**Autonomous Underwater Vehicle Navigation.**

We are aware that electromagnetic energy cannot propagate appreciable distances in the ocean except at very low frequencies. As a result, GPS-based and other such HF mode of navigation does not work. Acoustic energy however propagates well and hence acoustic transponders can be used as “beacons” to guide the motion of an AUV. Two types of systems have been primarily employed: Long Baseline (LBL) and Ultra Short Baseline (USBL).

The general principle behind the working of the LBL and SBL system are: An array of transponders is deployed and surveyed into the position. The vehicle sends out an acoustic signal which is then returned by each beacon as it is received. Position is determined by measuring the travel time between the vehicle and each beacon, measuring or assuming the local sound profile, and knowing the geometry of the beacon array. With this information, the relative distances between the vehicle and each array node can be calculated. There are two primary techniques for the same, which we will discuss later in the text, called FIX COMPUTATION (SPHERICAL POSITIONING) and KALMAN FILTER.
A variant is **HYPERBOLIC NAVIGATION**, in which the vehicle does not actively ping but instead listens to an array of beacons whose geometry is known. Each beacon pings in a specific sequence relative to the others at its specified frequency. By knowing which beacon pings when and the geometry of the array, the vehicle can reconstruct where it must be in space in order to hear the ping sequence as recorded. This system has the advantage of saving the vehicle the power expenditure of active pinging, and is especially useful for multiple AUV operations.

There are many sources of errors which make the AUV navigation. Some prominent are listed below.
- Errors in the assumed array geometry.
- Errors in the assumed sound profile.
- Reflection or multipath errors will result in incorrect time-of-flight values and hence erroneous position fixes.

We would be looking into some of the sources of error and how to get over them.

**TRANSPONDER / BEACON GEOMETRICS – ACCURACY ANALYSIS.**

The geometry of the transponders/ beacons with respect to the pinger affects tremendously the achievable performance of the positioning system. It is important to be able to assess a priori the performance of a given configuration, for instance as way to help in the design of favorable geometrics. An analytical lower bound on the accuracy of the position estimates can be obtained through the **CRAMER-RAO BOUND (CRB)**. Consider the set-up where the sender (vehicle) is synchronized with the receivers, and its depth \( z \) is assumed to be known. Let \( \epsilon = [ x' \ y']^T = [ x-x'' \ y-y'']^T \) belongs to \( \mathbb{R}^2 \) denote the vector of (horizontal) position estimation errors that are expected to be obtained with an unbiased estimator. For the problem at hand, the CRB states that the variance of the estimation error satisfies
where \( \text{tr}() \) is the matrix trace operator. The CRB for different configurations is illustrated in figures below. Interestingly, these ideas can be extended to another class of problems in which the attitude of the underwater vehicle needs to be estimated.

CRAMER-RAO BOUND (CRB) for different transponder/beacon geometries. The lower bound on the variance \( \text{tr}() \) factor is coded in gray scale. In all the cases, the covariance was set to \( R = \sigma I_m \) where \( m \) is the number of transponders/beacons and \( \sigma = 1 \) m. The hydrophones depth were set to \( z = 0 \) for all the transponders/beacons and the target depth was \( z = -50 \) m.

**ACOUSTIC NAVIGATION.**

Acoustic navigation of AUVs can be very challenging as it’s requires autonomous processing of travel time measurements affected by noise, drop outs and outliers. The predominant source of spurious measurements is the presence of multiple acoustic propagation paths between source and receiver. This phenomenon of multipath is the result of reflections off the ocean surface and/or bottom and
refraction of sound waves due to changes in sound speed with water depth. The result is that a single ping by an acoustic beacon can be detected by the AUV as a complex sequence of arrivals. The direct path is masked by destructive interference, resulting in a spurious travel time value. Hence, an approach has been pursued that uses a Kalman filter to integrate dead reckoning with acoustic range measurements to an array of transponders pre-deployed in the operating environment. Initialization and outlier rejection strategies, essential for the Kalman filtering to be successful in the presence of spurious measurements, are an important part of the algorithm.

The operation of the SBL system is as follows: In autonomous mode, the vehicle pings at a regular rate at the master frequency. Each acoustic beacon replies at its own frequency. If the speed of sound is known, then the elapsed time between the initial ping and detection of the reply yields the range between the vehicle and the beacon. The navigation problem is then to determine the vehicle position, given round-trip travel times measured from the vehicle to each beacon and the location of each beacon as determined in calibration (through an additional acoustic transponder that is hung from a surface ship and interrogates the array from various locations.)

