Heat Exchanger Efficiency

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This paper provides the solution to the problem of defining thermal efficiency for heat exchangers based on the second law of thermodynamics. It is shown that corresponding to each actual heat exchanger, there is an ideal heat exchanger that is a balanced counter-flow heat exchanger. The ideal heat exchanger has the same UA, the same arithmetic mean temperature difference, and the same cold to hot fluid inlet temperature ratio. The ideal heat exchanger's heat capacity rates are equal to the minimum heat capacity rate of the actual heat exchanger. The ideal heat exchanger transfers the maximum amount of heat, equal to the product of UA and arithmetic mean temperature difference, and generates the minimum amount of entropy, making it the most efficient and least irreversible heat exchanger. The heat exchanger efficiency is defined as the ratio of the heat transferred in the actual heat exchanger to the heat that would be transferred in the ideal heat exchanger. The concept of heat exchanger efficiency provides a new way for the design and analysis of heat exchangers and heat exchanger networks.
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1 Introduction

The concept of efficiency is used in many areas, particularly engineering, to assess the performance of real components and systems. Efficiency is a comparison between the actual (real) and ideal (best) performances and is typically defined to be less than or at best equal to 1. The ideal behavior is generally known from modeling, and the limitations dictated by physical laws, particularly the second law of thermodynamics. Knowing the ideal performance, the actual performance can be determined if expressions for the efficiency as a function of the system characteristics and the operating conditions are known. Efficiency provides a clear and intuitive measure of a system's performance by showing how close an actual system comes to the best that it can be and if further improvements are feasible and justified. Despite much effort, the application of the second law to heat exchangers has not yielded a consistent method for assessing the performance of heat exchangers.

Two of the more widely used approaches for analyzing heat exchangers are the log-mean temperature difference method (LMTD) and effectiveness NTU (ε-NTU) method. In the LMTD method

$$F = \frac{q}{UA\mathrm{LMTD}} \tag{1}$$

where the term in the denominator is the maximum rate of heat transfer, which takes place in a counter-flow heat exchanger having the same UA and the same inlet and exit temperatures as the heat exchanger under consideration. Expressions and charts are available to determine F for different heat exchangers. These correlations are typically a function of two parameters P and R that depend solely on the inlet and exit temperatures. The LMTD approach is generally used for solving heat exchanger problems where the inlet and the exit temperatures are known and the size of the heat exchanger is to be determined (sizing problems).

In the ε-NTU approach, the heat exchanger effectiveness is defined as

$$\varepsilon = \frac{q}{C_{\min}(T_1 - t_1)} \tag{2}$$

where the term in the denominator is the absolute maximum heat that can be transferred from a fluid at T_1 to another fluid at t_1. This maximum amount of heat transfer can only occur in a heat exchanger whose area approaches infinity. Expressions and charts are available to determine the effectiveness of different heat exchangers, and are typically a function of two variables (C_r and NTU). The ε-NTU method is mostly used in situations where the size of the heat exchanger and the inlet temperatures are known and the heat transfer rate and the fluid exit temperatures are sought (the rating problem), although sizing problems can also be solved with this method.

The author recently introduced the concept of heat exchanger efficiency [1–4]. The heat exchanger efficiency is defined as the ratio of the actual rate of heat transfer in the heat exchanger (q) to the optimum rate of heat transfer (q_{opt}):

$$\eta = \frac{q}{q_{\mathrm{opt}}} = \frac{q}{UA(\bar{T} - \bar{t})} \tag{3}$$

The optimum (maximum) rate of the heat transfer is the product of UA of the heat exchanger under consideration and its arithmetic mean temperature difference (AMTD), which is the difference between the average temperatures of hot and cold fluids. The rate of heat transfer in any heat exchanger with the same UA and AMTD is always less than the optimum value of the heat transfer rate ($\eta \leq 1$) [1]. Furthermore, the optimum heat transfer rate takes place in a balanced counter-flow heat exchanger [1].

The efficiency of a number of commonly used heat exchangers is given by the general expression

$$\eta = \frac{\tanh\,(\mathrm{Fa})}{(\mathrm{Fa})} \tag{4}$$

where Fa, the fin analogy number, is the nondimensional group that characterizes the performance of different heat exchangers. This is a remarkable expression in that the efficiency of a wide variety of heat exchangers has the same functional form as the efficiency of a constant area insulated tip fin. The expressions for Fa for some of the commonly used heat exchangers are given in Table 1.

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Table 1 Fin analogy number of various heat exchangers

Counter	Parallel	Single stream	Single shell
$\mathrm{Fa}=\mathrm{NTU}(1-C_r)/2$	$\mathrm{Fa}=\mathrm{NTU}(1+C_r)/2$	$\mathrm{Fa}=\mathrm{NTU}/2$	$\mathrm{Fa}=\mathrm{NTU}\sqrt{1+C_r^2}/2$

The efficiency expressions for cross flow heat exchangers are more complex than Eq. (4), however, for several cross-flow heat exchangers, Eq. (4) can still be used with a high degree of accuracy by using a generalized fin analogy number [2,4]. It is also important to note that the parallel flow and counter-flow heat exchangers represent the low and high limits of efficiency for a given NTU and C_r, respectively.

Figure 1 is a plot of the heat exchanger efficiency as a function of the fin analogy number. The maximum efficiency (heat transfer) occurs for Fa=0, which, from Table 1, only happens for a balanced ($C_r=1$) counter-flow heat exchanger, or a balanced counter-flow heat exchanger has the efficiency of 100%. For a given Fa, the efficiency is obtained from Eq. (4) or Fig. 1 and the heat transfer can be determined from Eq. (3).

