Vedic Mathematics

By

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Introduction

Vedic Mathematics is a great boon to the modern world from our ancient sages. It is an ancient technique, revived by His Holiness Jagadguru Bharathi Krishna Tirthaji. Vedic Mathematics is based on 16 Sutras (Formulas) and 16 Upa-Sutras (Sub-Formulas). These are in words and are very much useful in solving the problems. The Vedic methods are simple, short and speedy. The answers can be worked out in 2 or 3 lines. This saves a lot of time, space and energy. Answers can be computed mentally without using pen and paper.

Vedic mathematics has many advantages:

- Addition, subtraction, multiplication, etc. can be done from left to right as against traditional right to left.
- The answers can be verified easily and quickly.
- Multiplication, division, squares and square roots, cubes and cube roots, reciprocals etc. can be done easily and accurately.
- Accuracy will be far better than calculators. Calculators have their own limitations.
- Alternate methods are possible to solve a problem.
- It works well with arithmetic, algebra, geometry, trigonometry, and calculus.

Vedic mathematics is very much useful to the students studying in schools, colleges and every person in general. It is also helpful for those who are preparing for competitive and entrance examinations. It is well tested by scientists and engineers at NASA and taught in UK and other European countries.

Basics

Mathematics is a game of numbers. Numbers contain digits 1 – 9 and 0. To learn and understand mathematics we must know the meaning of the terms used and the rules of various operations.

Complement

If sum of two digits is equal to 10, then each digit is said to be the complement of the other from 10. e.g., 4 + 6 = 10. Here, 4 is the complement of 6 from 10 and vice versa.

Complements of digits from 10 and 9 are shown below. These must be studied carefully and remembered. This will help in understanding Vedic Mathematics clearly and properly.

Complements from 10

Digit	1	2	3	4	5	6	7	8	9
Complement	9	8	7	6	5	4	3	2	1

Complements from 9

Digit	0	1	2	3	4	5	6	7	8
Complement	9	8	7	6	5	4	3	2	1

Addition of two digits

Sum of two positive digits is equal to the sum of the digits with positive sign.

E.g., $2 + 3 = 5$, $6 + 3 = 9$, etc.

Sum of two negative digits is equal to the sum of the digits with negative sign.

E.g.,$(-2) + (-3) = -5$, $(-6) + (-3) = -9$, etc.

Sum of a positive digit and a negative digit is equal to the difference between the digits with the sign of larger digit. E.g., $2 + (-3) = -1$, $(-6) + 3 = -3$, etc.

Subtraction of two digits

Subtraction is nothing but negative addition. To subtract one digit from another digit, change the sign of the digit to be subtracted and add the two digits.

E.g., $2 - 3 = 2 + (-3) = -1$, $6 - 3 = 6 + (-3) = 3$, etc.

Multiplication of two digits

Product of two positive digits is equal to the product of the digits with positive sign.

E.g., $2 \times 3 = 6$, $6 \times 3 = 18$, etc.

Product of two negative digits is equal to the product of the digits with positive sign.

E.g., $(-2) \times (-3) = 6$, $(-6) \times (-3) = 18$, etc.

Product of a positive digit and a negative digit is equal to the product of the digits with the negative sign.

E.g., $2 \times (-3) = -6$, $(-6) \times 3 = -18$, etc.

Subtraction from 100, 1000, 10000, …

In Vedic mathematics, powers of 10, viz., 10, 100, 1000, … are used as base. When a number is subtracted from a base we get the complement of that number from that base.

E.g., $100 - 36 = 64$. Here, 64 is the complement of 36 from base 100.

$1000 - 36 = 964$. Here, 964 is the complement of 36 from 1000.

To find the complement of a number from a base we need not subtract the number from the base every time. We can use the Vedic Sutra "All from 9 and the last from 10", known as Nikhilam Sutra. It is also called "All from 9", in short. The following examples explain the use of this Sutra.

1. Consider the number 853 and base 1000.

Applying All from 9,	8 from 9 is 9 – 8 = 1
	5 from 9 is 9 – 5 = 4
And Last from 10,	3 from 10 is 10 – 3 =7

Thus, 147 is the complement of 853 from base 1000. Note that 147 + 853 = 1000.

2. Consider the number 52 and base 1000. Here, 1000 has 3 zeros. So, we rewrite 52 as 052 and apply Nikhilam.

All from 9,	0 from 9 is 9 – 0 = 9
	5 from 9 is 9 – 5 = 4
Last from 10,	2 from 10 is 10 – 2 =8

Thus, 948 is the complement of 52 from base 1000. Again, note that 948 + 52 = 1000.

Similarly, the complement of any number can be computed. The calculations shown above are only guidelines. We need not write all the steps. The calculations can be done mentally after a little practice. Try some more cases by yourselves.

Addition

In Mathematics addition and subtraction are simple and easy operations. But, addition is easier than subtraction. These topics are covered here for the reason that Vedic Mathematics allows us to add, subtract and multiply numbers from left to right or right to left. This is the greatness of Vedic Mathematics.

Left to right

We write the numbers from left to right. Also, we read numbers from left to right. But, in traditional mathematics addition, subtraction and multiplication are done from right to left. Would it not be fine if we can do these operations from left to right? In some problems we need to know first 2 or 3 digits. Here we shall see how this could be done!

Addition

As said earlier addition is the simplest and easiest operation. We shall see here how addition could be done from left to right. The method is given below.

Method

- Write the numbers one below the other.
- Add the digits column by column from left to right or right to left.

- If the sum exceeds 9, retain last digit and carry over other digits to left.
- Compute the final answer.

Example 1: Add 548 + 236 + 198.

5 4 8	5 4 8	5 4 8
2 3 6	2 3 5	2 3 6
+ 1 9 8	+ 1 9 8	+ 1 9 8
8	8 6	8 6 2
	1	1 2
		9 8 2

Working from left to right (Mental work)

Step 1: 1 + 2 + 5 = 8. Write as shown

Step 2: 9 + 3 + 4 = 16. – " –

Step 3: 8 + 6 + 8= 22. – " –

Add the two rows to get the final answer.

Example 2: Add 5678 + 3728 + 4379.

5 6 7 8	5 6 7 8	5 6 7 8	5 6 7 8
3 7 2 8	3 7 2 8	3 7 2 8	3 7 2 8
+ 4 3 7 9	+ 4 3 7 9	+ 4 3 7 9	+ 4 3 7 9
2 6	2 6 6	2 6 6	2 6 6 5
1 1	1 1 1	1 1 1	1 1 1 2
			1 3 7 8 5

Working from left to right (Mental work)

Step 1: 4 + 3 + 5 = 12. Write as shown

Step 2: 3 + 7 + 6 = 16. "

Step 3: 7 + 2 + 7 = 16. "

Step 4: 9 + 8 + 8 = 25. "

Add the two rows to get the final answer.

Subtraction

Subtraction is little harder than addition. But it is made simpler using Vedic Mathematics techniques. Instead of subtracting numbers directly we convert the number to be subtracted into Vinculum and then subtract. The method adopted is explained below.

Method:

- Write numbers one below the other.
- Check the lower digit from left to right. If it is larger than the upper digit, put a star on the its previous digit to increase its value by 1.e.g., 1–>1+1=2,2–> 2+1=3, etc.
- If the lower digit is smaller than the upper digit write the difference between digits.
- If the lower digit is larger than the upper digit write the complement of the difference between the digits. (*Alternately, add the complement of the lower digit to the upper digit*)

Example1: Subtract: 224 – 192.

	2	2	4
-	1*	9	2
	0	3	2

The answer is = 32.

Working from left to right (Mental work)

Step 1: 9 > 2. A (*) is put on 1. So, 1* = 1+1=2.

Step 2: 2 – (1 + 1) = 0, (Write this in 1st column)

Step 3: 2 < 9. So no (*), 2 – 9 + 10 = 3. (2nd column)

(Alternately, complement of 9 from 10 is 1 and 1 + 2 = 3)

Step 4: 4 > 2. So, 4 – 2 = 2, (3rd column)

Example 2: Subtract: 322 – 197.

	3	2	2
-	1*	9*	7
	1	2	5

The answer is = 125.

Working from left to right (Mental work)

Step 1: 9 > 2. A (*) is put on 1. So, 1* = 1+1=2.

7 > 3. A *(*) is put on 9. So, 9* = 9+1=10.

Step 2: 3 – (1 + 1) = 1, (Write this in 1st column)

Step 3: 2 < 9*.So, 2 – (9 + 1) + 10 = 2. (2nd column)

(Alternately, complement of 10 from 10 is 0 and 0 + 2 = 2)

Step 4: 2<7So, 2 – 7 + 10 = 5, (3rd column)

(Alternately, complement of 7 from 10 is 3 and 3 + 2 = 5)

Final answer is 125.

Multiplication

In mathematics multiplication is harder than addition and subtraction. In Vedic Mathematics we have simpler methods. Before taking up them we will discuss some special cases.

Multiplication by 11

Multiplication of a number by 11 is very easy. It is as good as addition. This method is explained below.

Method

- Sandwich the given number between zeros.
- Starting from left end add the digits taking them in pairs.
- If the total exceeds 9, retain the first digit and carry over the other digits to the left.

Example1: Multiply 135 x 11

Write the multiplicand as shown and add the digits in pair.

$$\begin{array}{c} 135 \times 11 \\ \hline 0\,1\,3\,5\;\;0 \\ \hline 1\,4\,8\,5 \\ \hline \end{array}$$

Thus, 135 x 11 = 1485

Working from left to right (Mental work)

Addition of digits in pair is shown below.

0 + 1 = 1,

1 + 3 = 4,

3 + 5 = 8,

5 + 0 = 5

Example 2: Multiply

58403 x 11

$$\begin{array}{l} \;\;58403 \times 11 \\ \hline \;\;0\,5\,8\,4\,0\,3\,0 \\ \hline \;\;5\,3\,2\,4\,3\,3 \\ \;\;1\,1 \\ \hline \;\;6\,4\,2\,4\,3\,3 \\ \hline \end{array}$$

Thus, 58403 x 11 = 642433

Working from left to right (Mental work)

Addition of digits in pair is shown below.

0 + 5 = 5, 5 + 8 = 13, 8 + 4 = 12,

4 + 0 = 4, 0 + 3 = 3, 3 + 0 = 3.

The above method can be extended to multiply any number by numbers like 111, 1111, 11111, etc. Sandwich the given number between a pair of zeros and add the digits in threesome (taking three at a time).

Example 3: Multiply 123 x 111

$$\begin{array}{c} 123 \times 111 \\ \hline 0\,0\,1\,2\,3\,0\,0 \\ \hline 1\,3\,6\,5\,3 \\ \hline \end{array}$$

Thus, 123 x 111 = 13653

Working from left to right (Mental work)

Addition of digits in three-some is shown below.

0 + 0 + 1 = 1, 0 + 1 + 2 = 3, 1 + 2 + 3 = 6,

2 + 3 + 0 = 5, 3 + 0 + 0 = 3

Example 4:

Multiply 5786 x 111

$$\begin{array}{l} \;\;5786 \times 111 \\ \hline \;\;0\,0\,5\,7\,8\,6\,0\,0 \\ \hline \;\;5\,2\,0\,1\,4\,6 \\ \;\;1\,2\,2\,1 \\ \hline \;\;6\,4\,2\,2\,4\,6 \\ \hline \end{array}$$

Thus, 5786 x 111 = 642246

Working from left to right (Mental work)

Addition of digits in threesome is shown below.

0 + 0 + 5 = 5, 0 + 5 + 7 = 12,

5 + 7 + 8 = 20, 7 + 8 + 6 = 21,

8 + 6 + 0 = 14, 6 + 0 + 0 = 6.

Multiplication by 12

This is similar to the one discussed earlier. But, there is a slight difference. This employs the Vedic Sutra "*Ultimate and twice the penultimate*". According to this, we must add twice the penultimate digit to the ultimate digit.

Consider the number 32. Here, penultimate digit is 3 and the ultimate digit is 2. By the above Sutra, the required sum = 3 x 2 + 2 = 8.

Example 1: Multiply 123 x 12

123 x 12

0 1 2 3 0

1 4 7 6

Thus, 123 x 12 = 1476

Working from left to right (Mental work)

Write the multiplicand as shown and add twice the penultimate digit to ultimate digit.

0 x 2 + 1 = 1, 1 x 2 + 2 = 4,

2 x 2 + 3 = 7, 3 x 2 + 0 = 6.

Example 2: Multiply 396 x 12

396 x 12

0 3 9 6 0

3 5 4 2

1 2 1

4 7 5 2

Thus, 396 x 12 = 4752

Working from left to right (Mental work)

Write the multiplicand as shown and add twice the penultimate digit to ultimate digit.

0 x 2 + 3 = 3, 3 x 2 + 9 = 15,

9 x 2 + 6 = 24, 6 x 2 + 0 = 12.

This method can be extended for multiplication with 13, 14, 15, … , 19 with little modification. Instead of *twice* we have to take *three* times, *four* times, etc. Try this!

Ekadhikena Multiplication

This is another simple method. The Vedic Sutra used in this method is "One more than the previous one" – Ekadhikena. Two different cases arise here.

Case I – Last digits adding to 10.

The numbers used in this method must obey the following conditions.

Both the numbers must have the same previous digit(s).

The sum of the last digits must be 10.

Numbers like 54 and 56, 42 and 48, 23 and 27, 34 and 36 form the examples.

Method

- Divide the answer space into LHS and RHS by Place a slash (/) or a colon (:).
- Use Ekadhikena (Previous digit +1) to the digit on the LHS.

- Write the product of last digits on the RHS.
- Take care to see that RHS has two digits, as it should be. (Digit rule)
- Remove the slash or colon.

Example 1:

Multiply 51 x 59

Same previous digit: 5,

Sum of last digits: 1+9=10.

51 x 59

5 x (5+1) / 1 x 9

30 / 09

Thus, 51x59=3009.

Note that 0 has been added on RHS.

Example 2:

Multiply 66 x 64

Same previous digit: 6,

Sum of last digits: 6+4=10.

