Robust Fuzzy Control of an Active Magnetic Bearing Subject to Voltage Saturation

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Abstract

Based on a recently proposed model for the active-magnetic-bearing (AMB) switching mode of operation, this paper presents a robust Takagi-Sugeno (T-S) -model-based fuzzy-control strategy to stabilize the AMB with fast response speed subject to control-voltage saturation and parameter uncertainties. The sufficient conditions for the existence of such a controller are derived in terms of linear matrix inequalities.
INTRODUCTION

- Active magnetic bearings (AMB) are being increasingly used in a variety of rotating machines, artificial heart pumps, compressors, high-speed milling spindles, flywheel energy-storage systems, etc. With the on-contact nature, AMBs offer many appealing advantages over conventional mechanical bearings, such as low power loss, elimination of oil supply and lubrication, long life, low weight.
II. AMB MODEL

A simplified AMB system, as shown in Fig. 1, is studied in this paper. Neglecting gravity, the mechanical subsystem of the AMB model is governed by

\[ m\ddot{z}(t) = F_1(t) - F_2(t) \quad (1) \]

Where \( m \) is the rotor mass. \( Z(t) \) represents the position of the rotor center. \( F_1(t), F_2(t) \) denote the forces produced by the two electromagnets.
II. AMB MODEL

$$F_i(t) = \frac{\Phi_i^2(t)}{\mu_o A_g}, \quad i=1, 2 \quad (2)$$

Where $\mu_o$ is the permeability of air, $A_g$ is the cross-sectional area of the electromagnet, $\Phi(t)$ is the total magnetic flux of the $i$th electromagnet.

$$\Phi(t) = \Phi_0 + \phi_i(t) \quad i = 1, 2 \quad (3)$$

The electrical dynamics are given by

$$\dot{\Phi}_i(t) = \dot{\phi}_i(t) = v_i/N \quad i = 1, 2 \quad (4)$$
II. AMB MODEL

where \( N \) denotes the number of coil turns in the electromagnet and \( v_i \) is the input control voltage of the \( i \)th electromagnet. The coil resistance \( R_i \) was neglected here for simplicity.

AMB model has the equivalent form

\[
\ddot{z}(t) = \frac{1}{m\mu_0 A_g}(2\Phi_0\phi(t) + \phi(t)|\phi(t)|) \tag{5}
\]

Where \( \Phi_0 = \Phi_0 + \min\{\phi_1(0), \phi_2(0)\} \)
II. AMB MODEL

Defining the state variables $x_1(t) = z(t)$, $x_2(t) = \dot{z}(t)$, and $x_3(t) = \phi(t)$ and the change of control variable $v(t) = u(t)$ and considering that the input voltage to the original AMB model is amplitude-limited, the state-space equation of the nonlinear time-varying AMB model can be written as

$$\dot{x}(t) = A(t)x(t) + B\bar{u}(t)$$  \hspace{1cm} (6)

$$A(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \beta_0 + \beta_1 \int (\phi(t)) \\ 0 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (7)

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/N \end{bmatrix}$$  \hspace{1cm} (8)
III. T-S FUZZY MODEL OF AMB

Consider the magnetic- material saturation in practice, the generalized control flux $\phi(t) x_3(t)$ will be bounded by its minimum value $\phi_{\min}$ and its maximum value $\phi_{\max}$, and hence the nonlinear function $\int (\phi(t))$ will be bounded by its minimum value $f_{\min}$ and its maximum value $f_{\max}$.

Using the idea of "sector nonlinearity", the nonlinear function $\int (\phi(t))$ can be represented by

$$\int (\phi(t)) = M_1(\xi(t)) f_{\max} + M_2(\xi(t)) f_{\min}$$

(9)
III. T-S FUZZY MODEL OF AMB

\[ M_1(\xi(t)) = \frac{f(\phi(t)) - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}} \]  \hspace{1cm} (10)

\[ M_2(\xi(t)) = \frac{f_{\text{max}} - f(\phi(t))}{f_{\text{max}} - f_{\text{min}}} \]  \hspace{1cm} (11)

Then, under the assumption on bounds of the generalized control flux \( \phi(t) \subset [\phi_{\text{min}}, \phi_{\text{max}}] \), we can exactly represent the nonlinear time varying AMB model with the T-S fuzzy model as

\[ \dot{x}(t) = \sum_{i=1}^{2} h_i(\xi(t)) A_i x(t) + B \tilde{u}(t) \]  \hspace{1cm} (12)

where \( A_1 \) and \( A_2 \) are matrices that are obtained by replacing \( \int(\phi(t)) \) in matrix \( A(t) \) with \( f_{\text{max}} \) and \( f_{\text{min}} \), respectively.
Since the generalized control flux $\phi(t)(x_3(t))$ can be measured or estimated, the premise variable $\xi(t)$ can be obtained, and the T-S fuzzy model can be constructed.

