Final Report
Engineering Design Project
Design of Rhombic Drive for Stirling Engine

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ABSTRACT

Stirling engines are used to convert temperature differences into useful mechanical work. They have an edge over other engines in terms of efficiency. However its practical applications are limited by its large size and high installation cost. In a direction to reduce its size, a rhombic drive for beta type Stirling engine was designed with the objective of optimizing overlap volume. Further, Schmidt Analysis was carried out to link the thermodynamic parameters with the obtained kinematic parameters.

Key words: Stirling engine, Alpha type, Beta type, Gamma type, Overlap Volume, Schmidt Analysis.

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1. Introduction

1.1 Stirling Engine

Stirling engine is a closed-cycle regenerative hot air engine. It uses a working fluid which is permanently contained within the system. Power is generated by heating and cooling of the working fluid (usually hydrogen, helium or air). Invented by Robert Stirling in 1816, the Stirling engine has the potential to be much more efficient than a gasoline or diesel engine. Its efficiency theoretically can go up to the full Carnot efficiency. It is classified as an external combustion engine, though heat can also be supplied by non-combusting sources such as solar or nuclear energy. A Stirling engine operates through the use of an external heat source and an external heat sink having a sufficiently large temperature difference (of the order of 300-500K) between them. They are used particularly in cases where the primary objective is *not* to minimize the capital cost per unit power, but rather to minimize the cost per unit energy generated by the engine. In recent years, the advantages of Stirling engines have become increasingly significant, given the general rise in energy costs, energy shortages and environmental concerns such as climate change. The applications include water pumping, space-based astronautics and electrical generation from plentiful energy sources that are incompatible with the internal combustion engine, such as solar energy, agricultural waste and domestic refuse. If supplied with mechanical power, it can also function as a heat pump.

Though Stirling engine is advantageous over other engines in terms of efficiency, minimum pollution, diversity of the heat source used and simple design, some of its disadvantages like its size and initial cost still restrict its application to certain areas. Thus, there is a need for reducing its size, which becomes the most crucial issue. It can be done by proper design of its drives, and optimizing the relative dimensions of its components.

1.2 Stirling cycle

A Stirling engine uses the **Stirling cycle**, which is unlike the cycles used in internal-combustion engines. The gases used inside a Stirling engine never leave the engine. There are no exhaust valves that vent high-pressure gases, as in a gasoline or diesel engine, and there are no explosions taking place. Because of this, Stirling engines are very quiet. The Stirling cycle uses an external heat source, which could be anything from gasoline to solar energy. Stirling cycle is a thermodynamic cycle with two isochors (constant volume) and two isotherms (constant temperature).

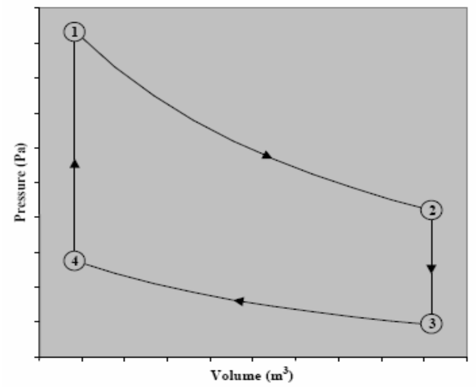

Figure 1: Stirling Cycle

- Points 1 to 2, Isothermal Expansion.
- Points 2 to 3, Constant-Volume (isochoric) heat-removal.
- Points 3 to 4, Isothermal Compression.
- Points 4 to 1, Constant-Volume (isochoric) heat-addition.

A real Stirling cycle is quasi-elliptical accounting for smooth motion.

1.3 Types of Stirling engine

Stirling engines are classified to three types with viewpoint of working space.

An **alpha Stirling** contains two separate power pistons in separate cylinders, one hot piston and one cold piston. The hot piston cylinder is situated inside the higher temperature heat exchanger and the cold piston cylinder is situated inside the low temperature heat exchanger. The two cylinders are connected by a regenerator.

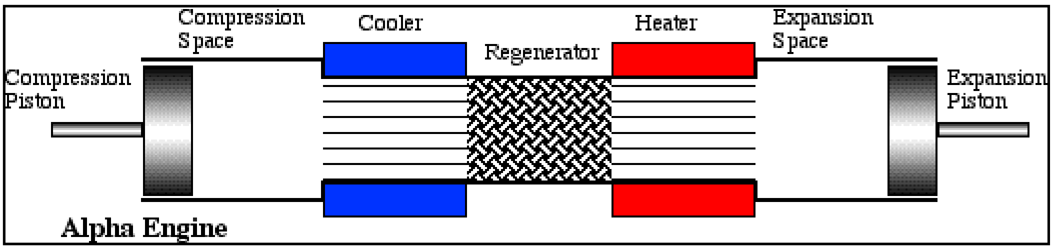

Figure 2: Alpha type Stirling engine

The **beta Stirling** has a displacer and a power piston with the same cylinder on the same shaft. The displacer piston is a loose fit and does not extract any power from the expanding gas but only serves to shuttle the working gas from the hot heat exchanger to the cold heat exchanger. When the working gas is pushed to the hot end of the cylinder it expands and pushes the power piston. When it is pushed to the cold end of the cylinder it contracts and the momentum of the machine, pushes the power piston the other way to compress the gas.