The analogy with fins provides additional insight into the concept of heat exchanger efficiency. For a constant area fin, the efficiency is given by

$$\eta=\frac{\tanh\left(\sqrt{\dfrac{h\bar{p}L^2}{kA_x}}\right)}{\left(\sqrt{\dfrac{h\bar{p}L^2}{kA_x}}\right)} \tag{5}$$

The heat transfer rate from a fin can be written as

$$q=\sqrt{kA_x h\bar{p}}\tanh\left(\frac{hA}{\sqrt{kA_x h\bar{p}}}\right)(T_b-T_\infty) \tag{6}$$

Rearranging Eqs. (3) and (4), the rate of heat transfer rate for a counter-flow heat exchanger becomes

$$q=\frac{2C_{\min}}{1-C_r}\tanh\left(\frac{UA}{\dfrac{2C_{\min}}{1-C_r}}\right)(\bar{T}-\bar{t}) \tag{7}$$

Although Eq. (5) indicates that increasing the fin length or the heat transfer coefficient leads to a reduction in the efficiency of a fin, the total amount of heat transfer actually increases with increasing these two parameters as seen from Eq. (6). In the limit, an infinitely long fin has an efficiency of zero, even though it still transfers a finite amount of heat. The same behavior can be seen for a heat exchanger. As the overall heat transfer coefficient or the area of the heat exchanger increases, the fin analogy number (Fa) increases, leading to a reduction in the heat exchanger efficiency. However, as can be seen from Eq. (7), the rate of heat transfer actually increases. Like a fin, an infinitely large heat exchanger has an efficiency of zero, even though it transfers a finite amount of heat.

Figure 2 is a plot of heat exchanger efficiency as a function of capacity ratio for a given NTU (=3). As can be seen, the efficiency of a counter-flow heat exchanger increases with capacity ratio, while the efficiency of shell and tube and parallel flow heat exchangers actually decreases with increasing capacity ratio.

The heat exchanger efficiency is based on the arithmetic mean temperature difference (AMTD) of the heat exchanger as the driving temperature potential and can be calculated from the knowledge of the inlet temperatures (the maximum temperature difference in the heat exchanger) and NTU and efficiency through

$$\bar{T}-\bar{t}=\frac{(T_1-t_1)}{1+\eta\,\mathrm{NTU}\left(\dfrac{1+C_r}{2}\right)} \tag{8}$$

Substituting from Eq. (8) in Eq. (3) results in

$$q=\eta UA(\bar{T}-\bar{t})=\frac{1}{\dfrac{1}{\eta\,\mathrm{NTU}}+\left(\dfrac{1+C_r}{2}\right)}C_{\min}(T_1-t_1) \tag{9}$$

Note that the fraction on the right hand side of Eq. (9) is the effectiveness of the heat exchanger and thus establishes the relation between efficiency and effectiveness.

Using the concept of heat exchanger efficiency for analyzing heat exchanger rating and sizing problems is demonstrated by two examples [5] in the Appendix. As can be seen, both types of problems can be conveniently solved using the concept of heat exchanger efficiency without the need for charts or complicated performance equations. Furthermore, the heat exchanger effi-

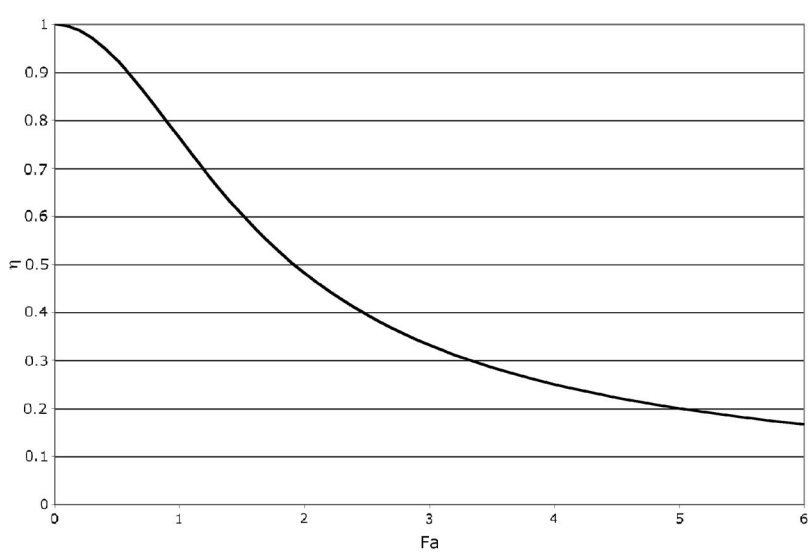

Fig. 1 Heat exchanger efficiency variation with capacity ratio

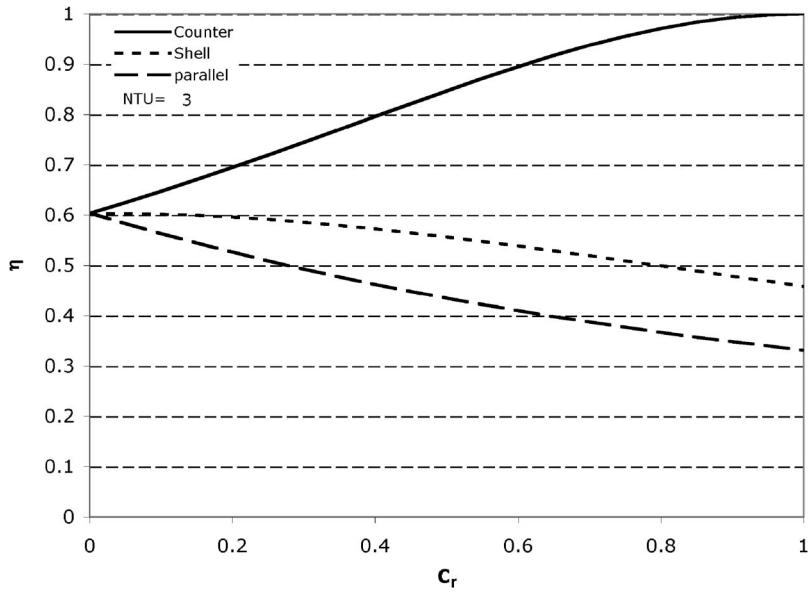

Fig. 2 Heat exchanger efficiency variation as with capacity ratio

ciency given by Eq. (4) is only a function of one nondimensional variable (Fa), whereas effectiveness (ε) depends on two parameters (C_r and NTU), and the LMTD correction factor (F) depends also on two parameters (P and R). The simple algebraic form of Eq. (4) and its dependence on one single nondimensional group simplify heat exchanger calculations and greatly facilitate comparison of different heat exchangers. The concept of thermal efficiency provides a new and more convenient approach for analyzing heat exchangers.