66 x 64

6 x (6+1) / 6 x 4

42 / 24

Thus, 66 x 64=4224.

Example 3:

Multiply 123 x 127

Same previous digits: 12,

Sum of last digits: 3+7=10.

123 x 127

12 x (12+1) / 3 x 7

156 / 21

Thus, 123 x 127=15621.

Case II – Squaring of numbers ending with 5.

When the numbers are equal and end with 5, they satisfy both the conditions. Then the product of the numbers gives the square of that number. Thus, the square of a number ending with 5 can be computed as above. In all such cases RHS will always be 25. These computations can be done mentally.

Example 1:

Find the square of 35.

Previous digit = 3

35^2

3 x (3+1) / 25

12 / 25

Thus, 35^2 = 1225.

Example 2:

Find the square of 75.

Previous digit = 7

75^2

7 x (7+1) / 25

56 / 25

Thus, 75^2 = 5625.

Example 3:

Find the square of 155.

Previous digits = 15

155^2

15 x (15+1) / 25

240 / 25

Thus, 155^2 = 24025.

Ekanyunena Multiplication

Ekanyunena is a Vedic Sutra which states that "One less than the previous one". This Sutra is useful in multiplying a number with multipliers having only 9's (9, 99, 999, 9999, ….). This is also a quick method and done mentally within no time. The answer can be written in one line.

Method

- Place a slash (/) or a colon (:) to separate answer into two parts, LHS and RHS.
- Subtract 1 from the multiplicand (Ekanyunena) and write it on LHS.
- Write the complement of multiplicand (from the base of multiplier) on RHS.

- Remove the slash or colon.

Three different situations need our attention.

Case I: Both multiplicand and multiplier having same number of digits.

Example 1: Multiply 4 x 9

4 x 9

4 – 1 / 6

3 6

Thus, 4 x 9 = 36

Working from left to right (Mental work)

LHS = 4 – 1 = 3 (Ekanyunena)

RHS = 10 – 4 = 6 (Complement of 4 from 10)

Example 2: Multiply 76 x 99

76 x 99

76 – 1 / 24

7524

Thus, 76 x 99 = 7524

Working from left to right (Mental work)

LHS = 76 – 1 = 75 (Ekanyunena)

RHS =100–76= 24 (Complement of 76 from 100)

Example 3: Multiply 353 x 999

353 x 999

353 – 1 / 647

352647

Thus, 353 x 999 = 352647

Working from left to right (Mental work)

LHS = 353 – 1 = 352 (Ekanyunena)

RHS=1000 – 353=647 (Complement of 353 from 1000)

Case II: Multiplier having more number of digits than multiplicand.

In this case, we equalize the number of digits both in multiplicand and multiplier by prefixing required number of zeros with the multiplicand.

Example 1: Multiply 6 x 99

06 x 99

6-1 / 94

594

Thus, 6 x 99 = 594

Working from left to right (Mental work)

LHS = 6 – 1 = 5 (Ekanyunena)

RHS = 100–06 = 94 (complement of 06 from 100)

(We can directly write the complement of 06 = 94)

Example 2: Multiply 35 x 999

035 x 999

35-1 / 965

34965

Thus, 35 x 999 = 34965

Working from left to right (Mental work)

LHS = 35 – 1 = 34

RHS = 1000 – 035 = 965

(Complement of 035 = 965)

Example 3: Multiply 98 x 99999

00098 x 99999

98-1 / 99902

9799902

Thus, 98x99999=9799902

Working from left to right (Mental work)

LHS = 98 – 1 = 97

RHS = 10000 – 00098 = 99902

(Complement of 00098 = 99902)

Note: Apply All from 9 last from 10 Rule in all these cases.

Case III: Multiplicand having more number of digits than multiplier.

In this case, we split the multiplicand into two parts.

i. Right Hand Part having same No.of digits as of Multiplier.

ii. Left Hand Part having remaining digits.

We subtract Left Hand Part digits also (in addition to 1 as usual) from the multiplicand and write it on LHS and complement of Right Hand Part digits on RHS.

Example 1: Multiply 72 x 9

7/2 x 9

72 – 1 – 7 / 8

71 – 7 / 8

648

Thus, 72 x 9 = 648

Working from left to right (Mental work)

LHS = 72 – 1 –7 = 64

RHS = 10 – 2 = 8 (complement of 2 from 10)

Example 2: Multiply 123 x 99

1/23 x 99

123 – 1 – 1 / 77

122 – 1 / 77

12177

Thus, 123 x 99 = 12177

Working from left to right (Mental work)

LHS = 123 – 1 – 1 = 121

RHS = 100 – 23 = 77

Example 3: Multiply 7936 x 99

79/36 x 99

7936 – 1 – 79 / 64

7935 – 79 / 64

785664

Thus, 7936 x 99 = 785664

Working from left to right (Mental work)

LHS = 7936 – 1 – 79 = 7856

RHS = 100 – 36 = 64

Nikhilam Multiplication - Base System

As pointed earlier the powers of 10, i.e., 10, 100, 1000, etc. are taken as base. In Vedic Mathematics numbers can be expressed in base system. This helps us to make various computations easier.

The difference between the number and the nearest base is called deviation. If the number is less than the base, the deviation will be negative. On the other hand, if h number is more than the base, the deviation will be positive. The combination of the number and its deviation forms the base system.

The following table gives us an idea of writing the number in base system.

Number	Base	Deviation	Base System
8	10	8 – 10 = – 2	8 – 2
12	10	12 – 10 = +2	12 + 2
96	100	96 – 100 = – 4	96 – 4
106	100	106 – 100 = + 6	106 + 6
86	100	86 – 100 = – 14	86 – 14
112	100	112 – 100 = + 12	112 + 12

Nikhilam Multiplication

As the deviation is obtained by Nikhilam sutra we call the method as Nikhilam multiplication. This is a special method to multiply two numbers near a base or one number near the base and the other a little away from the base. The method of multiplication is given below.

Method

- Write the numbers one below the other in base system.
- Divide the answer space into LHS and RHS by placing a slash (/) or a colon (:).
- Add or subtract one number with the deviation of the other number and write it on the LHS. (i.e., cross-sum or cross-difference)
- Write the product of the deviations on RHS.
- The number of digits on RHS must be same as the number of zeros in the base. If less, prefix the answer with the zeros. If more, transfer extra digits to the RHS. (Digit Rule)
- Take due care to the sign (+/–) while adding or multiplying the numbers.
- Remove the slash.

Three different cases are possible.

Case I: Both the numbers below the base.

Here, both deviations are negative and the product of the deviations will be positive.

Example 1: Multiply 6 x 8.

Working from left to right (Mental work)

6 – 4

x 8 – 2

---

4 / 8

---

4 8

Thus, 6 x 8 = 48

Base = 10, RHS Digits = 1

LHS = 6 – 2 = 4 or 8 – 4 = 4

RHS = - 4 x – 2 = 8

Note: Numbers are written in Base System.

Example 2: Multiply 92 x 97.

Working from left to right (Mental work)

92 – 08

x 97 – 03

---

89 / 24

8924

Thus, 92 x 97 = 8924

Base = 100, RHS Digits = 2

LHS = 92 – 03 = 89 or 97 – 08 = 89

RHS = - 08 x – 03 = 24

Note the number of digits in deviation.

Example 3: Multiply 91 x 99.

Working from left to right (Mental work)

91 – 09

x 99 – 01

---

90 / 09

9009

Thus, 91 x 99 = 9009

Base = 100, RHS Digits = 2

LHS = 91 – 01 = 90 or 99 – 09 = 90

RHS = - 09 x – 01 = 09 (Digit Rule)

Example 4:

Multiply 993 x 996.

Working from left to right (Mental work)

993 – 007

x 996 – 004

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989 / 028

989028

Thus, 993 x 996 = 989028

Base = 1000, RHS Digits = 3

LHS = 993 – 004 = 989 or 996 – 007 = 89

RHS = - 007 x – 004 = 028 (Digit Rule)

Note the number of digits in deviation.

Example 5: Multiply 79 x 84.

Working from left to right (Mental work)

79 – 21

x 84 – 16

---

63 / 36

3/

6636

Thus, 79 x 84 = 6636

Base = 100, RHS Digits = 2

LHS = 79 – 16 = 63 or 84 – 21 = 63

RHS = - 21 x – 16 = 336

3/36 –> 3 is carried over left. (Digit Rule)

Example 6: Multiply 64 x 96.

Working from left to right (Mental work)

```
  64 – 36
x 96 – 04
---------
  60 / 44
   1/
---------
  6144
---------
```

Thus, 64 x 96 = 6144

Base = 100, RHS Digits = 2

LHS = 64 – 04 = 60 or 96 – 36 = 60

RHS = - 36 x – 04 = 144

1/44 –> 1 is carried over to left. (Digit Rule)

Case II: Both the numbers above the base.

Here, both deviations are positive and the product of the deviations will also be positive.

Example 1: Multiply 12 x 14.

Working from left to right (Mental work)

```
  12 + 2
x 14 + 4
---------
  16 / 8
---------
  168
---------
```

Thus, 12 x 14 = 168

Base = 10, RHS Digits = 1

LHS = 12 + 4 = 16 or 14 + 2 = 16

RHS = 2 x 4 = 8

Example 2: Multiply 15 x 18.

Working from left to right (Mental work)

```
  15 + 5
x 18 + 8
---------
  23 / 0
   4 /
---------
  270
---------
```

Thus, 15 x 18 = 270

Base = 10, RHS Digits = 1

LHS = 15 + 8 = 23 or 18 + 5 = 23

RHS = 5 x 8 = 40

Example 3: Multiply 101 x 108.

Working from left to right (Mental work)

```
  101 + 01
x 108 + 08
---------
  109 / 08
---------
  10908
---------
```

Thus, 101 x 108 = 10908

Base = 100, RHS Digits = 2

LHS = 101 + 08 = 109 or 108 + 01 = 109

RHS = 01 x 08 = 08. (Digit Rule)

Example 4: Multiply 111 x 123.

Working from left to right (Mental work)

```
  111 + 11
x 123 + 23
---------
  134 / 53
```

Base = 100, RHS Digits = 2

LHS = 111 + 23 = 134 or 123 + 11 = 134

RHS = 11 x 23 = 253.

2 /

13653

Thus, 111 x 123 = 13653

2/53 –> 2 is carried over to left. (Digit Rule)

Example 5:

Multiply 1007x1012.

1007 + 007

x 1012 + 012

1019 / 084

1019084

Thus, 1007x1012=1019084

Working from left to right (Mental work)

Base = 1000, RHS Digits = 3

LHS = 1007 + 012 = 1019 or 1012 + 007 = 1019

RHS = 007 x 012 = 084. (Digit Rule)

Example 6:

Multiply 1003 x 1854.

1003 + 003

x 1854 + 854

1857 / 562

2 /

1859562

Thus,1003x1857=1859562

Working from left to right (Mental work)

Base = 1000, RHS Digits = 3

LHS = 1003 + 854 = 1857 or 1854 + 003 = 1857

RHS = 003 x 854 = 2562.

2/562 –> 2 is carried over to left. (Digit Rule)

Case III: One number below the base and one number above the base.

Here, one deviation is negative, another deviation is positive; product of deviations will be negative.

Example 1: Multiply 8 x 12.

8 – 2

x 12 + 2

10 / –4

10–1 / 10 –4

96

Thus, 8 x 12 = 96

Working from left to right (Mental work)

Base = 10, RHS Digits = 1

LHS = 8 + 2 = 10 or 12 – 2 = 10

RHS = – 2 x + 2 = – 4

Transfer 1x10 from LHS to RHS. (10 is the base)

Then we have (10 – 1)/(10 – 4) = 96

(Reduce LHS by 1 and write the complement of RHS from base 10).

Example 2: Multiply 94 x 102.

94 – 06
x 102 + 02
96 / –12
96–1/100–12
9598

Thus, 94 x 102 = 9598

Working from left to right (Mental work)

Base = 100, RHS Digits = 2

LHS = 94 + 02 = 96 or 102 – 06 = 96

RHS = – 06 x + 02 = – 12

Transfer 1x100 from LHS to RHS. (100 is the base)

Then we have (96 – 1)/(100 – 12) = 9598

Example 3:

Multiply 984 x 1008

984 – 016
x 1008 + 008
992 / –128
992–1/1000–128
991872

Thus, 984 x 1008 = 991872

Working from left to right (Mental work)

Base = 1000, RHS Digits = 3

LHS = 984 + 008 = 992 or 1008 – 016 = 992

RHS = – 016 x + 008 = –128

Transfer 1x1000 from LHS to RHS. (1000 is the base)

Then we have (992 – 1)/(1000 – 128) = 991872

Example 4: Multiply 7 x 17.

7 – 3
x 17 + 7
14 / –21
14–3/30–21
119

Thus, 7 x 17 = 119

Working from left to right (Mental work)

Base = 10, RHS Digits = 1

LHS = 7 + 7 = 14 or 17 – 3 = 14

RHS = – 3 x + 7 = – 21

Transfer 3x10 from LHS to RHS. (10 is the base)

Then we have (14 – 3)/(30 – 21) = 119

Example 5: Multiply 94 x 124.

94 – 06
x 124 + 24
118 / –144
118–2/200–144
11656

Thus, 94 x 102 = 9598

Working from left to right (Mental work)

Base = 100, RHS Digits = 2

LHS = 94 + 24 = 118 or 124 – 06 = 118

RHS = – 06 x + 24 = – 144

Transfer 2x100 from LHS to RHS. (100 is the base)

Then we have (118 – 2)/(200 – 144) = 11656

Example 6:

Multiply 734x1006.

734 – 266
x 1006 + 006
740 /–1596
740–2 / 2000–1596
738404

Thus, 734 x 1006 = 738404

Working from left to right (Mental work)

Base = 1000, RHS Digits = 3

LHS = 734 + 006 = 740 or 1006 – 266 = 740

RHS = – 266 x + 006 = –1596

Transfer 2x1000 from LHS to RHS. (1000 is the base)

Then we have (740 – 2)/(2000 – 1596) = 738404

Working with common base

Uphill now, we studied the multiplication of numbers near a base. We shall consider the case of multiplication of numbers having a common working base (WB). We define the base factor (BF) as,

$$BF = \frac{WorkingBase}{NormalBase}$$

Consider the multiplication of 43 x 46. Here, working base is 40 = 4 x 10, where 4 is BF, such that, 43 = 4 x 10 + 3 and 46 = 4 x 10 + 6. We proceed as usual. But, we have to multiply the LHS by BF, i.e., 4 here.