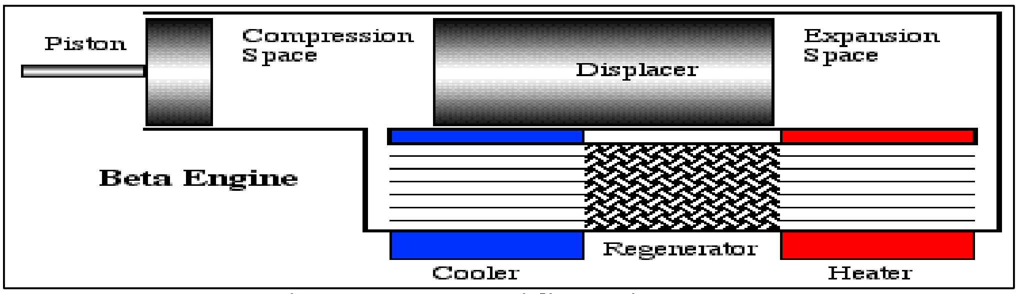

Figure 3: Beta type Stirling engine

The **gamma-type** has a displacer and a power piston with independent cylinders. The gas in the two cylinders can flow freely between them but remains a single body.

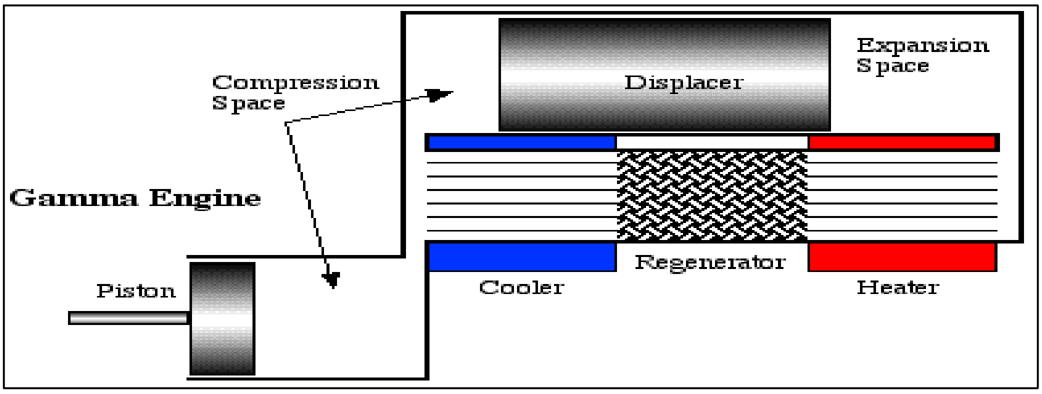

Figure 4: Gamma type Stirling engine

1.4 Working of a Beta Stirling Engine and the role of a Rhombic drive

The Beta type Stirling engine works in the following steps.

(a) Initially, the power piston (dark grey Fig 5.a) has compressed the gas, the displacer piston (light grey) has moved so that most of the gas is adjacent to the hot heat exchanger.
(b) The heated gas (Fig 5.b) increases its pressure and pushes the power piston along the cylinder. This is the **power stroke.**
(c) The displacer piston (Fig 5.c) now moves to shunt the gas to the cold end of the cylinder.
(d) The cooled gas (Fig 5.d) is now compressed by the flywheel momentum. This takes less energy since when it cooled its pressure also dropped.