Like efficiency, the LMTD correction factor (F) and the heat exchanger effectiveness (ε) are also less than 1, but the efforts to relate them to the second law have not been successful. The challenge in defining a second-law-based efficiency for heat exchangers is defining an ideal heat transfer process in heat exchangers. An isentropic process is the ideal process for many components and is used to define isentropic efficiency. This obviously cannot be applied to a heat exchanger whose function is to transfer heat. Since entropy production will not be zero (notwithstanding the unrealistic case of an infinitely large heat exchanger), minimization of entropy has been considered in heat exchanger analysis.

The application of this method to heat exchangers was first proposed by McClintock [6]. Bejan [7] introduced a nondimensional parameter, the number of entropy generation units N_s, as a measure of heat exchanger irreversibility. N_s is the ratio of the total amount of entropy generated in the heat exchanger as a result of irreversibilities associated with heat transfer and fluid friction, and the maximum capacity rate. Aceves-Saborio et al. [8] extended the irreversibility minimization method by including a term to account for the exergy of the heat exchanger material.

These approaches have found limited application in heat exchanger design, partly due to the fact that the global optimum often leads to a heat exchanger with infinite area [8]. The efforts in linking the effectiveness of a heat exchanger to its rate of entropy production have also not been successful. The minimum irreversibility does not appear to correlate with the effectiveness of the heat exchanger, as pointed out by Shah and Skiepko [9]. They showed that the heat exchanger effectiveness can be maximum or minimum at the minimum irreversibility operating point, concluding that effectiveness is not a measure of heat exchanger reversibility [9]. The analysis presented below is to show that the efficiency defined above is based on the second law of thermodynamics. It will be shown that the minimum irreversibility is associated with the maximum efficiency for heat exchangers, clarifying how the second law can be extended to heat exchangers.

Consider a heat exchanger having an area A and an overall heat transfer coefficient of U, where the hot and cold fluids enter at temperatures T_1 and t_1 with capacities C_h and C_c, respectively. The heat exchanger efficiency is evaluated from Eq. (4). The average temperature difference in the heat exchanger is fixed and is determined from Eq. (8). As shown above, a balanced counter flow heat exchanger where the hot and cold fluid capacities are equal to the $C_{\min}$ of the actual heat exchanger, having the same UA and AMTD will transfer the maximum amount of heat. The inlet temperatures of the hot and cold fluids of the balanced counter-flow heat exchanger are not specified; thus, infinitely many exchangers will transfer the same maximum amount of heat. The rest of the paper is to show among all these balanced counter-flow heat exchangers, the one having the same temperature ratio (t_1/T_1) as the actual heat exchanger also generates the minimum amount of entropy.

Therefore, corresponding to an actual heat exchanger, there is an ideal balanced flow heat where the hot and cold fluid capacities are equal to the $C_{\min}$ of the actual heat exchanger. The ideal and actual heat exchangers have the same UA, the same AMTD, and the same inlet temperature ratio (t_1/T_1). The ideal heat exchanger transfers the maximum amount of heat, equal to the product of UA and AMTD, while generating the minimum amount of entropy. Note that from Eq. (8) the inlet and exit temperatures of the fluids in the ideal heat exchanger are different from those of the actual heat exchanger. The expressions for the determination of the ideal temperatures will be presented later. Based on the second law, the ideal heat exchanger is, therefore, the most efficient (transferring the maximum amount of heat) and least irreversible heat exchanger (generating the minimum amount of entropy). This ideal heat exchanger is the reference against which other heat exchangers can be compared and their efficiency assessed.

2 Analysis

Assuming heat transfer from the surroundings to be zero, and the specific heats to be constant, the nondimensional rate of entropy generation for a heat exchanger is given by [9]

$$\sigma = \sigma_T + \sigma_f = \frac{C_h}{C_{\min}} \ln \frac{T_2}{T_1} + \frac{C_c}{C_{\min}} \ln \frac{t_2}{t_1} - \frac{C_h}{C_{\min}} \frac{R_h}{c_{p,h}} \ln \frac{P_2}{P_1} - \frac{C_c}{C_{\min}} \frac{R_c}{c_{p,c}} \ln \frac{p_2}{p_1} \tag{10}$$

where

$$\sigma = \frac{\dot{S}_{\text{gen}}}{C_{\min}} \tag{11}$$

The first two terms can be considered as the amount of entropy produced due to temperature change, which is generally as a result of heat transfer (σ_T), and the last two terms are due to pressure change (σ_P), which is generally due to flow irreversibilities, including friction.

In defining an ideal heat transfer process based on the second law, only irreversibilities caused by heat transfer need to be considered. These account for most of the irreversibilities as the pressure drop across the heat exchanger is typically small, and for incompressible fluids, the entropy change is only a function of temperature. Mohamed [10] also showed that the entropy generation number due to pressure is too low compared with that due to temperature ($\sigma_P \ll \sigma_T$), so it can be neglected. The irreversibility due to heat transfer is given by

$$\sigma_T = \frac{C_h}{C_{\min}} \ln \frac{T_2}{T_1} + \frac{C_c}{C_{\min}} \ln \frac{t_2}{t_1} \tag{12}$$

Eliminating the exit temperatures

$$\sigma_T = \frac{C_h}{C_{\min}} \ln\left(1 - \frac{q}{C_h T_1}\right) + \frac{C_c}{C_{\min}} \ln\left(1 + \frac{q}{C_c t_1}\right) \tag{13}$$

