Direct Current Motors
**Introduction and Working Principle**

A dc motor is used to convert the dc electrical power into mechanical power. These motors are used in Airplanes, Computers, robots, toys and mining industry, etc.

**Explanation**

Connect the armature terminal of a 2-pole dc motor to a dc voltage source. As a result, dc current flows through the armature which creates the alternate magnetic poles. Armature conductors under N pole are assumed to carry current upward (dots) and those under S-poles to carry current downwards.

Its operation is based on electromagnetic force. This force produce a powerful torque. Its magnitude is given by, \[ F = Bl\ell \]

The direction of the current will be reversed when the conductor passes from one pole to another pole. This process helps to develop a continuous and unidirectional torque of the motor.
Back or Counter EMF

When the armature rotates, conductor also rotates in the magnetic field and these conductors cuts by the magnetic flux. As a result, an emf will be induced in the conductors. The direction of the voltage is determined by FLHR. This induced voltage opposes the applied voltage and is known as back emf ($E_b$) or counter emf of the motor.

Shunt motor is used to explain the ($E_b$).

Net voltage across the armature circuit,

$$V_{net} = V_t - E_b$$

The current through armature circuit is,

$$I_a = \frac{(V_t - E_b)}{R_a}$$
**Importance of Back EMF**

The $E_b$ is used to control the additional load in a dc motor. The current through armature circuit is,

$$I_a = \frac{(V_t - E_b)}{R_a}$$

At the time of starting, armature is not rotating as a result $E_b$ is zero. For a small magnitude of the armature circuit resistance the current in the armature is extremely high,

$$I_a = \frac{(V_t - 0)}{R_a} \uparrow$$

The $I_a$ is $25 >$ full-load current of the motor. This high magnitude of $I_a \rightarrow$ increases $(F)$, which produces powerful torque $(T)$ which increases the $(N)$ of the motor. This increase in $(N)$ increases the $E_b$ of the motor. The $V_{\text{net}}$ across the armature decreases. As a result $I_a$ is also decreases, which decreases the torque $(T)$. As a result of this reduced torque, the motor will stop. Therefore, the back emf in a dc motor regulates the flow of the $I_a$ depending on the additional connected load.
Classification of DC Motor

Shunt Motor
The field winding is connected in parallel with the armature circuit.

\[ V_t = E_b + I_a R_a, \quad I_L = I_a + I_{sh} \]

Series Motor
In this field winding is connected in series with the armature circuit,

\[ I_a = I_L; \quad V_t = E_b + I_a (R_{se} + R_a); \]
Compound DC Motor

**Short shunt**

In short shunt compound $R_{se}$ is connected in parallel with the armature.

The voltage across the armature circuit is

$$E_b = V_t - I_L R_{se}$$

Current,

$$I_{sh} = \frac{E_b}{R_{sh}} \quad \& \quad I_L = I_a + I_{sh}$$

**Long shunt**

In long shunt compound motor $R_{se}$ is connected in series with the armature.

$$E_b = V_t + I_L R_{se}$$

Current,

$$I_{sh} = \frac{V_t}{R_{sh}} \quad \& \quad I_L = I_a + I_{sh}$$
**Mechanical Power of a DC Motor**

To find the mechanical power, the circuit of shunt motor is used.

\[ V_t = E_b + I_a R_a, \]

The power equation is, \[ V_t I_a = E_b I_a + I_a^2 R_a \]

\( V_t I_a \) = electric power supplied to the armature

\( I_a^2 R_a \) = armature cu losses

\( E_b I_a = P_{md} \) mechanical power developed by the armature

Since the \( R_a \) and \( V_t \) is constant. Therefore, to get the maximum power, differentiate the power equation w.r.t \( I_a \).

\( P_{md} \) developed by the armature will be maximum, when the \( E_b \) is equal to the half of the applied voltage.

\[ E_b = V_t / 2 \]
**Torque of DC Motor**

The tendency to produce the rotation of the conductor is known as torque. \[ T = F \times r \]

**Armature Torque**

Torque produces by the all the conductors of the armature denoted by \( T_a \).

