümit Özgüner (S'72–M'80) received the Ph.D. degree from the University of Illinois in 1975.

He has held research and teaching positions at IBM T. J. Watson Research Center, University of Toronto and Istanbul Technical University. He has been with The Ohio State University since 1981 where he is presently Professor of Electrical Engineering and Director of IVHS-OSU *Center for Safety, Reliability and Intelligent Control.* His areas of research interests are in decentralized and hierarchical control for large scale systems and mechatronic systems in general and applied automotive control, flexible structure control and IVHS. He is Technical Committee Chair on Intelligent Control for the IEEE Control Society and is chairing the Ohio Aerospace Institute Technet on Control. He has participated in the organization of many conferences and has been the General Chair of the 1994 International Symposium on Intelligent Control. His research has been supported by NSF, AFOSR, NASA LeRC and LaRc, Ford Motor Co., LLNL and Sandia Labs.

Dr. Ozgüner is the author of over 200 publications which have appeared in journals, as book chapters and in conference proceedings.

Peter Dix (S'86–M'90) received the B.S. degree in electrical engineering from Marquette University, Milwaukee, WI, in 1988 and the M.S. degree from Ohio State University in electrical engineering in 1990.

As a student, he worked for Motorola AIEG on automotive electronics and for Ohio State as a research associate on control of flexible structures. Since 1990, he has been employed by Ford Motor Company as a Product Design Engineer in the Antilock Brack/Traction Control Department of the Electronics Division. His work includes ABS control design, software design and vehicle modeling.

Behrouz Ashrafi received the B.S., M.S., and Ph.D. degrees in mechanical engineering from Drexel University, Philadelphia, PA, in 1983, 1985, and 1991, respectively.

Since 1992, Dr. Ashrafi has been a Product Design Engineer in the Electronics Division of Ford Motor Company, Dearborn, MI. His current interests include nonlinear control theory, stability, chaos, and application of estimation and control theory to advanced automotive vehicles.