Substituting for q from Eq. (8) results in

$$\sigma_T = \frac{1}{\frac{C_{\min}}{C_h}} \ln\left[1 - \frac{1}{\frac{1}{\text{NTU}\frac{C_{\min}}{C_h}\eta} + \frac{1}{2}\left(1 + \frac{C_h}{C_c}\right)} \frac{(T_1 - t_1)}{T_1}\right] + \frac{1}{\frac{C_{\min}}{C_c}} \ln\left[1 + \frac{1}{\frac{1}{\text{NTU}\frac{C_{\min}}{C_c}\eta} + \frac{1}{2}\left(1 + \frac{C_c}{C_h}\right)} \frac{(T_1 - t_1)}{t_1}\right] \tag{14}$$

from which the amount of entropy generated can be calculated. Either the cold or the hot fluid can have the minimum capacity. Assuming $C_c = C_{\min}$ results in

$$\sigma_T = \frac{1}{C_r} \ln\left[1 - C_r \frac{1}{\frac{1}{\text{NTU}\,\eta} + \frac{1 + C_r}{2}}\left(1 - \frac{t_1}{T_1}\right)\right] + \ln\left[1 + \frac{1}{\frac{1}{\text{NTU}\,\eta} + \frac{1 + C_r}{2}}\left(\frac{T_1}{t_1} - 1\right)\right] \tag{15}$$

and assuming $C_h = C_{\min}$, Eq. (14) becomes

$$\sigma_T = \ln\left[1 - \frac{1}{\frac{1}{\text{NTU}\,\eta} + \frac{1 + C_r}{2}}\left(1 - \frac{t_1}{T_1}\right)\right] + \frac{1}{C_r} \ln\left[1 + C_r \frac{1}{\frac{1}{\text{NTU}\,\eta} + \frac{1 + C_r}{2}}\left(\frac{T_1}{t_1} - 1\right)\right] \tag{16}$$

The product of NTU and η for four different heat exchangers can be obtained from Table 1 and Eq. (4) and is a function of NTU and capacity ratio. Therefore, the amount of entropy generated in the heat exchanger as given by Eqs. (15) and (16) is only a function of temperature ratio, NTU, and capacity ratio.

Figure 3 is a plot of entropy generated as a function of capacity ratio for different heat exchangers for both cases of the fluid with minimum capacity being the hot or the cold fluid. For a given NTU and inlet temperature ratio (0.7), the entropy generated is a decreasing function of capacity ratio for the three heat exchangers shown, assuming its lowest value for the balanced flow condition. For the conditions shown in Fig. 3(*a*) (NTU=3) at all capacity ratios, the counter-flow heat exchanger is the least irreversible heat exchanger, generating the least amount of entropy.

For the conditions shown in Fig. 3(*b*) (NTU=1.1) at all capacity ratios, the parallel flow heat exchanger generates the least amount of entropy. However, a parallel flow heat exchanger also has the lowest efficiency, transferring the least amount of heat compared to the others and hence the reason for lower generation of entropy. As can be seen in Fig. 3, the minimum entropy is generated for the balanced flow condition. For the balanced flow conditions, substituting $C_r = 1$ in Eqs. (15) and (16) and simplifying, results in

$$(\sigma_T)_{\min} = \ln\left[\frac{\left(1 + \frac{t_1}{T_1}\text{NTU}\,\eta\right)\left(1 + \frac{T_1}{t_1}\text{NTU}\,\eta\right)}{(1 + \text{NTU}\,\eta)^2}\right] \tag{17}$$

that is the minimum amount of entropy generated for any heat exchanger regardless of which fluid has the minimum capacity. The minimum entropy generated given by Eq. (17) is plotted in Fig. 4 as a function of the product of heat exchanger efficiency and the number of transfer units for several temperature ratios. As can be seen, the minimum entropy generated increases and reaches a maximum value and then decreases as the product ηNTU increases. This was first pointed out by Tribus and reported by Bejan [11] for a balanced counter-flow heat exchanger. This behavior is sometimes referred to as the entropy generation paradox and so far has only been reported for balanced counter-flow heat exchangers. A number of explanations [12,13] have been provided as to why such a behavior is observed for a counter-flow heat exchanger.

Equation (17) shows for all balanced flow heat exchangers, and not just counter-flow ones, that as the product ηNTU increases, the amount of entropy generated also increases to a maximum before decreasing. For all heat exchangers under the balanced flow condition, when NTU $\eta = 1$, the entropy generation reaches a maximum value of

$$(\sigma_T)_M = \ln\left[\frac{\left(1 + \frac{t_1}{T_1}\right)\left(1 + \frac{T_1}{t_1}\right)}{4}\right] \tag{18}$$

Since the efficiency of a balanced counter-flow heat exchanger is 1, maximum entropy generation for a counter-flow heat exchanger happens at NTU=1, which confirms the results of Bejan [11]. Substituting efficiency expressions of shell and tube and parallel flow heat exchangers in Eq. (17) shows that the maximum for a shell and tube heat exchanger occurs at NTU=1.2455, and for a parallel flow heat exchanger at NTU$\rightarrow\infty$.

Figure 5 shows the variation of the minimum entropy generated (Eq. (17)) normalized by its maximum (Eq. (18)) for three types of heat exchangers for a given value of the temperature ratio (0.1). For each value of NTU, efficiency is calculated from the expressions of Table 1, under the balanced flow condition. The general behavior seen in Fig. 5 is valid for all temperature ratios. The region near the point of maximum entropy is magnified to show the details of the behavior near this point. As can be seen, at a given NTU$\geq$1.2, a balanced counter-flow heat exchanger generates less entropy compared to the other heat exchangers, even though, having a higher efficiency, it transfers more heat. For small values of NTU, parallel flow or shell and tube heat exchang-