Example 1:

Multiply 43 x 46.

WB = 40, BF = 4.

43 + 3
x 46 + 6
4 x 49 / 18
196 / 8
1 /
1978
43x46=1978

Observe that the LHS is multiplied by the BF before applying digit rule.

Example 2:

Multiply 496 x 468.

WB = 500. BF = 5.

496 – 04
x 468 – 32
5 x 464 / 128
2320 / 28
1 /
232128
496x468=232128

Example 3:

Multiply 3988 x 4213.

WB = 4000, BF = 4.

3988 – 012
x 4213 + 213
4 x 4201 / –2556
16804 / –2556
16804–3/3000–2556
16801444
3988x4213=16801444

Transfer 3x1000 from LHS to RHS.

General Multiplication

So far we have discussed some special cases of multiplication. Vedic Mathematics gives a hint to multiply any two given numbers. The Vedic Sutra "Vertically and Crosswise" – Urdhva Tiryak – helps us to this goal. In this method multiplication can be done from left to right or from right to left.

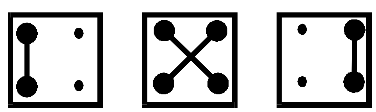
An (n x n) digit multiplication gives 2n or (2n – 1) digits. The format and the methods are given below.

Multiplication of 2 x 2 digit numbers.

Method

- Write the numbers one below the other.
- a b
  x c d
- Divide the answer space into three parts using slash (/) or colon (:).
- Step 1: Find (a x c) – Multiplying vertically on left side.
- Step 2: Find (a x d + b x c) – Multiplying crosswise and adding.
- Step 3: Find (b x d) – Multiplying vertically on right side.
- Write the respective products at appropriate places in the answer space.

The method can be remembered easily with the help of the following diagrams. Each dot denotes a digit and the lines represent the multiplication of digit pairs.



The following examples will illustrate the method.

Example 1: Multiply 12 x 13

```
   12
 x 13
 ------
 1 : 5 : 6
 ------
  156
 ------
```

Thus, 12 x 13 = 156

Working from left to right (Mental work)

Step 1: 1 x 1 = 1 – vertically left

Step 2: 1 x 3 + 2 x 1 = 3 + 2 = 5 – crosswise

Step 3: 2 x 3 = 6 – vertically right

Example 2: Multiply 34 x 72

```
   34
 x 72
 ------
 21 : 34 : 8
 ------
```

Working from left to right (Mental work)

Step 1: 3 x 7 = 21 - vertically left

Step 2: 3 x 2 + 4 x 7 = 6 + 28 = 34 - crosswise

Step 3: 4 x 2 = 8 - vertically right

Note: Number of digits in unit, ten, … places must be only one, except the

2 1 4 8

3

2 4 4 8

highest place. Extra digits may be transferred to next higher places.

Thus, 34 x 72 = 2448

Example 3: Multiply 68 x 56

Working from left to right (Mental work)

68

x 56

30 : 76 : 48

3 0 6 8

7 4

3 8 0 8

Step 1: 6 x 5 = 30 - vertically left

Step 2: 6 x 6 + 8 x 5 = 36 + 40 = 76 - crosswise

Step 3: 8 x 6 = 48 - vertically right

Thus, 68 x 56 = 3808

Example 4: Multiply 84 x 92

Working from left to right (Mental work)

84

x 92

72 : 52 : 8

7 2 2 8

5

7 7 2 8

Step 1: 8 x 9 = 72 - vertically left

Step 2: 8 x 2 + 4 x 9 = 16 + 36 = 52 - crosswise

Step 3: 4 x 2 = 8 - vertically right

Thus, 84 x 92 = 7728

Multiplication of 3 x 3 digit numbers.

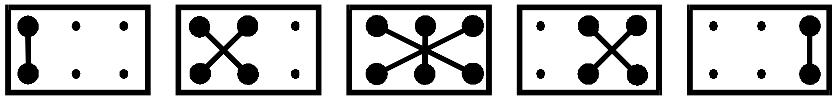
Method

- Write the numbers one below the other.
- a b c

  x d e f
- Divide the answer space into 5 parts using slash (/) or colon (:).
- Step 1: Find (a x d)
- Step 2: Find (a x e + b x d)
- Step 3: Find (a x f + b x e + c x d).
- Step 4: Find (b x f + c x e)
- Step 5: find (c x f)
- Write the respective products at appropriate places in the answer space.

The method can be remembered easily with the help of the following diagrams.

Each dot denotes a digit and the lines represent the multiplication of digit pairs.



The following examples will illustrate the method.

Example 1: Multiply 236 x 482

```
   236
 x 482
------------------------
 8 : 28 : 52 : 54 : 12
------------------------
 8 8 2 4 2
 2 5 5 1
------------------------
1 1 3 7 5 2
------------------------
```

Thus, 236 x 482 = 113752

Working from left to right (Mental work)

Step 1: 2x4 = 8

Step 2: 2x8 + 3x4 = 16+12 = 28

Step 3: 2x2 + 3x8 + 6x4 = 4+24+24=52

Step 4: 3x2 + 6x8 = 6 + 48 = 54

Step 5: 6x2 = 12

Example 2: Multiply 738 x 659

```
   738
 x 659
------------------------
 42 : 53 : 126 : 67 : 72
------------------------
 4 2 3 6 7 2
   5 2 6 7
   1
------------------------
 4 8 6 3 4 2
------------------------
```

Thus, 738 x 659 = 486342

Working from left to right (Mental work)

Step 1: 7x6 = 42

Step 2: 7x5 + 3x6 = 35+18 = 53

Step 3: 7x9 + 3x5 + 8x6 = 63+15+48=126

Step 4: 3x9 + 8x5 = 27 + 40 = 67

Step 5: 8x9 = 72

Example 3: Multiply 574 x 836

```
   574
 x 836
------------------------
 40 : 71 : 83 : 54 : 24
------------------------
 4 0 1 3 4 4
   7 8 5 2
------------------------
 4 7 9 8 6 4
------------------------
```

Thus, 574 x 836 = 479864

Working from left to right (Mental work)

Step 1: 5x8 = 40

Step 2: 5x3 + 7x8 = 15+56 = 71

Step 3: 5x6 + 7x3 + 4x8 = 30+21+32=83

Step 4: 7x6 + 4x3 = 42 + 12 = 54

Step 5: 4x6 = 24

Example 4: Multiply 972 x 638

```
   972
 x 638
------------------------
```

Working from left to right (Mental work)

Step 1: 9x6 = 54

Step 2: 9x3 + 7x6 = 27+42 = 69

54 : 69 : 105 : 62 : 16

5 4 9 5 2 6

6 0 6 1

1

6 2 0 1 3 6

Thus, 972 x 638 = 620136

Step 3: 9x8 + 7x3 + 2x6 = 72+21+12=105

Step 4: 7x8 + 2x3 = 56 + 6 = 62

Step 5: 2x8 = 16

Multiplication of 4 x 4 digit numbers.

Method

- Write the numbers one below the other.

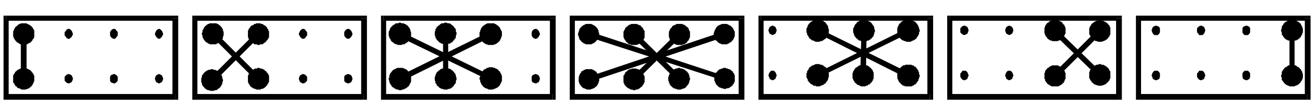
  a b c d

  x e f g h

- Divide the answer space into 7 parts using slash (/) or colon (:).
- Step 1: Find (a x e)
- Step 2: Find (a x f + b x e)
- Step 3: Find (a x g + b x f + c x e).
- Step 4: Find (a x h + d x e + b x g + c x f)
- Step 5: Find (b x h + c x g + d x f)
- Step 6: Find (c x h + d x g)
- Step 7: Find (d x h)
- Write the respective products at appropriate places in the answer space.

The method can be remembered easily with the help of the following diagrams.

Each dot denotes a digit and the lines represent the multiplication of digit pairs.



The following examples will illustrate the method.

Example 1: Multiply 2463 x 3728

2463

x 3728

6:26:50:75:65:54:24

6 6 0 5 5 4 4

2 5 7 6 5 2

9 1 8 2 0 6 4

Thus, 2463 x 3728 = 9182064

Working from left to right (Mental work)

Step 1: 2x3=6

Step 2: 2x7+4x3 =14+12=26

Step 3: 2x2+4x7+6x3=4+28+18=50

Step 4: 2x8+3x3+4x2+6x7=16+9+8+42=75

Step 5: 4x8+6x2+3x7=32+12+21=65

Step 6: 6x8+3x2=48+6=54

Step 7: 3x8=24

Example 2: Multiply 6378 x 7596

```
     6378
   x 7596
-----------------------------
   42:51:118:154:121:114:48
-----------------------------
    4 2 1 8 4 1 4 8
      5 1 5 2 1 4
      1 1 1 1
-----------------------------
   4 8 4 4 7 2 8 8
-----------------------------
```

Thus, 6378 x 7596 = 48447288

Working from left to right (Mental work)

Step 1: 6x7=42

Step 2: 6x5+3x7 =30+21=51

Step 3: 6x9+3x5+7x7=54+15+49=118

Step 4: 6x6+8x7+3x9+7x5=36+56+27+35=154

Step 5: 3x6+7x9+8x5=18+63+40=121

Step 6: 7x6+8x9=42+72=114

Step 7: 8x6=48

Example 3: Multiply 5743 x 6859

```
     5743
   x 6859
-----------------------------
   30:82:105:130:107:51:27
-----------------------------
   3 0 2 5 0 7 1 7
     8 0 3 0 5 2
     1 1 1
-----------------------------
   3 9 3 9 1 2 3 7
-----------------------------
```

Thus, 5743 x 6859 = 39391237

Working from left to right (Mental work)

Step 1: 5x6=30

Step 2: 5x8+7x6 =40+42=82

Step 3: 5x5+7x8+4x6=25+56+24=105

Step 4: 5x9+3x6+7x5+4x8=45+18+35+32=130

Step 5: 7x9+4x5+3x8=63+20+24=107

Step 6: 4x9+3x5=36+15=51

Step 7: 3x9=27

Summary

So far we studied various methods of multiplication. But, one may like to know which method suits best? The choice is personal. All methods give the same result as will be seen below. We must select the simplest and easiest method.

Let us consider a problem worked out by different methods. Suppose we want to multiply **->**

95 x 95.

By Ekadhika Multiplication.

Multiply 95 x 95.

9 is common. 5 + 5 = 10.

Ekadhika is applicable.

End digit = 5,

Previous digit = 9

$$\begin{array}{c} 95^2 \\ \hline 9 \times (9+1) \;/\; 25 \\ \hline 90 \;/\; 25 \\ \hline \end{array}$$

Therefore, 95^2 = 9025.

By Nikhilam Multiplication.

Multiply 95 x 95.

95 is nearer to Base=100.

Deviation = 95–100=–05

Nikhilam is applicable

$$\begin{array}{c} 95 - 05 \\ \times\; 95 - 05 \\ \hline 95\text{-}05 \;/\; 25 \\ \hline 9025 \\ \hline \end{array}$$

Therefore, 95 x 95 = 9025

By Urdhva Tiryak Method.

Multiply 95 x 95

$$\begin{array}{l} \quad 95 \\ \times\; 95 \\ \hline 81 : 90 : 25 \\ \hline 9025 \\ \hline \end{array}$$

Therefore, 95 x 95 = 9025

So, which method has to be preferred? Probably, Ekadhika method is best suited in this case. Is it not? Remember to work out the problem mentally and write the answer in one or line.

Division

Division is the hardest of all the arithmetical operations. The traditional method of division is always the same irrespective of the divisor. But in Vedic Mathematics there are different methods depending on the nature of the divisor.

The format for division in Vedic Mathematics is shown below.

$$\begin{array}{l|l} \text{Divisor} & \text{Dividend} \\ & \\ \hline & \text{Quotient / Remainder} \end{array}$$

We shall discuss various methods of division in Vedic Mathematics.

Division by 9.

Division by 9 is the simplest one in Vedic Mathematics. We divide the dividend by the devisor to get quotient and remainder. When the remainder is equal to or greater than 9 we re-divide the remainder by 9, carry over this quotient to the quotient side and retain the final remainder in the remainder side. We shall study the method by some illustrations.

Example 1: Divide 12 ÷ 9.

Dividend = 12,

Devisor = 9.

$$\begin{array}{r|l} 9 & 1 \;/\; 2 \\ & 0 \quad 1 \\ \hline & 1 \;/\; 3 \end{array}$$

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1^{st} digit and add. This gives quotient, Q = 1.

Step 4: Write the result (i.e., 1) under 2^{nd} digit and add to get 2 + 1 = 3. This gives the Remainder, R= 3.Thus, 12 ÷ 9 → Q = 1, R = 3.

Example 2:

Divide 123 ÷ 9.

Dividend = 123,

Devisor = 9.

9	1 2 / 3
	0 1 3
	1 3 / 6

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1st digit and add. This gives quotient, Q = 1.

Step 4: Write the result (i.e., 1) under 2nd digit and add to get 2+1=3. Division is over and Q = 13.

Step 5: Write this result (i.e., 3) under 3rd digit and add 3+3=6.This gives the Remainder, R= 6.

Thus, 12 ÷ 9 –> Q = 13, R = 6.

Example 3:

Divide 2340 ÷ 9.

Dividend = 2340,

Devisor = 9.

9	2 3 4 / 0
	0 2 5 9
	2 5 9 / 9
	1 / 0
	2 6 0 / 0

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1st digit and add. This gives quotient, Q = 2.