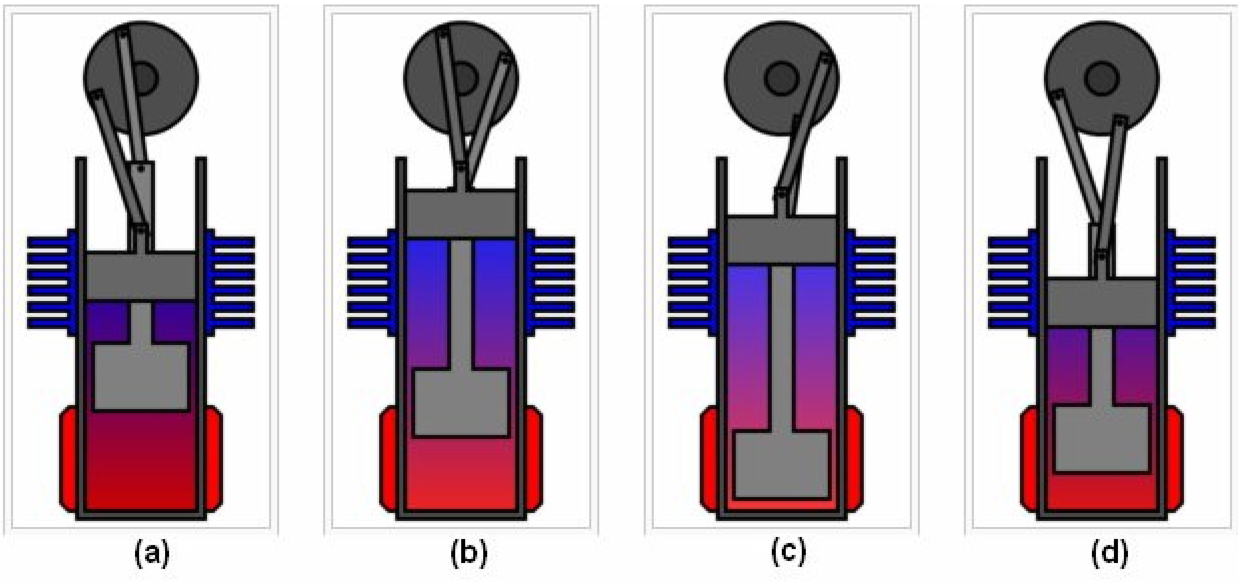

Figure 5: Step-wise operation of Beta-type Stirling engine

The **rhombic drive** is a specific method of transferring mechanical energy generated by the Beta type Stirling engine (Fig 6). The drive's complexity and tight tolerances causing a high cost of manufacture is a hurdle for the wide-spread usage of this drive.

In its simplest form, the drive utilizes a jointed rhomboid to convert linear work from a reciprocating piston to rotational work. The connecting rod of the piston is rigid as opposed to a normal reciprocating engine which directly connects the piston to the crankshaft with a flexible joint in the piston.

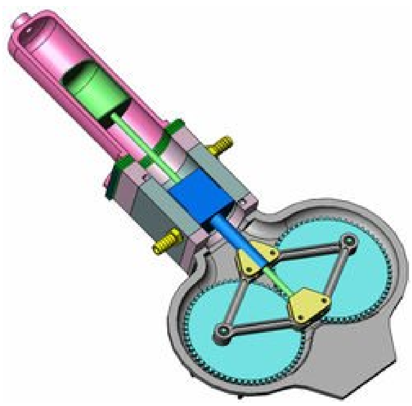

Figure 6: Rhombic drive

When force is applied to the piston, it pushes down; at the same time, the outer corners of the rhomboid push out. They push on two cranks/flywheels which cause them to rotate, each in opposite directions. As the wheels rotate the rhombus progresses its change of shape from being flattened in the direction of the piston axis at top dead centre to being flattened in the perpendicular direction to the piston axis at bottom dead centre.

2. Analysis of Rhombic Drive Mechanism for Stirling engines

Stirling Engines belong to a class of engines that operate on temperature difference. The Rhombic drive is a very popular mechanism that can be used to drive a β type Stirling Engine which consists of a Power piston and a Displacer. The Stirling engine provides power by driving the power piston. The gas used in the Stirling cycle (typically Hydrogen) moves across the displacer to and fro from the hot chamber to the cold chamber and vice versa.

2.1 Expressions for expansion and compression volumes

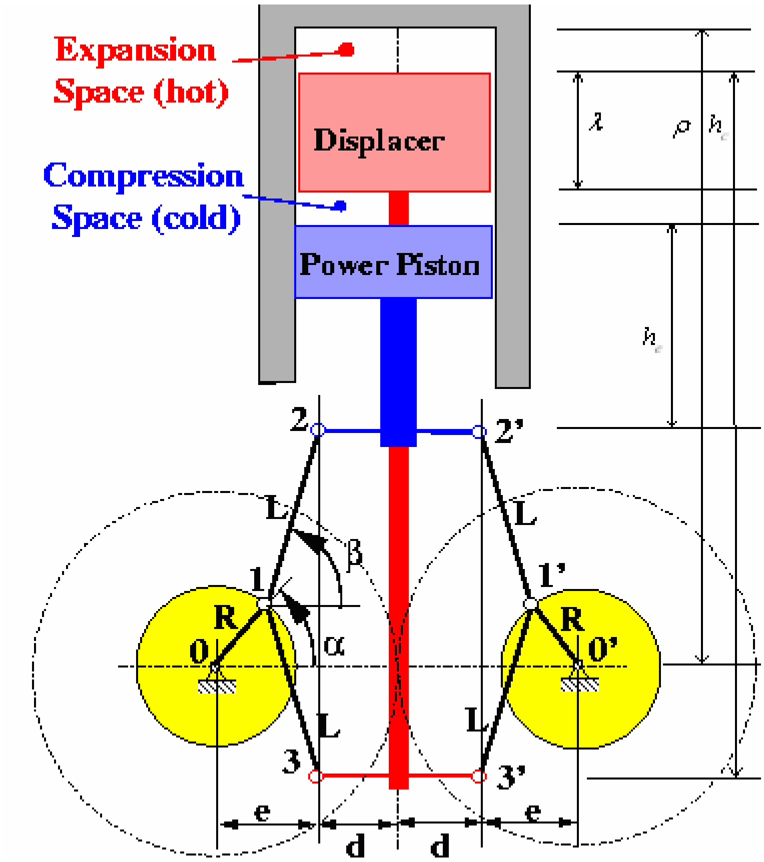

Figure 7: Schematic of a Beta-type with Rhombic engine

The Fig 7 represents the schematics of a rhombic drive with power piston connected to the upper and displacer to the lower section. The bars 12, 1'2', 13, and 1'3' have identical length, L , and are connected to the cross-bars 22' and 33' by pin/hole connections. Joints 1 and 1' are pin-hole connections as well. The crank throws 01 and 0'1' have identical length, r, and the crank centers 0 and 0' are at equal distance, $d+e$, from the piston axis. The size of the cross bars, 22' and 33', d , is has no bearing on the analysis of the variation of the height of the expansion space, V_e , and the compression space, V_c , as the angle α changes.