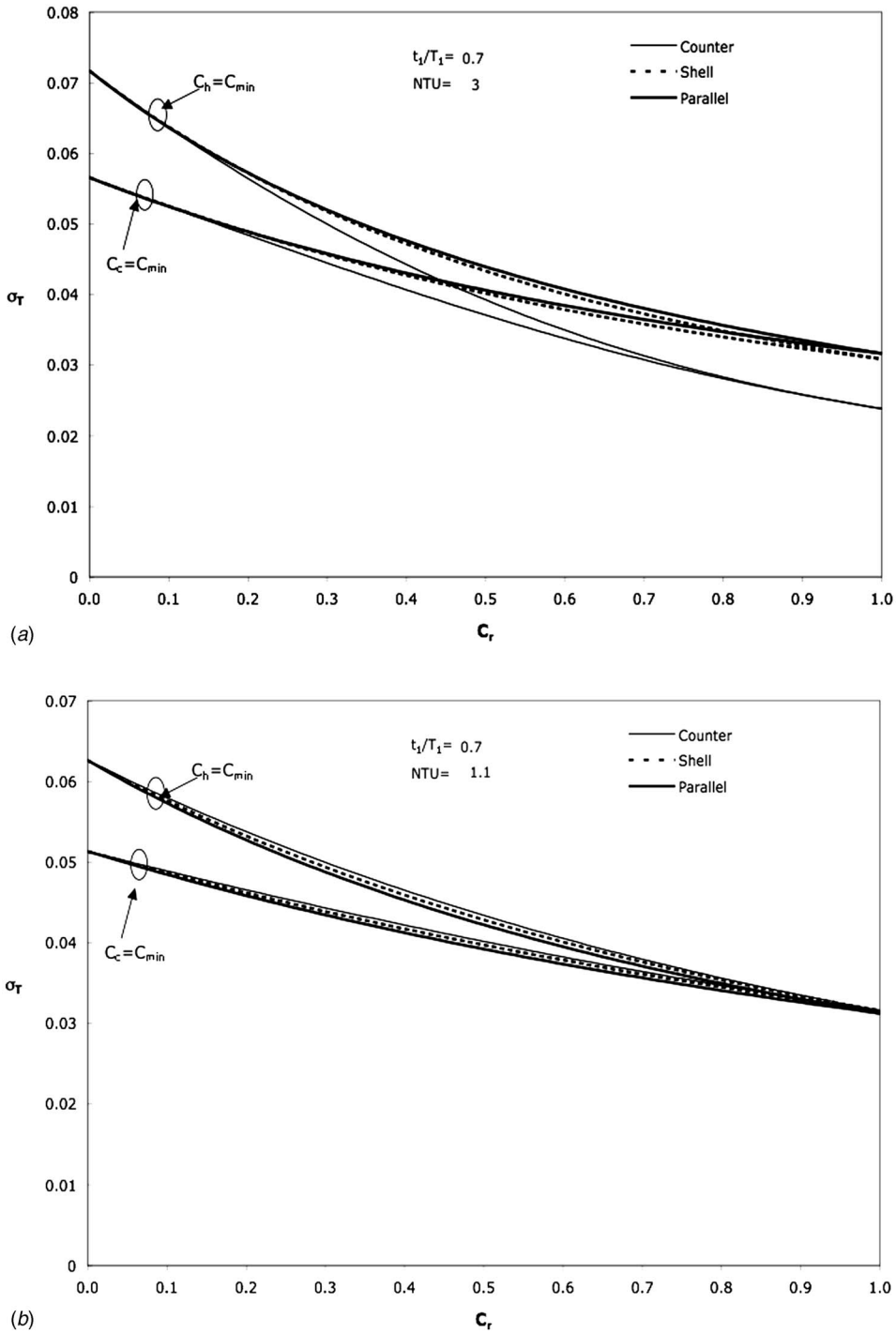

Fig. 3 Entropy generation with capacity ratio for different heat exchangers

ers generate less entropy than the counter-flow heat exchanger, due to their low efficiency (Fig. 2) and low heat transfer rates.

The minimum amount of entropy generated for a balanced counter-flow heat exchanger ($\eta=1$) is obtained from

$$(\sigma_T)_{\min} = \ln\left[\frac{\left(1+\frac{t_1}{T_1}\mathrm{NTU}\right)\left(1+\frac{T_1}{t_1}\mathrm{NTU}\right)}{(\mathrm{NTU}+1)^2}\right] \tag{19}$$

The point where the crossover, for example, for parallel flow exchanger, occurs is found by equating Eq. (17) evaluated for balanced counter-flow heat exchanger ($\eta=1$) to the same equation, evaluated for a balanced parallel flow heat exchanger ($\eta=\tanh \mathrm{NTU}/\mathrm{NTU}$)

$$\frac{\left(1+\frac{t_1}{T_1}\mathrm{NTU}\right)\left(1+\frac{T_1}{t_1}\mathrm{NTU}\right)}{(\mathrm{NTU}+1)^2} = \frac{\left(1+\frac{t_1}{T_1}\tanh \mathrm{NTU}\right)\left(1+\frac{T_1}{t_1}\tanh \mathrm{NTU}\right)}{(\tanh \mathrm{NTU}+1)^2} \tag{20}$$