Step 4: Write the result (i.e., 2) under 2nd digit and add to get 3+2=5.

Step 5: Write this result (i.e., 5) under 3rd digit and add to get 4+5=9.

Step 6: Write this result (i.e., 9) under the 4th digit and add to get 0 + 9 =9. Remainder, R= 9.

As the remainder is equal to 9, we re-divide it by 9 and compute new quotient and remainder.

Thus we get, 2340 ÷ 9 –> Q = 260, R = 0.

Example 4:

Divide 3509 ÷ 9.

Dividend = 3509,

Devisor = 9.

9	3 5 0 / 9
	0 3 8 / 8
	3 8 8 / 17
	1 / 8
	3 8 9 / 8

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1st digit and add. This gives quotient, Q = 3.

Step 4: Write the result (i.e., 3) under 2nd digit and add to get 5+3=8.

Step 5: Write this result (i.e., 8) under 3rd digit and add to get 0+8=8.

Step 6: Write this result (i.e., 8) under the 4th digit and add to get 9 + 8 =17. Remainder, R= 17.

As the remainder is greater than 9, we re-divide it by 9 and compute new quotient and remainder.

Thus we get, 3509 ÷ 9 –> Q = 389, R = 8.

Division By 8.

This method is similar to the one with division by 9. The only difference is, we write 2, the complement of 8, below 8. Every time we multiply the quotient digit by 2 and add it to the next digit. When the remainder is equal to or greater than 8 we re-divide the remainder by 8, carry over this quotient to the quotient side and retain the final remainder on the remainder side.

Example 1:

Divide 13 ÷ 8.

Dividend = 13,

Devisor = 8.

$$\begin{array}{r|l} 8 & 1\ /\ 3 \\ 2 & 0\ \ \ 2 \\ \hline & 1\ /\ 5 \end{array}$$

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1^{st} digit and add. This gives quotient, Q = 1.

Step 4: Now, 1 x 2 = 2, 3 + 2 = 5. Remainder, R= 5.

Thus, 13 ÷ 8 $\rightarrow$ Q = 1, R = 5.

Example 2:

Divide 257 ÷ 8.

Dividend = 257,

Devisor = 8.

$$\begin{array}{r|l} 8 & 2\ 5\ /\ 7 \\ 2 & 0\ 4\ \ 18 \\ \hline & 2\ 9\ /\ 25 \\ & \ \ \ 3\ /\ 1 \\ \hline & 3\ 2\ /\ 1 \end{array}$$

Working (Mental)

Step 1: Write the devisor and dividend as shown.

Step 2: Set off last digit for the remainder by a slash (/) or colon (:).

Step 3: Enter 0 under 1^{st} digit and add. This gives quotient, Q = 2.

Step 4: Now, 2 x 2 = 4, 5 + 4 = 9.

Step 5: Again, 9 x 2 = 18, 7 + 18 = 25.

As the remainder is greater than 8, we re-divide it by 8 and compute new quotient and remainder.

Thus we get, 257 ÷ 8 –> Q = 32, R = 1.

We can extend this method for division with 7, 6, . . .

Next we shall consider the division with divisors having more than one digit, and when the divisors are slightly smaller or greater than the base.

Nikhilam Division

This method is useful for division with the devisors nearer and below the base. The method consists in finding the complement of the devisor using Nikhilam Sutra "All from 9". Hence, the method is called Nikhilam division.

Method

- Write the problem in the usual format.
- Find the complement of the devisor and write it below the devisor.
- Divide the dividend into quotient part and remainder part by a slash (/) or colon (:).
- The number of digits in the remainder part must be equal to the number of zeros in the base.

- Leave sufficient space between the dividend and quotient lines. The number of lines must be equal to the number of digits in the quotient part of the dividend.
- Bring down the 1^{st} digit of dividend as the 1^{st} digit of the quotient.
- Multiply the 1^{st} quotient digit with the complement of the devisor and write the product starting under 2^{nd} column.
- Add the digits in 2^{nd} column to get 2^{nd} quotient digit.
- Multiply the 2^{nd} quotient digit again with the complement of the devisor and write the product starting under 3^{rd} column.
- Add the digits in 3^{rd} column to get 3^{rd} quotient digit.
- Repeat this as many times as the digits in the quotient part of dividend.
- Add all digits under remainder part to get the final remainder. If this exceeds the devisor, re-divide.
- If the sum exceeds 9, carry over the excess to the left.

Example 1: Divide 1318 ÷ 88.

```
8 8 | 1 3 : 1  8
1 2 |   1   2
    |       4  8
    -------------
      1 4 : 8  6
```

Working (Mental)

Step 1: Complement of devisor 88 is 12.

Step 2: Split dividend as 13:18. Bring down 1.

Step 3: 1x12=12. Write it as shown.

Step 4: 3+1=4 and 4x12=48. Division over.

Step 5: Remainder = 18+20+48=86.

Hence, 1318 ÷ 88 –> Q = 14, R = 86.

Example 2: Divide 2316 ÷ 97.

```
9 7 | 2 3 : 1 6
0 3 |   0   6
    |       0 9
    ------------
      2 3 : 8 5
```

Working (Mental)

Step 1: Complement of devisor 97 is 03.

Step 2: Split dividend as 23:16. Bring down 2.

Step 3: 2x03=06. Write it as shown.

Step 4: 3+0=3 and 3x03=09. Division over.

Step 5: Remainder = 16+60+09=85.

Hence, 2316 ÷ 97 –> Q = 23, R = 85.

Example 3: Divide 38445 ÷ 896.

```
8 9 6 | 3  8 :  4  4   5
1 0 4 |    3    1  2
      |        11  0  44
      -------------------
        3  11: 16 6  49
      -------------------
        4 1 : 1709
      -------------------
        4 2 : 813
```

Working (Mental)

Step 1: Complement of devisor 896 is 104.

Step 2: Split dividend as 38:445. Bring down 3.

Step 3: 3x104=312. Write it as shown.

Step 4: 3+8=11 and 11x104=11 0 44.

Step 5: Remainder = 1600+60+49=1709.

1709 ÷ 896 = Q = 1, R = 813.

Final quotient = 3x10 + 11 + 1 = 42.

Hence, 38445 ÷ 896 –> Q = 42, R = 813.

Paravartya Division

This method is useful for division with the devisors nearer and above the base. The method is similar to Nikhilam division, but slightly different. It uses the Vedic Sutra "Transpose and apply" known as Paravartya. We leave the first digit, and change the sign of other digit or digits with minus (-) sign and place them below the divisor. This is the meaning of �può transpose and apply ’.

Method

- Write the problem in the usual format.
- Leaving the first digit, write other digit or digits using minus (-) sign and place them below the divisor.
- Divide the dividend into quotient part and remainder part by a slash (/) or colon (:).
- The number of digits in the remainder part must be equal to the number of zeros in the base.
- Leave sufficient space between the dividend and quotient lines. The number of lines must be equal to the number of digits in the quotient part of the dividend.
- Bring down the 1st digit of dividend as the 1st digit of the quotient.
- Multiply the 1st quotient digit with the complement of the devisor and write the product starting under 2nd column.
- Add the digits in 2nd column to get 2nd quotient digit.
- Multiply the 2nd quotient digit again with the complement of the devisor and write the product starting under 3rd column.
- Add the digits in 3rd column to get 3rd quotient digit.
- Repeat this as many times as the digits in the quotient part of dividend.
- Add all digits under remainder part to get the final remainder. If this exceeds the devisor, re-divide.
- If the sum exceeds 9, carry over the excess to the left.

Example 1: Divide 358 ÷ 12.

1 2	3 5 : 8
–2	–6
	2
	3 –1: 10
	2 9 : 10

Working (Mental)

Step 1: Drop 1 of devisor 12. New devisor is –2

Step 2: Split dividend as 35:8. Bring down 3.

Step 3: 3x–2= –6 . Write it as shown.

Step 4: 5+(–6)=–1 and–1x–2 = 2. Division over.

Step 5: Remainder =8+2=10.

Hence, 358 ÷ 12 –> Q =3x10–1=29, R = 10.

Example 2: Divide 24688 ÷ 102.

```
1 0 2 | 2  4   6 : 8  8
  0 –2|    0 –4
      |        0 –8
      |            0 –4
      |__________________
         2  4  2:  0  4
```

Working (Mental)

Step 1: Drop 1 of devisor 122. New devisor is 0 –2

Step 2: Split dividend as 246:88. Bring down 2.

Step 3: (0 –2)x2= 0 –4. Write it as shown.

Step 4: 4+0=4 and (0 –2)x4= 0 –8. Division over.

Finally, 6+(–4)=2 and (0 –2)x2= 0 –4.

Step 5: Remainder =88–80–04= 04.

Hence, 358 ÷ 12 –> Q =242, R = 4.

Example 3: Divide 36478 ÷ 1012.

```
1 0 1 2 | 3  6 :  4   7    8
0 –1 –2 |    0  –3  –6
        |         0  –6 –12
        |____________________
          3 6 :  4 6
```

Working (Mental)

Step 1: Drop 1 of devisor 1012. New devisor 0 –1 –2

Step 2: Split dividend as 36:478. Bring down 3.

Step 3: (0 –1 –2)x3 =0 –3 –6. Write it as shown.

Step 4: 6+0 =6 and (0 –1 –2)x6 =0 –6 –12.

Step 5: Remainder =478–360–60–12 = 46

Hence, 36478 ÷ 1012 –> Q =36, R = 46.

Straight Division

This is the general method of division applicable to all types of division. This uses the combination of Vedic Sutras: "Dhvajanka and Urdhva Tiryak". The meaning of the word Dhvajanka is "flag digit", since it is written as a flag little above the operator digit. The method is given below.

Method

- Separate the first digit of the devisor from other digits. The first digit is called operator and other digits are called flag digits.
- Flag digits are placed at a higher level above the operator.
- Divide the dividend into two parts – quotient part and remainder part – using a slash (/) or a colon (:).
- The number of digits in the remainder part must be same as the number of flag digits.
- Divide the first digit (or first two digits) by the operator to get a partial quotient and a remainder.
- This remainder with the next digit of the quotient part of dividend forms the gross dividend (GD).
- Find the product of flag digits and partial quotient to get the flag dividend (FD).
- FD is calculated using Urdhva Tiryak. Note the method of calculation carefully.
- The difference between GD and FD gives net dividend ND.
- Divide ND further by the operator to get the next partial quotient and remainder.
- Repeat the process with the remaining digits of the quotient part.

- Finally, evaluate the remainder (R=ND–FD).

The above steps are illustrated in the example below.

Division by devisors with 1 flag digit

Example 1: Divide 5284 ÷ 32

$$\begin{array}{r|cccc} 2 & 5 & 2 & 8: & 4 \\ 3 & & 2 & 2 & :1 \\ \cline{2-5} & 1 & 6 & 5 & :4 \end{array}$$

Here, 3 –> operator, 2 –> flag digit.

Note that the remainder part has only one digit. (Guess)

Working (Mental)

Step 1: 5 ÷ 3 = 1 R 2

(Read Q=1, R=2). Written as above.

GD = 22, FD = 2 x 1 = 2

ND = 22 – 2 = 20 ÷ 3 = 6 R 2, written as above.

Step 2: GD = 28, FD =2 x 6 = 12

ND = 28 – 12 = 16 ÷ 3 = 5 R1, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 3: GD = 14, FD = 2x5=10.

Remainder = 14 – 10 = 4.

Hence, 5284 ÷ 32 --> Q = 165, R = 4.

Example 2: Divide 3275 ÷ 23

$$\begin{array}{r|cccc} 3 & 3 & 2 & 7: & 5 \\ 2 & & 1 & 1 & :1 \\ \cline{2-5} & 1 & 4 & 2 & :9 \end{array}$$

Here, 2 –> operator, 3 –> flag digit.

Working (Mental)

Step 1: 3 ÷ 2 = 1 R 1, written as above.

GD = 12, FD = 3 x 1 = 3, ND = 12 – 3 = 9 ÷ 2 = 4 R 1, written as above.

Step 2: GD = 17, FD = 3 x 4 = 12

ND = 17 – 12 = 5 ÷ 2 = 2 R 1, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 3: GD = 15, FD = 3x2= 6.

Remainder = 15 – 6 = 9.

Hence, 3275 ÷ 23 –> Q = 142, R = 9.

Example 3: Divide 5678 ÷ 43

$$\begin{array}{cc|cccc} & 3 & 5 & 6 & 7: & 8 \\ 4 & & & 1 & 1 & :0 \\ \cline{3-6} & & 1 & 3 & 2 & :2 \end{array}$$

Here, 4 –> operator, 3 –> flag digit.

Working (Mental)

Step 1: 5 ÷ 4 = 1 R 1, written as above.

GD = 16, FD = 3 x 1 = 3, ND = 16 – 3 = 13 ÷ 4 = 3 R 1, written as above.

Step 2: GD = 17, FD = 3 x 3 = 9

ND = 17 – 9 = 8 ÷ 4 = 2 R 0, written as above.

Here, division is over.

Step 3: GD = 8, FD = 3x2=6.

Remainder = 8 – 6 = 2.

Hence, 5678 ÷ 43 –> Q = 132, R = 2.

Example 4: Divide 27543 ÷ 64

$$\begin{array}{cc|cccc} & 4 & 27 & 5 & 4: & 3 \\ 6 & & & 3 & 1 & :2 \\ \cline{3-6} & & 4 & 3 & 0: & 23 \end{array}$$

Here, 6 –> operator, 4 –> flag digit.

Working (Mental)

Step 1: 27 ÷ 6 = 4 R 3, written as above.

GD = 35, FD = 4 x 4 = 16

ND = 35 – 16 = 19 ÷ 6 = 3 R 1, written as above.

Step 2: GD = 14, FD = 4 x 3 = 12

ND = 14 – 12 = 2 ÷ 6 = 0 R 2, written as above.

Here, division is over.

Step 3: GD = 23, FD = 4x0=0.

Remainder = 23 – 0 = 23.

Hence, 27543 ÷ 64 → Q = 430, R = 23.