In the configuration shown, the left crank will turn clockwise and the right crank counter clockwise in order to achieve proper phase lag between expansion and compression space. They turn at the same angular velocity which can be accomplished by two intermeshing counter-rotating gears.

Modeling the Expansion space height V_e , we get

$$V_e = C_e - R\sin\alpha + L\sin\beta \qquad \ldots\ldots (1)$$

where $C_e = \rho - h_e$

Also we can model the compression space height V_c by writing the equation of variation of the top of the Power piston as

$V_e + \lambda + V_c + h_c + L\sin\beta + R\sin\alpha = \rho$

Substituting from equation (1) for V_e we get

$$V_c = C_c - 2L\sin\beta \qquad \text{……... (2)}$$

where $C_c = h_e - h_c - \lambda$

The values of these constants are evaluated from the minimum values of V_c and V_e or the clearance of the compression and expansion volume. For the convenience of this computation we may choose the clearance to be zero.

From the figure the constraint on angles α and β becomes evident and is

$$e = R\cos\alpha + L\cos\beta = \text{const} \qquad \text{……….. (3)}$$

Let us differentiate this equation w.r.t α. We get

$$\frac{de}{d\alpha} = 0 = -R\sin\alpha - L\sin\beta\frac{d\beta}{d\alpha} \Rightarrow$$

$$\frac{d\beta}{d\alpha} = -\frac{R}{L}(\frac{\sin\alpha}{\sin\beta}) \qquad \text{............ (4)}$$

Also for complete rotation of crank we assume $L > R + e$

Now let us evaluate the **minimum of** V_e and set it equal to zero so as to get the value of C_e.

Setting $\frac{dV_e}{d\alpha} = 0$ in equation (1) and using the result obtained in Equation (4) we conclude that V_e becomes minimum when $\sin(\alpha + \beta) = 0$ or $\alpha + \beta = \pi$

i.e. when in Figure the points 1 and 3 are exactly on opposite sides of crank center 0, with point 1 above and to the left of point 0.'

Therefore for the minimum we get from Equation (3)

$$\cos\beta = \frac{e}{L - R} \text{ and } \alpha + \beta = \pi \qquad \text{………… (5)}$$

Substituting Equation (5) in (1) we get the value of min $\{V_e\}$

$$\min\{V_e\} = 0 = C_e - (R - L)\sqrt{1 - (\frac{e}{L - R})^2}$$

$$\Rightarrow C_e = -(L - R)\sqrt{1 - (\frac{e}{L - R})^2}$$

Putting this back in Equation (1) we get the variation of the expansion volume as

$$V_e = -(L-R)\sqrt{1-(\frac{e}{L-R})^2} - R\sin\alpha + L\sin\beta \qquad \text{…………. (6)}$$

Similarly, the **maximum value of** V_e occurs at $\alpha+\beta=2\pi$ for which again $\sin(\alpha+\beta)=0$. The $\max\{V_e\}$ occurs when the points 1 and 3 are exactly in line with the crank centre '0' but this time both are below and to the right of point '0'.

Substituting $\alpha+\beta=2\pi$ in Equation (3) we get

$$\cos\beta = \frac{e}{L+R} \text{ and } \alpha+\beta=2\pi \qquad \text{……….. (7)}$$

Substituting Equation (7) into (6) we get

$$\max\{V_e\} = \sqrt{(L+R)^2-(e)^2} - \sqrt{(L-R)^2-e^2} \qquad \text{……….. (8)}$$

Assuming that $e \geq R$ now let us **determine the value of the constant** C_c on similar lines. **Minimizing the value of equation (2)** w.r.t α we get

$$\frac{dV_c}{d\alpha} = 0 \Rightarrow \cot\beta\sin\alpha = 0$$

$$\Rightarrow \text{ either } \beta \to \frac{\pi}{2} \text{ or } \alpha = 0$$

Putting $\beta = \frac{\pi}{2}$ in Equation (3) we get $\cos\alpha = \frac{e}{R} > 1$ ($\because e > R$ basic assumption)

Hence $\alpha = 0$ is required for minima. Substituting this in Equation (3) we get

$$\cos\beta = \frac{(e-R)}{L} \qquad \text{………… (9)}$$

Therefore Substituting Equation (9) into (2) we get

$$\min\{V_c\} = 0 = C_c - 2L\sqrt{1-(\frac{e-R}{L})^2}$$

$$\Rightarrow C_c = 2L\sqrt{1-(\frac{e-R}{L})^2}$$

Putting this back in equation (2) we get the expression for variation of compression volume V_c as

$$V_c = 2L\{\sqrt{1-(\frac{e-R}{L})^2} - \sin\beta\} \qquad \text{………… (10)}$$

Now let us find the maximum value of V_c **.** This happens when $\alpha = 180^\circ$. Substituting this value of α back into Equation (3) we get

$$\cos\beta = \frac{(e+R)}{L} \qquad \text{.......... (11)}$$

Substituting (11) back in equation (10) we get

$$\max(V_c) = 2\{\sqrt{L^2-(e-R)^2} - \sqrt{L^2-(e+R)^2}\} \qquad \text{........... (12)}$$