Equation (20) simplifies to

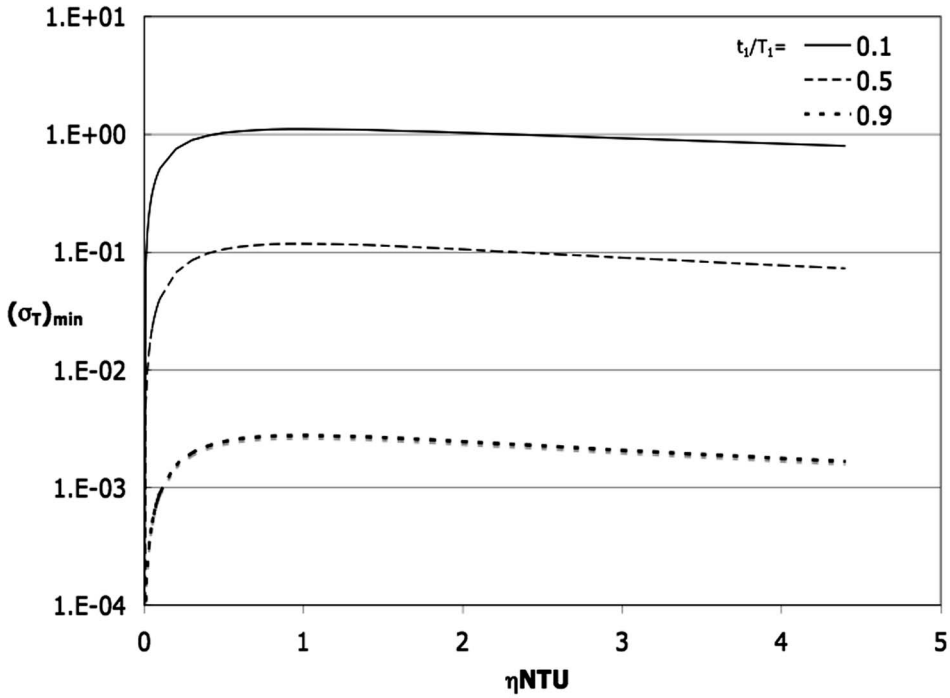

Fig. 4 Variation of entropy generation with ηNTU in balanced flow heat exchangers

$$\text{NTU} \tanh \text{NTU} = 1 \tag{21}$$

which results in the solution NTU=1.199679≅1.20. Note that at the crossover point, the parallel flow heat exchanger has an efficiency η=0.69%.

There is only one ideal heat exchanger corresponding to the specified heat exchanger under consideration. It is also helpful to further explore the idealized heat exchanger. As mentioned before, the ideal heat exchanger is a balanced counter-flow one, having the same *UA* as the actual heat exchanger where the hot and cold fluid capacities are equal to the $C_{\min}$ of the actual heat exchanger. The ideal and actual heat exchangers have the same AMTD, and the same temperature ratio (t_1/T_1). The ideal heat exchanger has an efficiency of 1 and transfers the maximum amount of heat given by

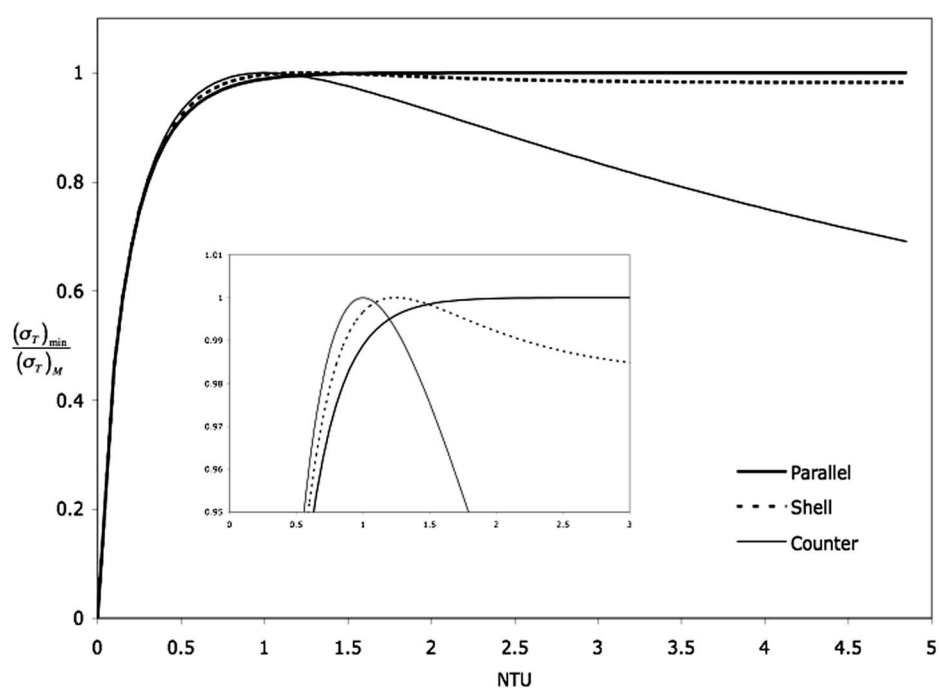

Fig. 5 Scaled entropy generation variation with NTU for different heat exchangers

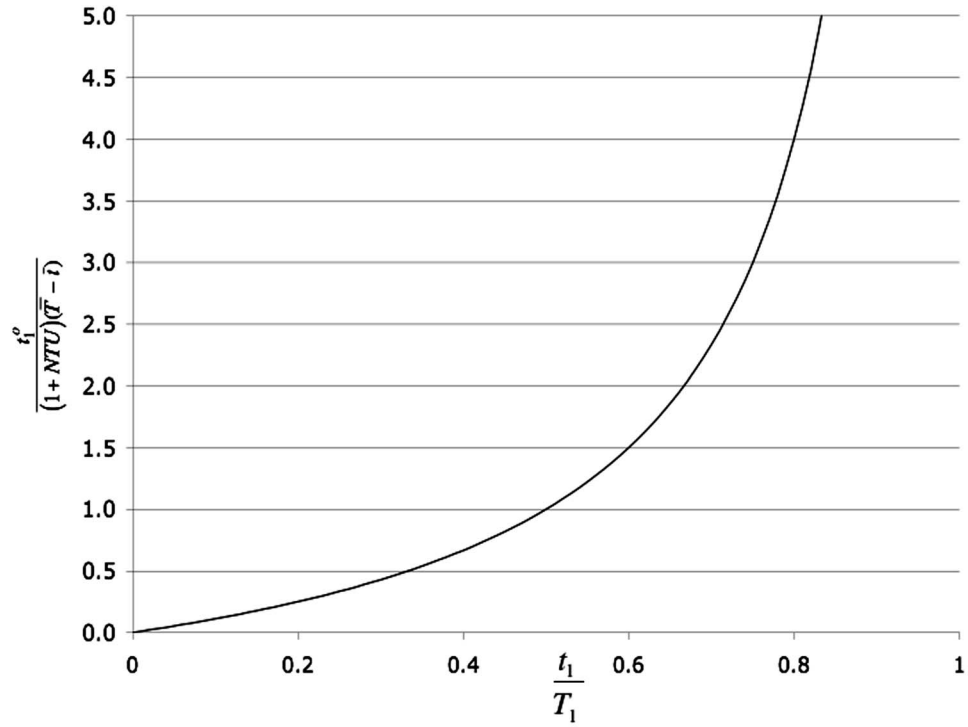

Fig. 6 Variation of the ideal temperatures as a function of the temperature ratio