Division by Devisors with 2 flag digits

Example 1: Divide 237963 ÷ 524

24	23	7	9:	63
5		3	4	:3
	4	5	4	: 67

Here, 5 –> operator, 24 –> flag digits.

Note that the remainder part has two digits. (Guess)

Working (Mental)

Step 1: 23 ÷ 5 = 4 R 3 , written as above.

GD = 37, FD = 2 x 4 + 4 x 0 = 8

ND = 37 – 8 = 29 ÷ 5 = 5 R 4, written as above.

Step 2: GD = 49, FD=2 x 5 + 4 x 4 = 26

ND = 49 – 26 = 23 ÷ 5 = 4 R 3, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 3: GD = 363,

FD = (2x4+4x5)/(2x0+4x4) = 28/16 = 296. (Guess how? 280 + 16)

Remainder = 363 – 296 = 67.

Hence, 237963 ÷ 524 –> Q = 454, R = 67.

Example 2: Divide 2999222 ÷ 713

13	29	9	9	2:	22
7		1	1	5	:4
	4	2	0	6	: 344

Here, 7 –> operator, 13 –> flag digits.

Working (Mental)

Step 1: 29 ÷ 7 = 4 R 1 , written as above.

GD = 19, FD = 1 x 4 + 3 x 0 = 4

ND = 19 – 4 = 15 ÷ 7 = 2 R 1, written as above.

Step 2: GD = 19, FD = 1 x 2 + 3 x 4 = 14

ND = 19 – 14 = 5 ÷ 7 = 0 R 5, written as above.

Step 3: GD = 52, FD = 1 x 0 + 3 x 2 = 6

ND = 52 – 6 = 46 ÷ 7 = 6 R 4, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 4: GD = 422,

FD = (1x6+3x0) / (1x0+3x6) = 6 / 18 = 78.

Remainder = 422 – 78 = 344.

Hence, 2999222 ÷ 713   Q = 4206, R = 344.

Example 3: Divide 35769 ÷ 512

	12	35 7: 69
5		5 :6
		6 9 :441

Here, 5 –> operator, 12 –> flag digits.

Working (Mental)

Step 1: 35 ÷ 5 = 6 R 5 , written as above.

(With higher quotient digit remainder becomes negative. Check?)

GD = 57, FD = 1 x 6 + 2 x 0 = 6

ND = 57 – 6 = 51 ÷ 5 = 9 R 6, written as above.

We shall calculate the remainder as below.

Step 2: GD = 669,

FD = (1x9+2x6) / (1x0+2x9) = 21 / 18 = 228.

Remainder = 669 – 228 = 441.

Hence, 35769 ÷ 512 –> Q = 69, R = 441.

Division by Devisors with 3 flag digits

Example 1: Divide 163758 ÷ 5314

	341	16 3: 758
5		1 :4
		3 0 : 3528

Here, 5 –> operator, 341 –> flag digits.

Working (Mental)

Step 1: 16 ÷ 5 = 3 R 1 , written as above.

GD = 13, FD = 3x3+4x0+1x0=9

ND = 13 – 9 = 4 ÷ 5 = 0 R 4, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 2: GD = 4758,

FD= (3x0+4x3+1x0)/(3x0+4x0+1x3)/(3x0+4x0+1x0)

= (0+12+0) / (0+0+3)/(0+0+0) : = 12 / 3 / 0 = 1230.

Remainder = 4758 – 1230 = 3528.

Hence, 163758 ÷ 5314 → Q = 30, R = 3528.

Example 2: Divide 563214 ÷ 4123

	123	5 6 3: 214
4		1 3 :4
		1 3 6 : 2486

Here, 4 –> operator, 123 –> flag digits.

Working (Mental)

Step 1: 5 ÷ 4 = 1 R 1 , written as above.

GD = 16, FD =1x1+2x0+3x0=1

ND = 16 – 1 = 15 ÷ 4 = 3 R 3, written as above.

Step 2: GD = 33, FD = 1x3+2x1+3x0=5

ND = 33 – 5 = 28 ÷ 4 = 6 R 4, written as above.

Here, division is over. We shall calculate the remainder as below.

Step 3: GD = 4214,

FD = (1x6+2x3+3x1)/(1x0+2x6+3x3)/(1x0+2x0+3x6)

= (6+6+3) / (0+12+9)/(0+0+18)

= 15 / 21 / 18 = 1728. (Guess how?)

Remainder = 4214 – 1728 = 2486.

Hence, 563214 ÷ 4123 –> Q = 136, R = 2486.

Example 3: Divide 12345678 ÷ 8234

	234	12 3 4 5: 678
8		4 9 11 :9
		1 4 4 9 : 2912

Here, 8 –> operator, 234 –> flag digits.

Working (Mental)

Step 1: 12 ÷ 8 = 1 R 4 , written as above.

GD = 43, FD = 2x1+3x0+4x0=2

ND = 43 – 2 = 41 ÷ 8 = 4 R 9, written as above.

Step 2: GD = 94, FD = 2x4+3x1+4x0=11

ND = 94 – 11 = 83 ÷ 8 = 9 R 11 written as above.

Step 3: GD = 115, FD = 2x9+3x4+4x1=34

ND = 115 – 34 = 81 ÷ 8 = 9 R 9 written as above.

Here, division is over. We shall calculate the remainder as below.

Step 4: GD = 9678,

FD = (2x9+3x9+4x4)/(2x0+3x9+4x9)/(2x0+3x0+4x9)

= (18+27+16) / (0+27+36)/(0+0+36)

= 61 / 63 / 36 = 6766. (Guess how?)

Remainder = 9678 – 6766 = 2912.

Hence, 12345678 ÷ 8234 –> Q = 1499, R = 2912.

Summary

So far we studied various methods of division. But, one may like to know which method suits best? The choice is personal. All methods give the same result as will be seen below. We must select the simplest and easiest method.

Let us consider a problem worked out by different methods. Suppose we want to divide 2345 by 98.

By Nikhilam Division Method.

Divide 2345 ÷ 98.

Modified divisor = 100–98 = 02

```
98 | 2 3 : 4 5
02 |   0   4
   |       0 6
   -----------
     2 3 : 91
```

By Paravartya Method

Divide 2345 ÷ 98.

We know 98 =100+(–2).

Modified divisor = 02

```
98 | 2 3 : 4 5
02 |   0   4
   |       0 6
   -----------
     2 3 : 91
```

By Straight Division Method

Divide 2345 ÷ 98.

Operator = 9

Flag digit = 8

```
  8 | 23  4 :  5
9   |      5 :11
    -------------
      2  3 : 9 1
```

So, either of the first two methods is preferable in this case.

Squares and Square Roots

When a number is multiplied by itself we get the square of that number.

E.g., 2 x 2 = 4, 3 x 3 = 9. We say that:

4 is the square of 2 and 2 is the square root of 4.

9 is the square of 3 and 3 is the square root of 9.

The squares of digits 1 – 9 are given below.

Digits	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

We notice that the digits equidistant from 5 have same end digits. We shall study the methods of squaring of a number.

Ekadhika method

We studied earlier the method of finding the square of numbers ending in 5 under Ekadhika multiplication. E.g., $35^2 = 3 \times 4 / 25 = 1225$,

$65^2 = 6 \times 7 / 25 = 4225$, etc.

If the number is large we can use Urdhva Tiryak multiplication in concurrence to achieve this. This is illustrated in the following examples.

Example 1: Find the square of 285.

$285^2 = 28 \times 29 / 25$

$$
\begin{array}{r}
2\ 8 \\
\times\ 2\ 9 \\
\hline
4\ 4\ 2 \\
3\ 7\ \ \\
\hline
8\ 1\ 2 \\
\hline
\end{array}
$$

Therefore, $285^2 = 812/25 = 81225$.

Example 2: Find the square of 1235.

$1235^2 = 123 \times 124 / 25$

But,

$$
\begin{array}{r}
1\ 2\ 3 \\
\times\ 1\ 2\ 4 \\
\hline
1\ 4\ 1\ 4\ 2 \\
1\ 1\ 1\ \ \\
\hline
1\ 5\ 2\ 5\ 2 \\
\hline
\end{array}
$$

Therefore, $1235^2 = 15252/25 = 1525225$.

Yavadunam Method

The Vedic Sutra "Deviate as much as deviation and add square of the deviation". This is known as Yavadunam Sutra. We shall see the application of this Sutra to find the square of numbers.

The answer consists of 2 portions as given here under.

- The excess (deviation) is added to the number. This forms the Left Portion of the answer.
- The square of the initial excess forms the Right Portion of the answer.

This is explained below with examples.

Example 1: Find the square of 12.

Here, Base = 10, Deviation = 12 – 10 = +2.

Hence, $12^2 = 12 + 2 / 2^2 = 144$.

Example 2: Find the square of 108.

Here, Base = 100, Deviation = 108 – 100 = +08.

$108^2 = 108 + 08 / 08^2 = 11664$. (Digit Rule)

Example 3: Find the square of 1015.

Base = 1000, Deviation = 1015 – 1000 = +015.

$1015^2 = 1015 + 015 / 015^2 = 1030225$.

Example 4: Find the square of 8.

Here, Base = 10, Deviation = 8 – 10 = – 2.

Hence, $8^2 = 8 - 2 / (-2)^2 = 64$.

Example 5: Find the square of 96.

Here, Base = 100, Deviation = 96 – 100 = – 04.

Hence, $96^2 = 96 - 4 / (-04)^2 = 9216$.

Example 6: Find the square of 985.

Here, Base = 1000,

Deviation = 985 – 1000 = – 015.

Hence, $985^2 = 985 - 015 / (-015)^2 = 970225$.

Straight Squaring (General Method)

To find the square of any number we use the Vedic Sutra Dwandva Yoga or Duplex System. Duplex of a number is formed (i) by squaring and (ii) by cross-multiplying. If the digit is single central digit, the Duplex is a square and for an even number of digits equidistant from the two ends the Duplex is twice the cross - product. Duplex of numbers are defined as follows:

No	Duplex	Example
a	$D(a) = a^2$	$D(2) = 2^2 = 4$
ab	D(ab) = 2ab	D(23) = 2x2x3=12
abc	$D(abc) = b^2 + 2ac$	$D(234) = 2^2 + 2x3x4 = 28$
abcd	D(abcd)=2ad+2bc	D(2345) = 2x2x5+2x3x4=44

We use Duplex system to find the square of numbers. Observe that for a n-digit number, the square of the number contains 2n or (2n-1) digits.

Squaring of 2 digit numbers

We have: $(a + b)^2 = a^2 + 2ab + b^2$

Or $(a + b)^2 = D(a) + D(ab) + D(b)$.

We use this property as follows.

$(ab)^2 = D(a) : D(ab) : D(b)$

This is clear from the following examples.

Example 1: Find the square of 36.

36^2

D(3):D(36):D(6)

9 : 36 : 36

9 6 6

3 3

Working (Mental)

Step 1: $D(3) = 3^2 = 9$

Step 2: D(36) = 2x3x6 =36

Step 3: $D(6) = 6^2 = 36$.

Hence, $36^2 = 1296$

1 2 9 6

Example 2: Find the square of 74.

74^2

D(7):D(74):D(4)

49 : 56 : 16

4 9 6 6

5 1

5 4 7 6

Working (Mental)

Step 1: D(7) = 7^2 = 49

Step 2: D(74) = 2x7x4 =56

Step 3: D(4) = 4^2 =36.

Hence, 74^2 = 5476

Example 3: Find the square of 85.

85^2

D(8):D(85):D(5)

64 : 80 : 25

6 4 0 5

8 2

7 2 2 5

Working (Mental)

Step 1: D(8) = 8^2 = 64

Step 2: D(85) = 2x8x5 =80

Step 3: D(5) = 5^2 =25.

Hence, 85^2 = 7225

Squaring of 3 digit numbers

We have: $(a + b + c)^2 = a^2 + 2ab + (b^2 + 2ac) + 2bc + c^2$

Or $(a+b+c)^2$ = D(a)+D(ab)+D(abc)+D(bc)+D(c).

We use this property as follows.

$(abc)^2$ = D(a) : D(ab) : D(abc): D(bc) D(c)

This is clear from the following examples.

Example 1: Find the square of 234.

234^2

D(2):D(23):D(234):D(34):D(4)

4 :12 : 25 : 24 : 16

4 2 5 4 6

1 2 2 1

5 4 7 5 6

Working (Mental)

Step 1: D(2) = 2^2 = 4

Step 2: D(23) = 2x2x3 =12

Step 3: D(234) = 3^2+2x2x4=25.

Step 4: D(34) = 2x3x4 =24

Step 5: D(4) = 4^2 =16.

Hence, 234^2 = 1296

Example 2: Find the square of 647.

647^2

D(6):D(64):D(647):D(47):D(7)

Working (Mental)

Step 1: D(6) = 6^2 = 36

Step 2: D(64) = 2x6x4 =48

Step 3: D(647) = 4^2+2x6x7=100.

3 6 8 0 6 9
4 0 5 4
1
4 1 8 6 0 9

Step 4: D(47) = 2x4x7 =56

Step 5: D(7) = 7^2 =49.

Hence, 647^2 = 418609

Example 3: Find the square of 795.

795^2

D(7):D(79):D(795):D(95):D(5)
49 :126 : 151 : 90 : 25
4 9 6 1 0 5
2 5 9 2
1 1
6 3 2 0 2 5

Working (Mental)

Step 1: D(7) = 7^2 = 49

Step 2: D(79) = 2x7x9 =126

Step 3: D(795) = 9^2+2x7x5=151.

Step 4: D(95) = 2x9x5 =90

Step 5: D(5) = 5^2 =25.

Hence, 795^2 = 632025

This method can be extended to other numbers up to any level.

Square Roots Of Perfect Squares

The square root of a perfect square can be found by division method. Before finding the square root of a number we must know 3 things.

- Number of digits in the square root.
  If the number has n-digits, the square root will have n/2 or (n+1)/2 digits.
- First digit of the square root must be known. The number is divided into groups of 2 digits from right to left. The nearest square root of the first group gives the first square root digit.
- The devisor must also be known.
  Twice the first square root digit is taken as devisor.
- While dividing we use Duplex of digits from second digit onwards.