The expressions of compression volume V_c and expansion volume V_e are plotted against time in Fig 8. (Also refer APPENDIX-I)

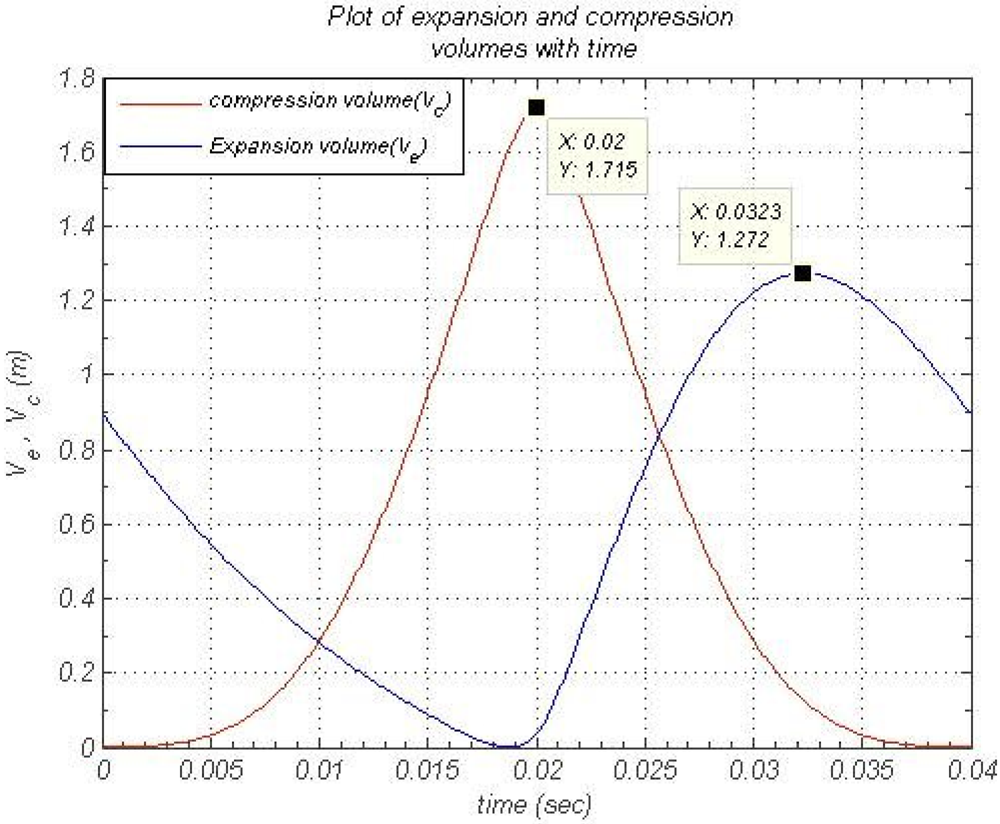

Figure 8: Plot of compression and expansion volume v/s time

One important point to be noted is that all these calculations for V_c are carried out for the condition when $e \geq R$. For $e < R$ a similar set of calculations can be carried out.

2.2 Some other terminologies

Let us now calculate two important terms of relevance to the Stirling engine.

(a) **VAR (Volume Amplitude ratio):**

$$VAR = \frac{\max\{V_c\}}{\max\{V_e\}} = \frac{2\{\sqrt{1-(\varepsilon-\rho)^2} - \sqrt{1-(\varepsilon+\rho)^2}\}}{\{\sqrt{(1+\rho)^2-\varepsilon^2} - \sqrt{(1-\rho)^2-\varepsilon^2}\}}$$

where $\varepsilon = \frac{e}{L}$ and $\rho = \frac{R}{L}$ (Note that $\varepsilon \geq \rho$) .

(b) **Volume Phase lag (VAP):**

It is generally assumed that V_c as a function of crank angle α lags behind V_e by 90° for optimum performance of a Stirling Engine. Maximum V_c occurred at $\alpha = 180^\circ$ & V_e in the range of $(270-360)^\circ$.

This indicates that clearly the engine has to rotate in such a way that the flywheel on the left hand side of the Figure rotates in the clockwise direction. Moreover rhombic drive needs different amount of time for the upstroke and the down stroke it is difficult to define phase lag here in a unique way.

Therefore looking separately at the phase lag based on occurrence of Max and Min of the expansion and compression volumes we get:

$$VPL \quad \text{(for max)}=180^\circ - \cos^{-1}(\frac{e}{L+R})$$

$$VPL \quad \text{(for min)}=180^\circ - \cos^{-1}(\frac{e}{L-R})$$

2.3 Overlap volume and its optimization

The last thing that we would evaluate in this exercise is the overlap length of the displacer and the power piston.