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\xi(t))(A_i + \Delta A_i)x(t) + B\bar{u}(t)$$

(13)

Where $\Delta A_i = I$ and $F_i$ represents the parameter uncertainties, $I$ and $E_i$ are known constant matrices with appropriate dimensions, and $F$ is an unknown matrix function bounded by $F^TF \leq I$. 
Our goal in this paper is to design a robust fuzzy feedback control law based on the so-called parallel-distributed compensation (PDC) scheme as

$$u(t) = \sum_{i=1}^{2} h_i (\xi(t))K_i x(t) = K_h x(t)$$

(13)

Where $K_h = \sum_{i=1}^{2} h_i (\xi(t))K_i$, $K_i$ is the state-feedback gain matrix to be designed, such that the equilibrium of the T-S fuzzy system
To design the controller, the following lemma will be used.

**Lemma 2**: For any matrices $X$ and $Y$ with appropriate dimensions, we have

$$X^TY + Y^TX \leq c X^TX + c^{-1} Y^TY$$

Where $c > 0$ is any scalar, T-S fuzzy system with controller is globally asymptotically stable with a decay rate if there exist matrices $Y_i$, $i=1,2$, scalars $c_1 > 0$ and $c_2 > 0$. 
V. NUMERICAL EXAMPLE

Similar to the work in this section, we validate the previous theoretical results through numerical simulations on both the AMB model, which is used for the control-law design, and the same high-fidelity model of 1-DOF magnetic-levitation system. The basic bearing parameter values are listed in table I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>$N$</th>
<th>$m$</th>
<th>$A_g$</th>
<th>$v_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>number of turns in coil</td>
<td>effective mass rotor</td>
<td>electromagnet pole area</td>
<td>maximum voltage</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>#</td>
<td>kg</td>
<td>mm</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>321</td>
<td>4.5</td>
<td>137</td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>
V. NUMERICAL EXAMPLE

- In this paper, the control input is limited as $u_{\text{min}} = 10\text{V}$, and the generalized control flux is limited to $200 \mu \text{Wb}$ such that the bounds of the nonlinear function is given as $f_{\text{min}} = 0$ and $f_{\text{max}} = 200 \phi_1(0) = 10 \mu \text{Wb}$ and $\phi_2(0) = 50 \mu \text{Wb}$

- The simulation program is realized by MATLAB/Simulink, and the simulation results for the system states $z(t), \dot{z}(t)$, and $\phi(t)$ and for the control voltage are shown in Fig. 2

- Where the simulation results of the “ideal” AMB model are compared with the “actual” (high fidelity) AMB model
V. NUMERICAL EXAMPLE

Fig. 2. Comparison of state trajectories and control voltages for ‘ideal’ and ‘actual’ AMB models with $\phi_1(0)=10\mu Wb$ and $\phi_2(0)=50\mu Wb$
V. NUMERICAL EXAMPLE

Fig. 2. Comparison of state trajectories and control voltages for ‘ideal’ and ‘actual’ AMB models with $\phi_1(0)=10\mu\text{Wb}$ and $\phi_2(0)=50\mu\text{Wb}$
- Now, consider one case that we change the initial conditions for control flux as $\phi_1(0)= 0\mu Wb$ and $\phi_2(0)= 0\mu Wb$

In this case, the same controller as designed earlier is applied to both the “ideal” and the “actual” models.

- The simulation results are shown in Fig. 3. It is shown in Fig. 3 that the position responses for two models all converge to zero “quickly.”
V. NUMERICAL EXAMPLE

Fig. 3. Comparison of state trajectories and control voltages for ‘ideal’ and ‘actual’ AMB models with $\phi_1(0)=0\mu\text{Wb}$ and $\phi_2(0)=0\mu\text{Wb}$
V. NUMERICAL EXAMPLE

Fig. 3. Comparison of state trajectories and control voltages for ‘ideal’ and ‘actual’ AMB models with $\phi_1(0) = 0 \mu\text{Wb}$ and $\phi_2(0) = 0 \mu\text{Wb}$
VI. CONCLUSION

In this paper, we present a robust fuzzy state-feedback control strategy for a recently developed low-bias AMB model subject to control-voltage saturation. Using the idea of “sector nonlinearity,” the nonlinear uncertain AMB model is represented by a T-S fuzzy model in a defined region. By means of the PDC scheme, a fuzzy state-feedback controller is designed to stabilize the obtained T-S fuzzy model with a given decay rate. At the same time, the control-voltage constraint is involved in the controller design process.