Example 1: Find the square root of 1156.

No of square root digits = 4/2 = 2.

Grouping of digits = 11'56.

First square root digit = 3 (3^2 < 11)

Modified Devisor = 3x2 = 6.

	11 5 6
6	2 1
	3 4.0

Working (Mental)

Step 1: 11 – 3^2 = 11 – 9 = 2

Step 2: 25 ÷ 6 = 4 R 1

Step 3: 16 – D(4) = 16 – 4^2 = 0

Hence, Square Root of 1156 = 34.

Example 2: Find the square root of 6241.

No of square root digits = 4/2 = 2.

Grouping of digits = 62'41.

First square root digit = 7 ($7^2 < 62$)

Modified Devisor = 7x2 = 14.

	62	4	1
14	13	8	
	7	9.0	

Working (Mental)

Step 1: 62 – 7^2 = 62 – 49 = 13

Step 2: 134 ÷ 14 = 9 R 8

Step 3: 81 – D(9) = 81 – 9^2 = 0

Hence, Square Root of 6241 = 79.

Example 3: Find the square root of 80656.

No of square root digits = (5+1)/2 = 3.

Grouping of digits = 8'06'56.

First square root digit = 2.

Modified Devisor = 2x2 = 4.

	8	0	6	5	6
4		4	8	6	1
	2	8	4 .	0	0

Working (Mental)

Step 1: 8 – 2^2 = 8 – 4 = 4

Step 2: 40 ÷ 4 = 8 R 8 (Guess)

Step 3: 86 – D(8) = 86 – 8^2 = 22

22 ÷ 4 = 4 R 6

Step 4: 65 – D(84) = 65 – 64 = 1

I ÷ 4 = 0 R 1

Step 5: 16 – D(4) = 16 – 16 = 0

Hence, Square Root of 80656 = 284.

Example 4: Find the square root of 119025.

No of square root digits =6/2 = 3.

Grouping of digits = 11:90:25.

First square root digit = 3.

Modified Devisor = 3x2 = 6.

	11	9	0	2	5
6		2	5	4	2
	3	4	5 .	00	

Working (Mental)

Step 1: $11 - 3^2 = 11 - 9 = 2$

Step 2: $29 \div 6 = 4$ R 5

Step 3: $50 - D(4) = 50 - 4^2 = 34$

$34 \div 6 = 5$ R 4

Step 4: $42 - D(45) = 42 - 40 = 2$

$2 \div 6 = 0$ R 2

Step 5: $25 - D(5) = 25 - 5 = 0$

Hence, Square Root of 119025 = 345.

Summary

So far we studied various methods of squaring. But, one may like to know which method suits best? The choice is personal. All methods give the same result as will be seen below. We must select the simplest and easiest method.

Let us consider a problem worked out by different methods. Suppose we want to find the square of 95.

By Ekadhika Method

Find the square of 95.

$95^2 = 9 \times 10 / 25$

$95^2 = 9025$

By Yavadunam Method

Find the square of 95.

Here, Base = 100,

Deviation = 95–100 = –05.

$95^2 = (95–05) / 05^2 = 9025$.

By Dwandva Yoga Method

Find the square of 95.

95^2

D(9):D(95):D(5)

81 : 90 : 25

9025

Naturally, Ekadhika method is best suited in this case.

Cubes and Cube Roots

When a number is multiplied by itself three times we get the cube of that number.

E.g., 2 x 2 x 2 = 8, 3 x 3 x 3 = 27, 4 x 4 x 4 = 64, etc. Thus,

8 is the cube of 2 and 2 is the cube root of 8,

27 is the cube of 3 and 3 is the cube root of 27,

64 is the cube of 4 and 4 is the cube root of 64.

The cubes of digits from 1 – 9 are given below.

Digits	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

Thus, the cubes of digits have unique end digits.

We shall next study the method of cubing a number.

Yavadunam Method

This method explained earlier can be used in this case also, with modifications. The answer consists of 3 portions as given here under.

- Twice the excess (deviation) is added to the number. This forms Left Portion of the answer.
- The product of new excess and the original excess forms the Middle Portion of the answer.
- The cube of the initial excess forms the Right Portion of the answer.

This is explained below with examples.

Example 1: Find the cube of 13.

Base = 10. Deviation = +3.

New excess =3 + 6=9

(Deviation + Twice deviation) Hence,

13^3

$13+6 : 9 \times 3 : 3^3$

19 : 27 : 27

1 9 7 7

2 2

2 1 9 7

Therefore, $13^3 = 2197$

Example 2: Find the cube of 106.

Base = 100. Deviation = +6.

New excess = 6+12=18

(Deviation + Twice deviation) Hence,

106^3

$106+12 : 18 \times 6 : 6^3$

118 : 108 : 216

1 1 8 0 8 1 6

1 2

1 1 9 1 0 1 6

Therefore, $106^3 = 1191016$

Example 3: Find the cube of 1003.

Base = 1000. Deviation = +3.

New excess =3 + 6 = 9

(Deviation + Twice deviation) Hence,

1003^3

$1003+6 : 9 \times 3 : 3^3$

1009 : 027 : 027

Example 4: Find the cube of 8.

Base = 10. Deviation = – 2.

New excess = – 2 – 4 = – 6

(Deviation + Twice deviation) Hence,

8^3

$8 - 4 : -6 \times -2 : (-2)^3$

4 : 12 : –8

1009027027

Therefore, 1003^3 = 1009027027

(Digit Rule)

4 : 11 : (10–8)

4 : 11 : 2

4 1 2

1

512

Therefore, 8^3 = 512

Example 5: Find the cube of 93.

Base = 100. Deviation = - 7.

New excess = - 7 - 14 = - 21 Hence,

93^3

93 - 14 : 147 : –343

79 : 147 : –343

79 : 143 : (400–343)

79 : 143 : 057

7 9 4 3 5 7

1 0

8 0 4 3 5 7

Therefore, 93^3 = 804357

Example 6: Find the cube of 995.

Base = 1000. Deviation = - 5.

New excess = - 5 – 10 = - 15 Hence,

995^3

995 - 15: 075 : –125

985 : 075 : –125

985 : 074 : (1000–125)

985074875

Therefore, 985^3 = 985074875 (Digit Rule)

Straight Cubing of 2 digit numbers

To find cube of any number directly we use the formula: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

We rewrite this as

a^3	a^2b	ab^2	b^3
	$2a^2b$	$2ab^2$	
a^3	$3a^2b$	$3ab^2$	b^3

The form makes it easy to compute the cube any 2 digit number. The following examples will show how this could be done.

Method

- Find the values of a^3, a^2b, ab^2, b^3 and write them as shown.
- Also double the vales of a^2b, ab^2 and write them under respective column.
- Compute the cube of the number from the result.

Example 1: Find the cube of 26.

Here, a = 2, b = 6.

Example 2: Find the cube of 84.

Here, a = 8, b = 4.

8	24	72	216
	48	144	
8	72	216	216

8 2 6 6
7 1 1
2 2

1 7 5 7 6

Hence, $26^3 = 17576$

Example 3: Find the cube of 33.

Here, a = 3, b = 3.

27	27	27	27
	54	54	
27	81	81	27

2 7 1 1 7
8 8 2

3 5 9 3 7

Hence, $33^3 = 35937$

512	256	128	64
	512	256	
512	768	384	64

5 1 2 8 4 4
6 8 6
7 3

5 9 2 7 0 4

Hence, $84^3 = 592704$

Example 4: Find the cube of 47.

Here, a = 4, b = 7.

64	112	196	343
	224	392	
64	336	588	343

6 4 6 8 3
3 8 4
3 5 3

1 0 3 8 2 3

Hence, $47^3 = 103823$

Cube Roots Of Perfect Cubes

The cube root of a perfect cube can be found by division method. Before finding the cube root of a number we must know 3 things.

- Number of digits in the cube root.
  If the number has n-digits, the cube root will have n/3 or (n+1)/3 or (n+2)/3 digits.
- First digit of the cube root must be known.
  The number is divided into groups of 3 digits from right to left. The nearest cube root of the first group gives the first cube root digit.
- The devisor must also be known.
  Thrice the square of first cube root digit is taken as the devisor.

Example 1:

Find the cube root of 13824.

No of cube root digits = 5+1/3 = 2.
Grouping of digits = 13 ' 824
First cube root digit = 2 ($2^3 < 13$)

Working (Mental)

Step 1: $13 - 2^3 = 13 - 8 = 5$. Write it as shown.

Step 2: 58 ÷ 12 = 4 R 10. Write it as shown.

Step 3: But, $3ab^2 = 3x2x4^2 = 96$

102 – 96 = 6 ÷ 12 = 0 R 6

Modified Devisor = $3x2^2 = 12$.

$$\begin{array}{r|l} & 13\ \ 8\ \ 2\ \ 4 \\ 12 & \ \ \ 5\ \ 10\ \ 6 \\ \hline & 2\,4\,.\,0\,0 \end{array}$$

Step 4: But, $b^3 = 4^3 = 64$

$64 - 64 = 0 \div 12 = 0$ R 0

Hence, Cube Root of 13824 = 24.

Example 2:

Find the cube root of 195112.

No of cube root digits = 6/3 = 2.

Grouping of digits = 195 ' 112

First cube root digit = 5 ($5^3 < 195$)

Modified Devisor = $3x5^2 = 75$.

$$\begin{array}{r|l} & 195\ \ \ 1\ \ \ 1\ \ \ 2 \\ 75 & \ \ \ \ \ 70\ \ 101\ \ 51 \\ \hline & 5\,8\,.\,0\,0 \end{array}$$

Working (Mental)

Step 1: $195 - 5^3 = 195 - 125 = 70$. As shown

Step 2: $701 \div 75 = 8$ R 101. As shown

Step 3: But, $3ab^2 = 3x5x8^2 = 960$

$1101 - 960 = 51 \div 75 = 0$ R 51

Step 4: But, $b^3 = 8^3 = 512$

$512 - 512 = 0 \div 75 = 0$ R 0

Hence, Cube Root of 195112 = 58.

Divisibility

If we divide one number by another number and get a whole number, we say that the first number is divisible by the second. This property of division is called divisibility. For example,

25 is divisible by 5.; 12 is divisible by 2, 3, 4, 6.

63 is divisible by 3, 7, 9. etc.

Divisibility Criteria - Here are divisibility Criteria for a first few integers:

Divisor	Criteria	Examples
2	All even numbers	12; 24; 136
3	Sum of digits is divisible by 3	15; 234
4	Last two digits divisible by 4	1932; 2016
5	Last digit is 0 or 5	15; 210; 305
6	Numbers divisible by 2 and 3	36; 1236
8	Last four digits divisible by 8	451936
9	Sum of digits divisible by 9	27; 171;

Test for divisibility

Suppose we want to find whether a number is divisible by prime numbers like 7, 13, 17, 19 . . . we have no clue in our modern mathematics. But, Vedic Mathematics gives us a nice method to do it.

Any number can be split into prime numbers. A number divisible by itself or 1 is called prime number. Otherwise, it is a composite number. To find the divisibility of one number by another number, we must try division by prime numbers. A number divisible by a composite number is also divisible by all its prime factors.

The prime numbers within 100 are given below:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 51, 53, 57, 59, 61, 67, 71, 73, 79, 83, 87, 89, 91 and 97.

Operators

We observe that the prime numbers have 1, 3, 7,9 as end digits. We can convert such numbers either to end with 1 or 9 using a multiplier.

Converting numbers ding with 9			**Converting numbers ending with 1**		
End digit	Multiplier	Example	End digit	Multiplier	Example
1	9	11x9=99	1	1	31x1=31
3	3	13x3=39	3	7	13x7=91
7	7	7x7=49	7	3	17x3=51
9	1	19x1=19	9	9	19x9=171

Now, we have two choices.

Division by numbers ending in 9 and Division by numbers ending in 1.

Therefore, we select two operators – one positive operator and another negative operator.

Positive operator is used with devisors ending in 9. The Vedic Sutra Ekadhika – One more than the previous, gets it. Here are some positive operators.

Devisor	Operator
19	1+1=2
29	2+1=3
39	3+1=4
49	4+1=5
etc	etc

Negative operator is used with devisors ending in 1. Just dropping end digit 1 gets it. Here are some negative operators.

Devisor	Operator
11	1
21	2

31 41 etc	3 4 etc

Testing Divisibility by a Positive Operator

Testing the divisibility of a number using a positive operator is very simple.

Find the operator first using Ekadhika.

Separate the last digit from the number.

Multiply this digit by the operator.

Add the result to the remaining number.

Apply this rule again and again as necessary.

If the final result is divisible then so is the first number.

Example 1: Is 171 divisible by 19?

Here, Operator = 1 + 1 = 2.

Now, 171 –> 17 + 1x2 = 17 + 2 = 19.

Hence, given number is divisible.

Example 2: Is 348 divisible by 29?

Here, Operator = 2 + 1 = 3.

Now, 348 –>34 + 8x3 = 34 + 24 = 58

58 –> 5 + 8x3 = 5 + 24 = 29.

Hence, given number is divisible.

Example 3: Is 1764 divisible by 49?

Here, Operator = 4 + 1 = 5.

Now, 1764 –> 176 + 4x5 = 176 + 20 = 196

196 –> 19 + 6x5 = 19 + 30 = 49.

Hence, the result.

Example 4: Test the divisibility of 3614 by 13.

Here, devisor ends with 3. Therefore, 13x3=39.

Operator = 3 + 1 = 4

Now, 3614 –> 361 +4x4 = 361 + 16 = 377.

377 –> 37 + 7x4 37 + 28 = 65.

65 –> 6 + 5x4 = 6 + 20 = 26.

26 –> 2 + 6x4 = 2 + 24 = 26. Repeated.

Hence, 3614 is divisible.

Example 5: Find if 13294 divisible by 23?

Here, devisor ends with 3. Therefore, 23x3=69.

Operator = 6 + 1 = 7.

Now, 13294 –> 1329 + 4x7 = 1329 + 28 = 1357.