Navigation calculations: Two approaches are possible, fix computation and filtering. In both cases, the vehicle dead reckons its position until the SBL data are available.

( The local earth frame origin is at the surface. The x-axis points north, the y-axis points east, and the z-axis points down. The vehicle position in this frame is \((x; y; z)\) and the coordinates of beacon \(i\) are \((x_i; y_i; z_i)\). The measured round-trip travel times between the vehicle and the beacons are \(t_i\) and the associated distances are \(d_i\). The distances are computed from the travel times by the approximate relation: \(d_i = c \frac{(t_i - \_i)}{2}\), where \(c\) is the average speed of sound1 and \(_i\) is the turnaround time of beacon \(i\) (typically 15 ms).

**FIX COMPUTATION:** For three beacons, it consists of computing the depth constrained analytical solution corresponding to the intersection of three spheres centered at the beacon locations with radii equal to the measured distances. Though we would be using the filtering approach in our algorithm, it is needed for initialization of the filter.

The analytical solution based on measurements consists of solving the following non-linear system if equations for vehicle co-ordinates. (In order to make the computation
easier, we first compute the solution in an intermediate frame obtained by shifting
the local earth frame at the location of beacon 1 and advantage is also taken of the
accurate knowledge of the vehicle depth)

\[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = d_i^2, \quad i = 1, 2, 3. \quad (1)\]

\[
x = \frac{b_1 c_1 - b_2 c_1}{a_1 b_2 - a_2 b_1} + x_1
\]

\[
y = \frac{a_2 c_1 - c_2 a_1}{a_1 b_2 - a_2 b_1} + y_1 \quad (2)
\]

With,

\[
a_1 = -2x_2'
\]

\[
b_1 = -2y_2'
\]

\[
c_1 = x_2'^2 + y_2'^2 + z_2'^2 - 2z'z_2' - d_2^2 + d_1^2
\]

\[
a_2 = -2x_3'
\]

\[
b_2 = -2y_3'
\]

\[
c_2 = x_3'^2 + y_3'^2 + z_3'^2 - 2z'z_3' - d_3^2 + d_1^2. \quad (3)
\]

Where the dashed co-ordinates denote the beacon co-ordinates in the frame of
beacon 1.

When measurements from four beacons are available one could do a least squares
minimization.

**KALMAN FILTER**: In this approach, the travel times and depth measurements are
used as they arrive to correct the predicted position. This method does not explicitly
compute a fix, but provides a position estimate which is a weighted combination of
dead reckoning measurements and absolute measurements. This leads to a
smoother track since the effect of a given set of travel times is only used to a certain
degree to correct the predicted position instead of completely resetting it.

The state and observation models used in the filter are given below. The state
equation correspond to dead reckoning, while the travel times and depth measurements are the measurements for the observation model. The position prediction stage of the filter is based on classical dead reckoning:

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}_{k+1} = \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}_k + A_v(\psi_k, \theta_k, \phi_k) V_k \Delta t + v_k
\]

(4)

\(A_v\) is the rotation matrix from the local earth frame \(R_0\) to the vehicle frame \(R_v\). It is the function of yaw, pitch and roll angles provided by the attitude and heading system. The number of components of the speed vector depends on the type of speed sensor available on board.

Depth measurements are readily obtained from a pressure sensor mounted on the vehicle, and hence the measurement equation for depth is simply \(D = z + wD\). The returns from each transponder arrive asynchronously, depending on their position relative to the vehicle. The travel times are processed one by one as soon as they arrive, whereas a depth measurement is available during each control cycle. There is no need to wait for a set of 3 or more travel times as it is the case for fix computation. Unlike fix computation, the observation equation for travel times is expressed as a function of the position at the ping and at the reception of the return:

\[
t_i = \frac{1}{c} \sqrt{(x_{ping} - x_i)^2 + (y_{ping} - y_i)^2 + (z_{ping} - z_i)^2} \\
+ \frac{1}{c} \sqrt{(x_{recept} - x_i)^2 + (y_{recept} - y_i)^2 + (z_{recept} - z_i)^2}
\]

(5)

This shows that the geometric constraint is not really a sphere but an ellipse with foci at the ping and reception locations. The vehicle position at the ping and at the reception can be expressed as a function of the current position by dead reckoning the vehicle displacement between the ping and the current position and the reception time and the current position.
OUTLIER REJECTION.

Two approaches for outlier rejection: In the time domain (travel time rejection) and in the spatial domain (fix rejection). Since erroneous travel times can be rejected individually, temporal outlier rejection is desirable. However, to reject data in the time domain, there must be an accurate a priori initial position estimate from which predicted arrival times can be generated for each of the beacons. Until the filter is initiated, such a dead-reckoned position estimate is not available, and so for initialization outlier rejection must occur in the spatial domain.