$$q = UA(\bar{T} - \bar{t}) \tag{22}$$

and generates the minimum amount of entropy given by Eq. (19). The inlet and exit temperatures of the ideal heat exchanger (optimum values) are higher than those of the actual heat exchanger. Using Eqs. (8) and (9), it can be shown that they can be determined from

$$t_1^{o^*} = \frac{t_1^o}{(1+\mathrm{NTU})(\bar{T}-\bar{t})} = \frac{\frac{t_1}{T_1}}{1-\frac{t_1}{T_1}} \tag{23}$$

$$t_2^{o^*} = \frac{t_2^o}{(1+\mathrm{NTU})(\bar{T}-\bar{t})} = t_1^{o^*} + \frac{\mathrm{NTU}}{1+\mathrm{NTU}} \tag{24}$$

$$T_1^{o^*} = \frac{T_1^o}{(1+\mathrm{NTU})(\bar{T}-\bar{t})} = 1 + t_1^{o^*} \tag{25}$$

$$T_2^{o^*} = \frac{T_2^o}{(1+\mathrm{NTU})(\bar{T}-\bar{t})} = t_1^{o^*} + \frac{1}{1+\mathrm{NTU}} \tag{26}$$

The closer the inlet and exit temperatures of the actual heat exchanger are to their optimum values, the closer the heat exchanger is to its ideal performance. Equation (23) is plotted in Fig. 6, and represents the ideal nondimensional inlet temperature of the cold fluid. The temperature ratio for Examples 1 and 2 is $t_1/T_1 = 0.668$, where temperatures are in Kelvin. The inlet and exit temperatures of the ideal heat exchanger can be determined from Eqs. (23)–(26). For example, the optimum inlet temperature of the cold fluid is 40.7°C as compared to the actual temperature of 16°C.

The methodology described in this paper can also be used for optimization of heat exchangers, by defining an objective function and relaxing one or more of the specified parameters, like the heat exchanger type, U, A, inlet temperatures, etc., but is beyond the scope of the present work.

3 Conclusions

The heat exchanger efficiency is defined as the ratio of the actual heat transfer in a heat exchanger to the optimum heat transfer rate. For some of the commonly used heat exchangers, the efficiency expressions have the same simple algebraic function, similar to the efficiency of a constant area fin with an insulated tip and are a function of a single nondimensional parameter called a fin analogy (Fa) number. For a given heat exchanger and its operating condition, there exists an ideal heat exchanger, which transfers the maximum amount of heat and generates the minimum amount of entropy. The actual heat transfer from the heat exchanger is obtained by multiplying its efficiency and the optimum heat transfer rate, given by the product UA and the arithmetic mean temperature difference. The ideal heat exchanger also generates the minimum amount of entropy. The concept of heat exchanger efficiency provides a new and more convenient way for the design and analysis of heat exchangers and heat exchanger networks.

Nomenclature

A = heat exchanger surface area, m^2
A_x = fin cross section area, m^2
AMTD = arithmetic mean temperature difference; AMTD$=(\bar{T}-\bar{t})$
C_c = heat capacity rate of the cold fluid $C_c=(\dot{m}c_p)_c$
C_h = heat capacity rate of the hot fluid $C_h=(\dot{m}c_p)_h$
$C_{\min}$ = minimum heat capacity rate $=\min(C_c, C_h)$
$C_{\max}$ = maximum heat capacity rate $=\max(C_c, C_h)$
c_p = constant pressure specific heat
C_r = capacity ratio $C_r = C_{\min}/C_{\max}$
F = LMTD correction factor
Fa = fin analogy number
h = heat transfer coefficient
k = thermal conductivity

L = length
LMTD = log-mean temperature difference; LMTD$=((T_1-t_2)-(T_2-t_1))/\ln((T_1-t_2))/(T_2-t_1))$
NTU = number of transfer units NTU$=UA/C_{\min}$
P = pressure of hot fluid
p = pressure of cold fluid
P = $P=(t_2-t_1)/(T_1-t_1)$
$\bar{p}$ = circumference
q = rate of heat transfer
q_{opt} = optimum rate of heat transfer; $q_{\text{opt}}=UA(\bar{T}-\bar{t})$
R = $R=(T_1-T_2)/(t_2-t_1)$
$\dot{S}_{\text{gen}}$ = rate of entropy production
T = hot fluid temperature
t = cold fluid temperature
$\bar{T}$ = average temperature of the hot fluid $\bar{T}=(T_1+T_2)/2$
$\bar{t}$ = average temperature of the cold fluid $\bar{t}=(t_1+t_2)/2$
T^o = temperature of the hot fluid in the ideal heat exchanger
t^o = temperature of the cold fluid in the ideal heat exchanger
U = overall heat transfer coefficient, W/m^2 K
η = heat exchanger efficiency
ε = heat exchanger effectiveness $\varepsilon=q/C_{\min}(T_1-t_1)$
σ = nondimensional entropy generation rate $\sigma=\dot{S}_{\text{gen}}/C_{\min}$

Subscripts and Superscripts

1 = inlet
2 = outlet
* = nondimensional

Appendix

Example 1. Water at a rate of 10,000 kg/hr is used to cool oil from 160°C to 94°C on the shell side of a single shell and four-tube paths heat exchanger. Water having a specific heat of 4182 J/kg K enters the tubes at 16°C and exits at 84°C. If the overall heat transfer coefficient is 355 W/m^2 K, determine the heat exchanger area.