1357 –> 135 + 7x7 = 135 + 49 = 184.

184 –> 18 + 4x7 = 18 + 28 = 46.

46 –> 4 + 6x7 = 4 + 42 =46. Repeated.

Hence, 13294 is divisible by 23.

Example 6: Find if 3171 divisible by 7?

Here, devisor ends with 7. Therefore, 7x7=49.

Operator = 4 + 1 = 5.

Now, 3171 –> 317 + 1x5 = 317 + 5 = 322.

322 –> 322 + 2x5 = 32 + 10 = 42.

42 –> 4 + 2x5 = 4 + 10 = 14. Divisible by 7.

Hence the result.

Testing Divisibility by Negative Operator

Testing the divisibility of a number using a positive operator is very simple.

Find the operator first using Ekadhika.

Separate the last digit from the number.

Multiply this digit by the operator.

Subtract the result from remaining number.

Apply this rule again and again as necessary.

If the final result is divisible then so is the first number.

Example 1: Find if 1331 divisible by 11?

Here, Operator = 1.(End digit is dropped.)

Now, 1331 –> 133 – 1x1 = 133 – 1 = 132.

132 –> 13 – 2x1 = 13 – 2 = 11.

11 –> 1 – 1x1 = 1 – 1 = 0.

Hence, 1331 is divisible by 11.

Example 2: Is 5376 divisible by 21?

Here, Operator = 2.(End digit is dropped.)

Now, 5376 –> 537 – 6x2 = 537 – 12 = 525.

525 –> 52 – 5x2 = 52 – 10 = 42.

42 –> 4 – 2x2 4 – 4 = 0.

Hence, the result.

Example 3: Test the divisibility of 3844 by 31.

Here, Operator = 3.(End digit is dropped.)

Now, 3844 –> 384 – 4x3 = 384 – 12 = 372.

372 –> 37 – 2x3 = 37 –6 = 31.

31 –> 3 – 1x3 = 3 – 3 = 0.

Therefore, 3844 is divisible by 31.

Example 4: Find if 27249 divisible by 31?

Here, Operator = 3.(End digit is dropped.)

Now, 27249 –> 2724 – 9x3 = 2724 – 27 =2697.

2697 –> 269 – 7x3 = 269 – 21 = 248.

248 –> 24 – 8x3 = 24 – 24 = 0.

Hence, the result.

Example 5: Is 4318 divisible by 17?

Here, devisor ends with 17. Therefore, 17x3=51.

Therefore, Operator = 5.

Now, 4318 –> 431 – 8x5 = 431 – 40 = 391.

391 –> 39 – 1x5 = 39 – 5 = 34.

34 –> 3 – 4x5 = 3 – 20 = – 17, divisible by 17.

Hence, the result.

Example 6: Is 3171 divisible by 7?

Here, devisor ends with 7. Therefore, 7x3=21.

Therefore, Operator = 2.

Now, 3171 –> 317 – 1x2 = 317 – 2 = 315.

315 –> 31 – 5x2 = 31 – 10 = 21.

21 –> 2 – 1x2 = 0.

Hence, the result.

> Note: Compare this problem with the same one under positive operator above. Both give the same answer. This shows either of the method could be used. Selection of easier method is our choice. Suppose we want to test for divisibility by 21 or 63 (21=3x7, 63=3x3x7). First we check for divisibility by 3 (digit sum divisible by 3). If it does, then try for 7.

Now let us take an example where divisibility is not possible, to show the strength of this test.

Example: Is 3245 divisible by 7?

(i) We test this by positive operator.

Here, 7 x 7 = 49. Operator = 5.

Now, 3245 –> 324 – 5x5 =324 – 25 = 299.

299 –> 29 – 9x5 = 29 – 45 = – 16.

This is not divisible by 7.

(ii) We now test this by negative operator.

Again, 7 x 3 = 21. Operator = 2.

3245 –> 324 – 5x2 = 324 – 10 = 314.

314 –> 31 – 4x2 = 31 – 8 = 23.

This is also not divisible by 7.

Therefore, it is clear that we can go for either of the methods.

Recurring Decimals

A decimal with a sequence of digits that repeats itself indefinitely is called recurring decimal.

The Vedic Sutra Ekadhika helps us to convert Vulgar fractions of the type 1/p9, where p = 1, 2, 3, . . . 9, into recurring decimals. The number of decimal places before repetition is the difference of numerator and denominator. The devisor can be found using Sutra "One more than the previous one".

Conversion of a few vulgar fractions is shown below. The method is simple and does not require actual division. It is only simple division with small figures.

Convert 1/19 into recurring decimal.

Here, number of decimal places is (19 – 1) = 18.

The denominator is 19, and the previous one is 1.

Hence, Ekadhika = (1 + 1) = 2.

The method of division is as follows:

Now, 1/19 = 1/20 = 0.1/2

Divide 1 by 2, answer is 0, remainder 1

–> $0.{}_{1}0$ (Remainder is put before the answer)

Next 10 ÷ 2 is 5 –> .05

Next 5 ÷ 2 is 2 remainder 1 –> $0.{}_{1}05{}_{1}2$

Next 12 (remainder 1) ÷ 2 is 6 –> $0.{}_{1}0526$

Next 6 ÷ 2 is 3 –> $0.05{}_{1}263$

Next 3 ÷ 2 is 1 remainder 1 –> $0.{}_{1}05{}_{1}263{}_{1}1$

Next 11 ÷ 2 is 5 remainder 1 –> $0.{}_{1}05{}_{1}263{}_{1}1{}_{1}5$

and so on...

Continue like this till the end result got is

$1/19 = 0.{}_{1}05_{1}263_{1}5_{1}7_{1}89_{1}47_{1}3_{1}68421$

Therefore, 1/19 = 0.052631578947368421

Convert 11/19 into recurring decimal.

The denominator = 19. Previous digit = 1. Ekadhika is 1+1=2.

$11/19 = 11/20 = 1.1/2 = 0.{}_{1}5_{1}7_{1}89_{1}47_{1}3_{1}68421$

Therefore, 11/19 = 578947368421.

Convert 1/7 into recurring decimal.

Here, number of decimal places is (7 – 1) = 6.

1/7 = 7/49. The denominator is 49.

Previous digit = 4. Ekadhika is 4+1=5.

$7 / 49 = 7/50 = 0.7/5 = 0.{}_{2}1_{1}4_{4}2_{2}8_{3}57$

Therefore, 1/7 = 0.142857.

Convert 9/39 into recurring decimal.

The denominator = 39. Previous digit = 3. Ekadhika is 3+1=4.

$9/39 = 9/40 = 0.9/4 = 0.{}_{1}23_{3}0769$. Therefore, 9/39 = 0.230769.

Equations

Simple Equations

A simple equation consists of one variable or unknown. The variable may be present on one side or both sides. To solve such problems the Vedic Sutra "Transpose and apply" (Paravartya Sutra) comes to our rescue. We can transpose the terms from left to right or from right to left. In such cases, positive(+) sign becomes negative(–) sign and vice versa. We can also change numerator to denominator or denominator to numerator. Then multiplication(×) sign becomes division(÷) sign and vice versa.

Paravartya Method

We shall consider the following types of equation under simple equations.

Type 1: ax + b = cx + d. Solution: $x = \frac{d-b}{a-c}$

Example 1: Solve: 3x + 5 = 5x – 3.

Here, a = 3, b = 5, c = 5, d = –3.

$$\therefore \quad x = \frac{d-b}{a-c} = \frac{-3-5}{3-5} = 4$$

Example 2: Solve: (2x/3) + 1 = x - 1

--> $2x + 3 = 3x - 3$

Here, a = 2, b = 3, c = 3, d = –3.

$$\therefore \; x = \frac{d-b}{a-c} = \frac{-3-3}{2-3} = 6$$

Type 2: $(x + a)(x + b) = (x + c)(x + d)$

Solution: $x = \frac{cd - ab}{(a+b)-(c+d)}$. Note: If ab = cd, then x = 0.

Example 1: Solve: $(x + 1)(x + 2) = (x - 3)(x - 4)$

Here, a = 1, b = 2, c = –3, d = –4.

$$\therefore \; x = \frac{cd - ab}{(a+b)-(c+d)} = \frac{(-3\times -4)-(1\times 2)}{(1+2)-(-3-4)} = 1$$

Example 2: Solve: $(x + 3)(x + 2) = (x - 3)(x - 2)$

Here, a = 3, b = 2, c = –3, d = –2.

$$\therefore \; x = \frac{cd - ab}{(a+b)-(c+d)} = \frac{(-3\times -2)-(3\times 2)}{(3+2)-(-3-2)} = 0$$

Type 3: $\frac{ax+b}{cx+d} = \frac{m}{n}$ Solution: $x = \frac{md - bn}{an - mc}$

Example: Solve: $\frac{2x+3}{5x+4} = \frac{1}{2}$

Here, a = 2, b = 3, c = 5, d = 4, m = 1, n = 2.

$$\therefore \; x = \frac{md - bn}{an - mc} = \frac{1\times 4 - 3\times 2}{2\times 2 - 1\times 5} = \frac{-2}{-1} = 2$$

Type 4: $\frac{m}{x+a} + \frac{n}{x+b} = 0$ Solution: $x = \frac{-(mb + na)}{(m+n)}$

Example: Solve: $\frac{2}{x+2} + \frac{3}{x+7} = 0$

Here, a = 2, b = 7, m = 2, n = 3.

$$\therefore \; x = \frac{-(mb + na)}{(m+n)} = \frac{-(2\times 7 + 3\times 2)}{(2+3)} = \frac{-20}{5} = -4$$

Sunyam Samya Method

Next we shall explore another Vedic Sutra "Common group is zero" (Sunyam Samya Sutra). The group may be a single variable or an expression and it should be equated to zero. We shall consider the following cases.

Type 1: (i) $ax + bx = cx + dx$, and (ii) $m(x + a) = n(x + a)$.

Example 1: $2x + 3x = 5x + x$.

Here, x is a common factor (group). Therefore, $x = 0$.

Example 2: $3(x + 2) = 5(x + 2)$.

Here, $(x + 2)$ is again a common factor. Hence, $x + 2 = 0$ or $x = -2$, is the solution.

Type 2: $(x + a)(x + b) = (x + c)(x + d)$. If $a \times b = c \times d$ then $x = 0$.

Example: $(x + 2)(x + 6) = (x - 3)(x - 4)$

Here, $2 \times 6 = -3 \times -4 = 12$. Hence, $x = 0$.

Type 3: $\frac{N_1}{D_1} + \frac{N_2}{D_2} = 0$. N_1 and N_2 are numerators, D_1 and D_2 are denominators.

If $N_1 = N_2$, $D_1 + D_2 = 0$, is the solution. From this the value of x can be found.

Example: Solve: $\frac{1}{2x+3} + \frac{1}{x-6} = 0$

Here, $D_1 + D_2 = 2x + 3 + x - 6 = 0$ or $3x - 3 = 0$. $\therefore$ $x = 1$, is the solution.

Type 4: $\frac{N_1}{D_1} = \frac{N_2}{D_2}$. N_1 and N_2 are numerators, D_1 and D_2 are denominators

If $N_1 + N_2 = D_1 + D_2 = k(D_1 + D_2)$, where k is a number, then

$N_1 + N_2 = D_1 + D_2 = 0$, is the solution. From these the value of x can be found.

Example 1: Solve: $\frac{3x+4}{2x+11} = \frac{2x+11}{3x+4}$

Here, $N_1 + N_2 = 3x + 4 + 2x + 11 = 5x + 15$

$D_1 + D_2 = 2x + 11 + 3x + 4 = 5x + 15$

Therefore, $N_1 + N_2 = D_1 + D_2 = 0$ or $5x + 15 = 0$. $\therefore$ $x = -3$ is the solution.

Example 2: Solve: $\frac{2x+3}{4x+5} = \frac{x+1}{2x+3}$

Here, $N_1 + N_2 = 2x + 3 + x + 1 = 3x + 4$

$D_1 + D_2 = 4x + 5 + 2x + 3 = 6x + 8 = 2(3x + 4)$

Therefore, $N_1 + N_2 = D_1 + D_2 = 0$ or $3x + 4 = 0$. $\therefore$ $x = -4/3$ is the solution.

Type 5: $\frac{N_1}{D_1} = \frac{N_2}{D_2}$. N_1 and N_2 are numerators, D_1 and D_2 are denominators

If $N_1 + N_2 = D_1 + D_2$ and $N_1 \sim D_1 = N_2 \sim D_2$ then

$N_1 + N_2 = D_1 + D_2 = 0$ and $N_1 \sim D_1 = N_2 \sim D_2$ are two solutions.

From these the values of x can be found.

Example: Solve: $\frac{3x+4}{6x+7} = \frac{5x+6}{2x+3}$

$N_1 + N_2 = 3x + 4 + 5x + 6 = 8x + 10$

$D_1 + D_2 = 6x + 7 + 2x + 3 = 8x + 10$

$N_1 \sim D_1 = 3x + 4 - 5x - 6 = -2x - 2 = -(x + 1)$

$N_2 \sim D_2 = 6x + 7 - 2x - 3 = 4x - 4 = 4(x + 1)$

Since $N_1 + N_2 = D_1 + D_2 = 8x + 10$ and $N_1 \sim D_1 = N_2 \sim D_2 = x + 1$,

we get $8x + 10 = 0$ or $x = -5/4$ and $x + 1 = 0$ or $x = -1$.

Hence, $x = -5/4$ and $x = -1$ are the solutions

Type 6: $\frac{1}{D_1} + \frac{1}{D_2} = \frac{1}{D_3} + \frac{1}{D_4}$. D_1, D_2, D_3 and D_4 are denominators

If $D_1 + D_2 = D_3 + D_4$, then $D_1 + D_2 = D_3 + D_4 = 0$.

From this the value of x can be found.

Example: Solve: $\frac{1}{x-7} + \frac{1}{x-9} = \frac{1}{x-6} + \frac{1}{x-10}$

Here, $D1 + D_2 = x - 7 + x - 9 = 2x - 16$

$D_3 + D_4 = x - 6 + x - 10 = 2x - 16$

Since $D_1 + D_2 = D_3 + D_4$, $2x - 16 = 0$. $\therefore$ $x = 8$ is the solution.