From Equation (8)

$$\max\{V_e\} + \lambda = \lambda + \sqrt{(L+R)^2 - (e)^2} - \sqrt{(L-R)^2 - e^2} \qquad \text{………… (13)}$$

Also using Equation (6) and (10) we get

$$V_c + \lambda + V_e = -\sqrt{(L-R)^2 - (e)^2} + 2\sqrt{L^2 - (e-R)^2} - R\sin\alpha - L\sin\beta + \lambda \qquad \text{................ (14)}$$

Minimizing this volume we get

$$\frac{d(V_c + \lambda + V_{\,e})}{d\alpha} = 0 \Rightarrow \sin(\alpha - \beta) = 0 \quad \text{\{using the result in Equation (4)\}}$$

$\therefore \alpha = \beta$ is the condition when the minimum volume above the power piston is achieved. Substituting this in Equation (3) we get

$$\cos\alpha = \cos\beta = \frac{e}{L+R} \qquad \text{………….. (15)}$$

Substituting Equation (15) into (14) we get

$$\min\{V_c + \lambda + V_e\} = -\sqrt{(L-R)^2 - (e)^2} + 2\sqrt{L^2 - (e-R)^2} - \sqrt{(R+L)^2 - e^2} + \lambda \qquad \text{…… (16)}$$

Therefore overlap region {from Equation (13) & (16)} between the displacer and the power piston equals

$$\max\{V_e + \lambda\} - \min\{V_c + \lambda + V_e\} = 2\{\sqrt{(R+L)^2 - e^2} - \sqrt{L^2 - (e-R)^2}\} \qquad \text{……… (17)}$$

Values of R, L and e have to be chosen such that we get maximum possible overlap ensuring that the cylinder volume is properly utilized. This is accomplished by having a surface plot of Overlap volume v/s e/L and R/L (Fig 9).

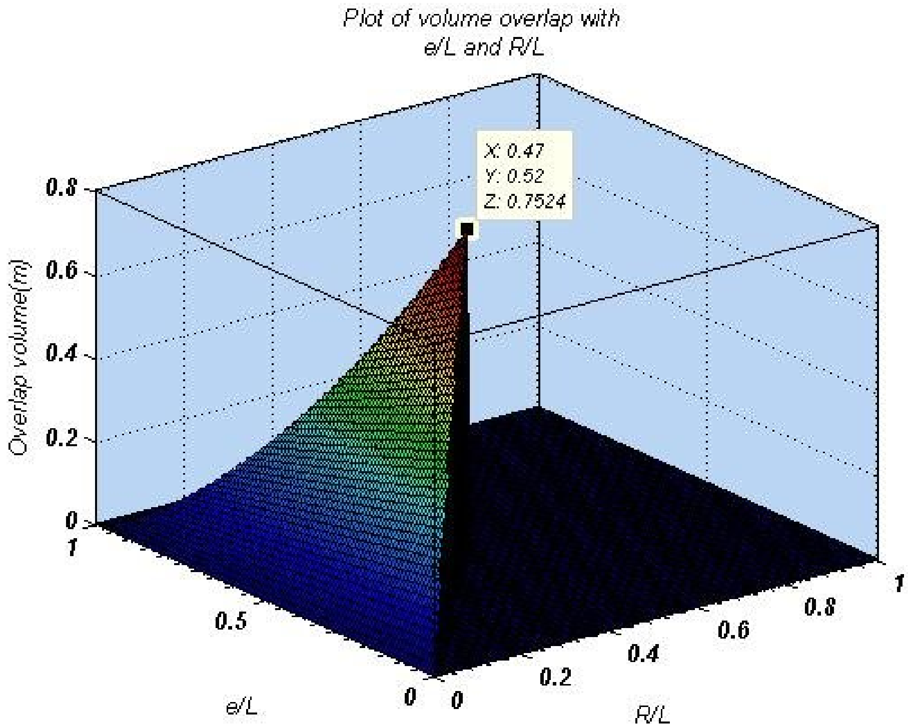

Figure 9: Surface plot of overlap volume v/s e/L & R/L

The constraints under which the surface plots were obtained are $L > R + e$ and $(e/L) \geq 1.1(R/L)$. We incorporate the factor 1.1 so as to ensure that the sides of the rhombus never become perpendicular to the base, in other words we ensure that $\beta < 90^{\circ}$. From the plots we get $\frac{R}{L} = 0.47$ **and** $\frac{e}{L} = 0.52$**.** (Also refer APPENDIX-I)

3. Schmidt's Analysis

Schmidt's analysis involves calculation of the pressure of the Stirling engine as a function of crank angle. It is based on the Isothermal conditions. The calculation of pressure leads to the determination of the compression and the expansion work done by the engine. The total work comes out from the two works and the optimization of the work done can be done thereafter.
Schmidt's analysis involves several assumptions. These assumptions may differ in their validity for different working conditions. Keeping in view the scope of research in the project, we more or less agree with all the assumptions stated in the Schmidt's analysis. So, first we'll deal with the basics of Schmidt's analysis and then we move on to modify it according to our requirements.