**Spatial Domain (During Initialization)**: The presence of outliers in the travel time measurements makes the initialization of the algorithm difficult. Although the vehicle initial position is roughly known before the first SBL returns are detected, it is necessary to obtain a valid and accurate initial fix on which rejection of following SBL data will be based.

Some methods to get over this are:

A). When 3 or more beacons are used, the beacons are used, the validity of measurements could be checked by thresholding the residual error at the estimated position:

\[
\frac{1}{2} \sum_{i=1}^{n} \left( d_i - \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \right)^2 < \tau
\]  

(6)

The choice of the threshold value depends on the accuracy with which beacon coordinates are known and on the noise on the measured distances.

B). Compute fixes with subsets of available travel times during an SBL cycle and look for consistency in them. If four beacons are available, then four 3-beacon fixes which can be produced are less subject to errors due to baseline miscalibration than the three 2-beacon fixes associated with the three-beacon navigation.

Initialization can then be obtained by first determining the direction of motion of the vehicle and testing the following fixes for consistency with the estimated direction. Assuming linear initial motion for the vehicle, the direction of motion can be determined by a median fit rather than a least square fit. This option gives less weight to outliers than the least square fit does.
The initialization procedure then goes as follows: fixes are computed if three travel times are available during an SBL cycle. If four travel times are available, the three beacon fix with the least residual is used. A line is fitted to the N most recent fixes by minimizing the absolute deviation of the fixes with respect to the line. When a new fix is obtained, two thresholds are calculated: threshold $T_1$ is the distance traveled by the vehicle since the line parameters were last computed, based on the vehicle speed. Threshold $T_2$ is the maximum authorized lateral deviation from the line for the new fix, and is defined by (7). For a fix to be considered the initial position, it is necessary that two fixes lie closer to the line than $T_2$ and be separated by less than $T_1$. $T_2$ then checks alignment consistency perpendicularly to the direction of motion, whereas $T_1$ checks consistency of successive fixes along the line. The validated fix is used to initialize the filter state vector. An estimate of the variance of the initial fix is derived by differentiating Equation (2) with respect to the measured distances, based on an assumed variance $\sigma_t$ for valid travel times. This initial variance is used to initialize the position error covariance matrix. PK

$$T_2 = \beta + T_1 \tan(\alpha). \quad (\alpha = 1 \text{ degree and } \beta = 5) \quad (7)$$

**Time Domain (During Filtering):** In the case of initialization, waiting several SBL cycles before being able to declare a fix as a valid initial fix was not too restrictive. It is however no longer the case now as the vehicle sets in motion. Once an initial accurate fix is obtained, the short term accuracy of dead reckoning measurements allow one to predict more or less where the vehicle should be. Rejection of outliers after initialization should take advantage of this.

Certain methods transform the position uncertainty (due to dead reckoning from the initial position) to the travel time domain, defining validation regions for each of the times of flight. A KALMAN FILTER such as the one described above directly performs this transformation.

When the vehicle dead reckons, its position is predicted using the state equation (4). The associated error covariance is also predicted by:

$$P_{k+1/k} = P_{k/k} + J_k C J_k^T + Q_k \quad (8)$$
where $C$ is a diagonal matrix containing the covariance of the speed components and orientation angles, $J$ is the Jacobian Matrix of the state vector w.r.t. the speed and orientation and $Q$ is the covariance matrix of the state noise.

After propagation of the position error, the uncertainty in position is transformed into uncertainty in the predicted travel time using the observation equation. Outliers are then gated using the well-known Mahalanobis distance. When a new travel time arrives, its variance is also calculated. The new time is validated or discarded by computing its normalized innovation squared and comparing it with a threshold. :

$$
n_k^t S_k^{-1} n_k < \gamma
$$

$$
n_k = t_k - \hat{t}_k
$$

$$
S_k = H_k P_{k+1/k} H_k^t + \sigma_t^2
$$

where $H$ is the travel time Jacobian. A validated measurement is then used in the estimation step of the Kalman Filter to correct the current predicted position.

Kalman Filtering and Time domain Outlier Rejection results in a smoother track. One source of error in is that the vehicle motion between the ping and the reception of the returns is not taken into account. If a speed sensor is put up, even this can be removed. Else an assumption of speed will introduce new errors into the Kalman Filter. These errors are greatest in turns and in ascent/descent. A more appropriate approach would have to postpone decisions concerning the validity of measurements until more data is acquired by setting up a tree of data interpretation hypotheses, but their computational cost is very high.