Simultaneous Equations

Solving simultaneous equations by conventional method is tedious. But Vedic mathematics gives us a simple and elegant method to find the solution. We discuss three categories under Vedic mathematics.

Category 1: Equal ratios of coefficients of x (or y) and constants.

To solve simultaneous equations of this type, we use the Vedic Sutra: " If one is in ratio, the other one is zero". i.e., If the ratio of coefficients of one variable is equal to the ratio of the constants then the other variable is zero. In such a case, we get two simple equations in one variable. The solution of either equation gives the value of this variable.

Example 1: Solve: $2x + 3y = 6$

$5x + 6y = 12$

Ratio of coefficients of y: 3/6 = 1/2 and 6/12 = 1/2.

Therefore, $x = 0$, and $3y = 6$ or $6y = 12 \rightarrow y = 2$.

Thus, $x = 0$, $y = 2$.

Example 2: Solve: $12x + 5y = 36$

$36x + 6y = 108$

Ratio of coefficients of x: 12/36 = 1/3 and 36/108 = 1/3.

Therefore, $y = 0$, and $12x = 36$ or $36x = 108 \rightarrow x = 3$.

Thus, $x = 3$, $y = 0$.

Category 2: Coefficients of x and y are interchanged.

To solve simultaneous equations of this type, we use the Vedic Sutra: "by addition and by subtraction". We simply add and subtract the equations separately and simplify them to get two other equations. Again by adding and subtracting these two equations we get the values of the unknowns.

Example 1: Solve: $5x + 7y = 31$

$7x + 5y = 29$

By addition: $12x + 12y = 60 \rightarrow x + y = 5$

By subtraction: $-2x + 2y = 2 \rightarrow -x + y = 1$

By adding last two equations we get, $2y = 6$ or $y = 3$.

By subtracting the equations we get, $2x = 4$ or $x = 2$.

Thus, $x = 2$, $y = 3$.

Example 2: Solve: $43x - 27y = 16$

$27x - 43y = -16$

By addition: $70x - 70y = 0 \rightarrow x - y = 0$

By subtraction: $16x + 16y = 32 \rightarrow x + y = 2$

By adding last two equations we get, $2x = 2$ or $x = 1$.

By subtracting the equations we get, 2 y = 2 or y = 1.

Thus, x = 1, y = 1.

Category 3: General methods not covered above.

The method used under this category is given below. Let us consider the equations,

a x + b y = p

c x + d y = q

Method:

1. Write the coefficients and constants as shown below.

   b p a

   d q c

   Note the *order* carefully.

2. Find the cross-products, (bq – pd), (pc – aq) and (bc – ad)
3. Compute x and y from:

$$x = \frac{bq - pd}{bc - ad} \quad \text{And} \quad y = \frac{pc - aq}{bc - ad}$$

Example 1: Solve: 2x + 3y = 13

7x + 5y = 29

The coefficients are: 3 13 2

5 29 7

$$\therefore \qquad x = \frac{(3\times 29) - (13\times 5)}{(3\times 7) - (2\times 5)} = \frac{87 - 65}{21 - 10} = \frac{22}{11} = 2$$

$$y = \frac{(13\times 7) - (2\times 29)}{(3\times 7) - (2\times 5)} = \frac{91 - 58}{21 - 10} = \frac{33}{11} = 3$$

Example 2: Solve: 3x + 4y = 17

5x – 7y = 1

The coefficients are: 4 17 3

–7 1 5

$$\therefore \qquad x = \frac{(4\times 1) - (17\times -7)}{(4\times 5) - (3\times -7)} = \frac{4 + 119}{20 + 21} = \frac{123}{41} = 3$$

$$y = \frac{(17 \times 5) - (3 \times 1)}{(4 \times 5) - (3 \times -7)} = \frac{85 - 3}{20 + 21} = \frac{82}{41} = 2$$

Example 3: Solve: $4x - 3y = 11$

$7x - 5y = 20$

The coefficients are: $-3 \quad 11 \quad 4$

$-5 \quad 20 \quad 7$

$$\therefore \qquad x = \frac{(-3 \times 20) - (11 \times -5)}{(-3 \times 7) - (4 \times -5)} = \frac{-60 + 55}{-21 + 20} = \frac{-5}{-1} = 5$$

$$y = \frac{(11 \times 7) - (4 \times 20)}{(-3 \times 7) - (4 \times -5)} = \frac{77 - 80}{-21 + 20} = \frac{-3}{-1} = 3$$

Quadratic Equations

A second degree equation in one variable is called the quadratic equation. Often we come across such equations in practice. The normal method consists in factorizing the given expression into factors and finding the answer by equating each factor to zero. Sometimes it becomes inconvenient and cumbersome in factorizing. In such cases we may use the direct method given below.

Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are constants. The solution to this equation is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The following examples will illustrate the method.

Example 1: Solve: $x^2 + 5x + 6 = 0$

Here, $a = 1$, $b = 5$, $c = 6$.

Therefore, $x = \frac{-5 \pm \sqrt{25 - 4 \times 1 \times 6}}{2 \times 1} = \frac{-5 \pm 1}{2}$

$\therefore$ $x = -2$ or $x = -3$

Example 2: Solve: $x^2 - 2x - 15 = 0$

Here, $a = 1$, $b = -2$, $c = -15$.

Therefore, $x = \frac{2 \pm \sqrt{4 - (4 \times 1 \times -15)}}{2 \times 1} = \frac{2 \pm 8}{2}$

$\therefore$ $x = 5$ or $x = -3$

Example 3: Solve: $6x^2 - 5x - 6 = 0$

Here, a = 6, b = –5, c = –6.

Therefore, $x = \dfrac{5 \pm \sqrt{25-(4\times 6\times -6)}}{2\times 6} = \dfrac{5\pm 13}{12}$

∴ x = 3/2 or x = –2/3

Fractions

The concept of fractions is not properly understood by many. We are familiar with whole numbers. We can add, subtract, multiply and divide whole numbers. When it comes to fractions we feel confused. To get a clear picture of the situation it is found necessary to add a note on fractions.

A fraction consists of two numbers separated by a line or a slash.

- The top number is called the numerator.
- The bottom number is called the denominator.
- The denominator tells us how many parts a whole thing is divided into.
- The numerator tells us how many parts are taken out of it.

E.g., The fraction 3/8 tells us that the whole thing is divided into 8 parts and 3 parts are taken out of it. Here 3 is the numerator and 8 is the denominator.

Types of Fractions

We classify the fractions into the following types.

Proper fractions – A fraction in which numerator is less than denominator. e.g., 2/3, 3/7. 4/5,

Improper fractions – A fraction in which numerator is greater than denominator. e.g., 5/3, 9/7. 12/5,

Mixed fractions - A fraction consisting of two parts, an integer and a fraction. e.g., $2\frac{3}{4}, 5\frac{1}{3}, 7\frac{2}{5}$, etc.

Decimal fractions – A fraction in which the denominator is 10 or powers of 10, viz. 100, 1000, 10000, … e.g., 2/10, 23/100, 126/1000,

Compound fractions – A fraction of a fraction. e.g., 2/3 of 4/5 → (2/3)x(4/5)

(i) A mixed fraction can be converted into improper fraction.

e.g., $2\frac{3}{4} = \frac{4\times 2+3}{4} = \frac{11}{4}$.

(ii) An improper fraction can be converted into mixed fraction.

e.g., $\frac{11}{4}=\frac{8+3}{4}=2+\frac{3}{4}=2\frac{3}{4}$

Equivalent Fractions

Fractions that have the same value are called equivalent fractions. Equivalent fractions are obtained by multiplying or dividing both numerator and denominator of the fraction by the same number.

By multiplying both numerator and denominator successively by 2, 3, 4,..., we get equivalent fractions. E.g., $\frac{1}{2},\frac{2}{4},\frac{6}{12},\frac{12}{24}$.

Similarly, by dividing both numerator and denominator also successively by 2, 3, 4,..., we get equivalent fractions. E.g., $\frac{12}{24},\frac{6}{12},\frac{2}{4},\frac{1}{2}$.

A fraction must always be expressed in its simplest form. This is done by cancelling the factors common to both numerator and denominator.

Comparing the fractions

Two or more fractions can be compared with each other. To find the greater of the given fractions we must make their denominators equal. Then the fraction with greater numerator is the greatest.

Comparison of two fractions

Suppose we want to compare two fractions a/b and c/d. Cross multiply the numerator of one fraction by the denominator of other fraction. The fraction with greater cross-product is larger.

e.g., $\frac{a}{b}:\frac{c}{d}\Rightarrow\frac{a\times d:b\times c}{bd}$. If a×d > b×c then $\frac{a}{b}>\frac{c}{d}$ and vice versa.

Example 1: Which one is large 2/3 or 3/4?

$\frac{2}{3}:\frac{3}{4}\Rightarrow\frac{2\times4:3\times3}{3\times4}=\frac{8:9}{12}$. Since 9 > 8, 3/4 > 2/3.

Example 2: Which of the two is larger 3/5 or 4/7?

$\frac{3}{5}:\frac{4}{7}\Rightarrow\frac{3\times7:4\times5}{5\times7}=\frac{21:20}{35}$. Since 21 > 20, 3/5 > 4/7.

Comparison of three fractions

Suppose we want to compare three fractions a/b, c/d and e/f. Cross multiply the numerator of one fraction by the denominator of the other two fractions. The fraction with greater cross-product is larger.

e.g., $\frac{a}{b}:\frac{c}{d}:\frac{e}{f}\Rightarrow\frac{a\times d\times f:c\times b\times f:e\times b\times d}{bdf}$.

If $a \times d \times f > c \times b \times f > e \times b \times d$ then $\frac{a}{b} > \frac{c}{d} > \frac{e}{f}$ and vice versa.

Example: Which one is the greatest 3/5, 5/7, 7/9?

$$\frac{3}{5}:\frac{5}{7}:\frac{7}{9} \Rightarrow \frac{3\times7\times9:5\times5\times9:7\times5\times7}{5\times7\times9} = \frac{189:225:245}{315}.$$

Since 245 > 225 > 189, 7/9 is the greatest.

Note: By converting the fractions to decimals we can also find the greatest fraction. In the present case, 3/5 = 0.60, 5/7 = 0.71 and 7/9 = 0.77. Thus, 7/9 is the greatest.

Operations with fractions

Two fractions can be added, subtracted, multiplied or divided as is done with whole numbers. However, mixed fractions must be converted into improper fractions before such methods. The final result may be expressed as a mixed fraction after simplification.

Addition of fractions

We can add two or more fractions with each other.

$$\text{Sum of two fractions} = \frac{\text{Sum of cross - products}}{\text{Product of denominato rs}}$$ In symbols, $\frac{a}{b}+\frac{c}{d}=\frac{a\times d+b\times c}{bd}$

Example 1: Add 2/3 and 3/4.

Solution: $\frac{2}{3}+\frac{3}{4}=\frac{2\times4+3\times3}{3\times4}=\frac{17}{12}=1\frac{5}{12}$

Example 2: Add 5/8 and 7/9.

Solution: $\frac{5}{8}+\frac{7}{9}=\frac{5\times9+7\times8}{8\times9}=\frac{101}{72}=1\frac{29}{72}$

Subtraction of fractions

We can subtract two fractions from one another.

$$\text{Difference of two fractions} = \frac{\text{Difference of cross - products}}{\text{Product of denominators}}$$ In symbols, $\frac{a}{b}-\frac{c}{d}=\frac{a\times d-b\times c}{bd}$

Example 1: Subtract 4/7 from 5/6.

Solution: $\frac{5}{6}-\frac{4}{7}=\frac{5\times7-4\times6}{6\times7}=\frac{11}{42}$

Example 2: Subtract 1/4 from 2/3.

Solution: $\frac{2}{3}-\frac{1}{4}=\frac{2\times4-1\times3}{3\times4}=\frac{5}{12}$

Multiplication of fractions

We can multiply two or more fractions with each other.

$\text{Product of two fractions}=\frac{\text{Product of numerators}}{\text{Product of denominators}}$ In symbols, $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

Example 1: Multiply 2/3 and 3/4.

Solution: $\frac{2}{3}\times\frac{3}{4}=\frac{2\times3}{3\times4}=\frac{6}{12}=\frac{1}{2}$

Example 2: Multiply 6/7 and 7/9.

Solution: $\frac{6}{7}\times\frac{7}{9}=\frac{6\times7}{7\times9}=\frac{42}{63}=\frac{2}{3}$

Division of fractions

We can divide two fractions from one another.

For this, divisor is inverted (numerator and denominator exchanged) and multiplication is carried as usual.

In symbols, $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{a\times d}{b\times c}$

Example 1: Divide 5/7 by 3/5.

Solution: $\frac{5}{7}\div\frac{3}{5}=\frac{5}{7}\times\frac{5}{3}=\frac{5\times5}{7\times3}=\frac{25}{21}=1\frac{4}{21}$

Example 2: Divide 3/8 by 1/6.

Solution: $\frac{3}{8}\div\frac{1}{6}=\frac{3}{8}\times\frac{6}{1}=\frac{3\times6}{8\times1}=\frac{18}{8}=\frac{9}{4}=1\frac{1}{4}$

Converting a Fraction to a Decimal

Divide the numerator of the fraction by the denominator.

Round it to desired precision.

e.g., $\frac{2}{3}=0.6667$, $\frac{1}{7}=0.1429$, etc.

Converting a Decimal to a Fraction

Divide by appropriate power of 10 and simplify.

e.g., $0.56 = \frac{56}{100} = \frac{14}{25}$, $0.3333 = \frac{3333}{10000} = \frac{1}{3}$, etc.

Converting a Fraction to a percent

Multiply the fraction by 100 and attach % sign

e.g., $\frac{2}{5} = \frac{2}{5} \times 100 = \frac{200}{5} = 40\%$

Converting a percent to a Decimal

Remove the Percent sign and divide by 100.

e.g., $40\% = \frac{40}{100} = 0.40$