Basic Schmidt's analysis assumes both the volume of compression as well as the volume of expansion to vary sinusoidally with respect to the crank angle. This differs from our analysis of the rhombic drive which deducts a non-sinusoidal variation of one of the volumes. Other than this, all the assumptions remain same. Following are the assumptions:

- An ideal gas is used as working fluid and the ideal gas law connecting pressure p, volume V, mass m, and temperature T to each other via the specific gas constant R :
  p V = m R T
- The pressure P is the same everywhere inside the engine and varies only with time.
- The heat transfer conditions are sufficient to keep the gas inside the expansion space, volume V_e, and compression space, volume V_c, at constant temperature T_h and T_c respectively at all times. Therefore, the masses of gas inside the expansion and the heater space, respectively, are given by :
  $m_e = p\ V_e / (R\ T_h)$
  $m_c = p\ V_c / (R\ T_c)$
- The heat transfer conditions are sufficient to keep the temperature distribution inside the regenerator, volume V_r , linear, varying from T_c where the regenerator is connected to the kooler to T_h at the heater side. Therefore the mass of gas inside the regenerator space is given by :
  $m_r = p\ V_r \ln(T_h/T_c) / (R (T_h - T_c))$
- The volume of the compression space varies as follows:
  $V_c = K_1 - 2L\sin\beta$
  Where K_1 is given by:
  $$K_1 = 2L\sqrt{1-\left(\frac{e-R}{L}\right)^2}$$
- The volume of the expansion space varies as follows:
  $V_e = K_2 - R\sin\alpha + L\sin\beta$
  $$K_2 = (R-L)\sqrt{1-\left(\frac{e}{L-R}\right)^2}$$
  And α & β are related as follows:
  $e = R\cos\alpha + L\cos\beta$
- The total mass of gas inside the engine M_{tot} does not vary in time and is the sum total of masses of gas in the compression, expansion and the regenerator space.
  $m_{tot}=m_c+m_e+m_r$
- **Important:** The discussion of regenerator and its role is not taken forward in the further calculations. Therefore any quantity related to it is taken to be zero.

3.1 Pressure

An equation for pressure P as function of angle α can now be found by substituting the equations for masses of compression and expansion spaces in the total mass and solving it for pressure. This gives us the total pressure of the system as follows:

$$m_{tot} = \frac{P}{R_g}\left[\frac{K_1}{T_c} + \frac{K_2}{T_h} - \frac{R}{T_h}\sin\alpha + \left(\frac{L}{T_h} - \frac{2L}{T_c}\right)\sin\beta\right]$$

Let

$$A = \frac{K_1}{T_c} + \frac{K_2}{T_h},\quad B = -\frac{R}{T_h},\quad C = \left(\frac{L}{T_h} - \frac{2L}{T_c}\right)$$

This gives us P as:

$$P = \frac{m_{tot} R_{gas}}{A + B\sin\alpha + C\sin\beta}$$

From the relation of α and β we have:

$$\sin\beta = \sqrt{1 - \left(\frac{e - R\cos\alpha}{L}\right)^2}$$

Also the area of cross section of the cylinder is:

$$A_{cyl} = \pi\frac{D^2}{4}$$

Thus

$$P = \frac{m_{tot} R_{gas} A_{cyl}}{A + B\sin\alpha + C\sqrt{1 - \left(\frac{e - R\cos\alpha}{L}\right)^2}}$$

We can carry out an optimization scheme to find out the maximum and minimum pressure of the system. This is not required for the purpose of finding the net work done by the system but is necessary for design consideration.

Also the average working pressure can be calculated by integrating (preferably numerical) the pressure function for one complete cycle of the crank.

3.2 Work

Work done is comprised of two components, work done by expansion space and work done by compression space. By definition:

$$W = \int PdV$$

Expansion space:

Work done is given by:

$$W_e = \int_0^{2\pi} PV_e d\alpha$$

Where:

$$V_e = K_2 - R\sin\alpha + L\sin\beta = K_2 - R\sin\alpha + L\sqrt{1 - \left(\frac{e - R\cos\alpha}{L}\right)^2}$$

Therefore:

$$dV_e = \left(\frac{\sin\alpha\left(e - R\cos\alpha\right)}{\sqrt{L^2 - \left(e - R\cos\alpha\right)^2}} - \cos\alpha\right)Rd\alpha$$

And P is given by the final equation for pressure.

Compression space:

Work done is given by:

$$W_c = \int_0^{2\pi} PdV_c$$

Where:

$$V_c = K_1 - 2L\sqrt{1-\left(\frac{e-R\cos\alpha}{L}\right)^2}$$

Therefore:

$$dV_c = 2R\frac{\sin\alpha\left(e-R\cos\alpha\right)}{\sqrt{L^2-\left(e-R\cos\alpha\right)^2}}d\alpha$$

3.3 Numerical Integration

Keeping in mind the maximum overlap we take some prototype values of the parameters associated with Stirling engine under our observation. Following is the set of calculations which gives us the work done.

L=10 cm

For maximum overlap,

$\frac{R}{L} = 0.47$, $\frac{e}{L} = 0.52$.