This example is based on problem 11.44 of Ref. [5] and an example of a sizing problem. Using the concept of heat exchanger efficiency, the solution is found as follows:

$$C_c=\frac{10{,}000}{3600}4182=11{,}617\ \text{W/K},$$

$$q=C_c(t_2-t_1)=11{,}617(84-16)=7.90\times10^5\ \text{W}$$

$$C_h=\frac{q}{T_1-T_2}=\frac{7.90\times10^5}{160-94}=11{,}970\ \text{W/K}\ ^\circ\text{C}$$

$$q=UA\,\eta(\bar{T}-\bar{t})=C_{\min}\text{NTU}\ \eta(\bar{T}-\bar{t})$$

$$\text{NTU}\ \eta=\frac{q}{C_{\min}(\bar{T}-\bar{t})}=\frac{7.90\times10^5}{11{,}617(77)}=0.8832$$

$$\eta=\frac{\tanh(\text{Fa})}{(\text{Fa})}$$

$$\eta=\frac{\tanh\left(\text{NTU}\dfrac{\sqrt{1+C_r^2}}{2}\right)}{\left(\text{NTU}\dfrac{\sqrt{1+C_r^2}}{2}\right)}$$

$$\text{NTU}\frac{\sqrt{1+C_r^2}}{2}=\tanh^{-1}\left(\eta\text{NTU}\frac{\sqrt{1+C_r^2}}{2}\right)$$

$$\text{NTU}=\frac{\tanh^{-1}\left(0.8832\dfrac{\sqrt{1+0.971^2}}{2}\right)}{\dfrac{\sqrt{1+0.971^2}}{2}}=1.030$$

$$\eta=\frac{0.8832}{1.030}=0.857$$

$$A=\frac{C_{\min}\text{NTU}}{U}=\frac{11{,}617\times1.030}{355}=33.71\ \text{m}^2$$

The area for the shell and tube heat exchanger calculated here matches the value given in the solutions manual for Ref. [5] by using the F correction factor.

Example 2. Water at a rate of 10,000 kg/hr and 16°C with specific heat of 4182 J/kg K is used to cool oil at 160°C in a shell and tube heat exchanger similar to Example 1, having an overall heat transfer coefficient of 355 W/m^2 K and an area of 33.71 m^2. If C_h=11,970 W/K, determine the rate of heat transfer.

This is the reverse problem, and an example of a heat exchanger rating problem, typically solved using the ε-NTU approach. Here the problem is solved using the concept of heat exchanger efficiency.

$$\text{Fa}=\text{NTU}\frac{\sqrt{1+C_r^2}}{2}=1.030\frac{\sqrt{1+0.971^2}}{2}=0.718$$

$$\eta=\frac{\tanh(\text{Fa})}{(\text{Fa})}=0.857$$

$$\bar{T}-\bar{t}=\frac{(T_1-t_1)}{1+\text{NTU}\ \eta\left(\dfrac{1+C_r}{2}\right)}=\frac{160-16}{1+1.030\times0.857\left(\dfrac{1+0.971}{2}\right)}$$
$$=77\ \text{K}$$

$$q=UA\,\eta(\bar{T}-\bar{t})=355\times33.71\times0.858\times77$$
$$=7.90\times10^5\ \text{W}$$

References

[1] Fakheri, A., 2003, "Arithmetic Mean Temperature Difference and the Concept of Heat Exchanger Efficiency," *Proceedings of the 2003 ASME Summer Heat Transfer Conference*, Las Vegas, Nevada, July 21–23, Paper No. HT2003-47360.
[2] Fakheri, A., 2003, "The Shell and Tube Heat Exchanger Efficiency and its Relation to Effectiveness," *Proceedings of the 2003 American Society of Mechanical Engineers (ASME) International Mechanical Engineering Congress and Exposition (IMECE)*, Washington, D.C., November 16–21.
[3] Fakheri, A., 2003, "An Alternative Approach for Determining Log Mean Temperature Difference Correction Factor and Number of Shells in Shell and Tube Heat Exchangers," J. Enhanced Heat Transfer, **10**(4), pp. 407–420.
[4] Fakheri, A., 2006, "Thermal Efficiency of the Cross Flow Heat Exchangers," *Proceedings of the 2006 American Society of Mechanical Engineers (ASME) International Mechanical Engineering Congress and Exposition (IMECE)*, Chicago, IL, November 5–10.
[5] Incropera, F. P., and DeWitt, D., 2002, *Fundamentals of Heat and Mass Transfer*, 4th ed., Wiley, New York.
[6] McClintock, F. A., 1951, "The Design of Heat Exchangers for Minimum Irreversibility," Presented at the ASME Annual Meeting, Paper No. 51-A-108.
[7] Bejan, A., 1977, "Concept of Irreversibility in Heat Exchanger Design: Counterflow Heat Exchangers for Gas-to-Gas Applications," ASME J. Heat Transfer, **99**(3), pp. 374–380.
[8] Aceves-Saborio, S., Ranasinghe, J., and Reistad, G. M., 1989, "Extension to

the Irreversibility Minimization Analysis Applied to Heat Exchangers," ASME J. Heat Transfer, **111**(1), pp. 29–36.
[9] Shah, R. K., and Skiepko, T., 2004, "Entropy Generation Extrema and Their Relationship With Heat Exchanger Effectiveness—Number of Transfer Unit Behavior for Complex Flow Arrangements," ASME J. Heat Transfer, **126**(6), pp. 994–1002.
[10] Mohamed, H. A., 2006, "Entropy Generation in Counter Flow Heat Exchangers," ASME J. Heat Transfer, **128**, pp. 87–92.
[11] Bejan, A., 1982, "Second-Law Analysis in Heat Transfer and Thermal Design," Adv. Heat Transfer, **15**, pp. 1–58.
[12] Sekulic, D. P., 1990, "The Second Law Quality of Energy Transformation in Heat Exchanger," ASME J. Heat Transfer, **112**, pp. 295–300.
[13] Ogiso, K., 2003, "Duality of Heat Exchanger Performance in Balanced Counter-Flow Systems," ASME J. Heat Transfer, **125**(3), pp. 530–532.