Thus we have:

e	r	K_1	K_2	T_h	T_c	A	B	C
5.2 cm	4.7cm	19.975cm	- 1.025cm	750 K	350 K	0.0557 cm/K	-0.00627 cm/K	-0.0438 cm/K

Assuming the working gas to be air,

M_{tot}= 0.009 Kg

P = 568 Kpa

$$P = \frac{m_{tot}R_{gas}}{\left(0.0557-0.00627\sin\alpha-0.0438\sqrt{1-\left(0.52-0.47\cos\alpha\right)^2}\right)*50.7*10^{-6}}$$

Numerical integration with trapezoidal rule gives: (Also refer APPENDIX-I)

W_c = 8.51 KJ

W_e = -5.48 KJ

Net Work done = 8.51-5.48 = 3.027 KJ

4. Conclusions

Satisfying the objective of the project, we conclude that the optimal values for e/L and R/L are 0.52 and 0.47 respectively. These values are obtained by an in-depth detailed analysis of rhombic drive mechanism of beta type Stirling engine with the help of computational tools like MATLAB through surface plots and graphs. The constraints under which the surface plots were obtained are $L > R + e$ and $(e/L) \geq 1.1(R/L)$. We incorporate the factor 1.1 so as to ensure that the sides of the rhombus never become perpendicular to the base, in other words we ensure that $\beta < 90^{\circ}$. For a constant angular velocity given to the crank of the rhombic drive mechanism, we found through plots that the compression volume exhibited a sinusoidal behavior, which was not the case with the expansion volume. Thus, a unique phase difference between expansion and compression volumes could not be defined. Further, a Schmidt's analysis was also carried out linking the thermodynamic parameters to the optimal design. For a sample set of values, the net work done was calculated.

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APPENDIX- I

Matlab code used to generate the optimal overlap volume

```
% Program to calculate maximum volume overlap and to plot
% the overlap volume with respect to the parameters e(e/L) and
% r(r/L), which are clear from the diagram as shown in the .pdf
% file.

clear all;
t0=cputime; %initial time of the cpu
e(1)=0;
r(1)=0;
for i=1:1:100 % loop for defining the range of variables
    e(i+1)=e(i)+0.01;
    for j=1:1:100
       r(j+1)=r(j)+0.01;
    end;
end;
overlap(1,1)=0;
L=1;% keeping the scale as L=1m.
for k=1:1:101% loop for calculating the overlap values
    for l=1:1:101
       if(e(k)+r(l)<=1 && e(k)>=1.1*r(l))%incorporating design constraints
           % 1.1 has been chosen instead of 1 to account for fabrication
           % errors.
           overlap(k,l)=2*L*(((1+r(l))^2-(e(k))^2)^(0.5)-(1-(e(k)-
r(l))^2)^(0.5));
           %just using the formula derived for overlap volume.
       else
           overlap(k,l)=0;
       end;
    end;
end;
max=0;
max_a=0;
max_b=0;
for a=1:1:101 % loop to calculate the maximum values of volume overlap and
corresponding value of variables
    for b=1:1:101
       if(overlap(a,b)>=max)
           max=overlap(a,b);
           max_a=a;
           max_b=b;
       end;
    end;
end;
%displaying the values
disp('The value of maximum volume overlap is =');
disp(max);
disp('The value of e/L at max overlap is =');
disp(e(max_a));
disp('The value of r/L at max overlap is =');
disp(r(max_b));

% plotting the overlap volume
[X Y]=meshgrid(e,r);
surf(X,Y,overlap);
```

```
% calculating the time taken by the cpu
disp('time taken by the cpu is');
disp(num2str(cputime-t0));
```

Matlab code used to generate the plots for the expansion and compression space

```
%program to plot V_c and V_e
%this is done for values of parameters for maximum overlap volume
clear all;
t0=cputime;
t=0:0.0001:0.04;
e=0.52;
r=0.47;
L=1;
omega=2*pi*1500/60;%taking rpm of the driver gear to be 1500 rpm
alpha=omega.*t;
beta=acos((e-r.*cos(alpha))/L);
V_c=2*L*((1-((e-r)/L)^2)^(0.5)-sin(beta));
V_e=-(L-r)*((1-(e/(L-r))^2)^(0.5))-r.*sin(alpha)+L.*sin(beta);
plot(t,V_e);
hold on;
plot(t,V_c);
disp('time taken by the computer');
disp(num2str(cputime-t0);
```

Matlab code for numerical integration of work done

```
clear all;
n=1;
for alpha=0:pi/6:2*pi %taking the spacing to be pi/6
    %writing the whole expression for compression pdV
     f(n)=(2*0.47*(5.2-4.7*cos(alpha))*sin(alpha))/((0.0557-
(0.00627*sin(alpha))-(0.0438*((0.52-0.47*cos(alpha))^2)^(1/2)))*((1-((0.52-
0.47*cos(alpha))^2))^(1/2)));
     % writing the whole expression for expansion pdV
    g(n)=((-4.7*cos(alpha)*(1-((0.52-0.47*cos(alpha))^2)^(1/2)))-
(0.47*sin(alpha)*(5.2-4.7*cos(alpha))))/((0.0557-(0.00627*sin(alpha))-
(0.0438*((0.52-0.47*cos(alpha))^2)^(1/2)))*((1-((0.52-
0.47*cos(alpha))^2))^(1/2)));
    n=n+1;
end;

% applying finally the trapezoidal rule
integral_comp=(pi/12)*(f(1)+2*f(2)+2*f(3)+2*f(4)+2*f(5)+2*f(6)+2*f(7)+2*f(8)+
2*f(9)+2*f(10)+2*f(11)+2*f(12)+f(13));
integral_exp=
(pi/12)*(g(1)+2*g(2)+2*g(3)+2*g(4)+2*g(5)+2*g(6)+2*g(7)+2*g(8)+2*g(9)+2*g(10)
+2*g(11)+2*g(12)+g(13));
```