Mechanical power developed by the armature is, \[ P_{md} = T_a \times \omega \]

Electrical power converted into mechanical power is, \[ P_{md} = E_b \times I_a \]

Expression of armature torque is, \[ T_a = 9.55 \left( \frac{E_b I_a}{N} \right) \]

**Shaft Torque**

Torque \( T_{sh} \), which is available at the shaft of the motor for work. If the motor runs at a speed of \( N \) rps. The motor O/P power \( P_o \) is \[ P_o = T_{sh}(\omega) \rightarrow T_{sh} \left( \frac{2\pi N}{60} \right) \rightarrow T_{sh} = 9.55 \frac{P_o}{N} \text{ (N-m)} \]
**Practice Problem**

A shunt motor takes a current of 40A from 230V supply and runs at a speed of 1100rpm. Find the torque developed by armature, if the armature and the shunt field resistances are 0.25Ω and 230Ω respectively.

A 6-pole shunt motor is energized by 230V dc supply. The motor has 450 conductors that are wound in lap configuration. It takes 30A current from the supply system and develops output power of 5560W. The current through the field windings is 3A and the flux per pole is 25mWb. The armature circuit resistance is 0.8Ω. Find the speed and the shaft torque.

A shunt motor takes a current of 45A from 220V do supply and runs at a speed of 1500rpm. Find the torque developed by the armature, if the armature and the shunt field resistances are 0.5 Ω and 65 Ω respectively.

A 6-pole shunt motor is energized by 230V dc supply. The motor has 500 conductors that are connected in lap wound. It takes 40A from the supply system and develops output power of 5.9kW. The current through the field windings is 4A and the flux per pole is 0.04Wb. The armature circuit resistance is 0.4 Ω. Calculate the shaft torque.
Speed of a DC Motor

The speed of dc motor changes slightly with the increase in load. The equation of speed can be find using the relation \( E_b = V_t - I_a R_a \)

General relation of induced voltage is, \( E_b = (P \varphi Z N)/60A \)

The expression for speed becomes, \( N = \frac{60A(V_t - I_a R_a)}{PZ \varphi} \)

\[ N = K \frac{(V_t - I_a R_a)}{\varphi} \rightarrow N = K \frac{E_b}{\varphi}, \quad \text{where} \quad K = \frac{60A}{PZ}, \]

In general, \( N = K \frac{E_b}{\varphi}; \quad N \alpha \frac{E_b}{\varphi} \)

The speed of a dc motor is directly proportional to the back emf and inversely proportional to the flux/pole.

Let \( N_1, \varphi_1 \) and \( E_{b1} \) be the initial values and \( N_2, \varphi_2 \) and \( E_{b2} \) be the final values.

\[ N_1 \alpha \frac{E_{b1}}{\varphi_1} \quad \text{and} \quad N_2 \alpha \frac{E_{b2}}{\varphi_2} \]
Combining the equations for initial and final speed, we have

\[ \frac{N_1}{N_2} = \frac{(E_b_1 \varphi_2)}{(E_b_2 \varphi_2)} \]

The flux in the **shunt motor** is constant with the change in \( V_t \). Because the field circuit resistance \( R_{sh} \) is connected in parallel with the armature circuit. Therefore the flux remain constant, \( \varphi_1 = \varphi_2 \)

\[ \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \]

For a **series motor**, flux is directly proportional to the \( I_a \), prior to saturation, the equation for series motor becomes,

\[ \frac{N_1}{N_2} = \frac{(E_{b1} I_{a2})}{(E_{b2} I_{a1})} \]
Practice Problems

A dc shunt motor takes a current of 5A at no-load with terminal voltage 230V & runs at 1000rpm. The armature and Field resistances are 0.2Ω & 230Ω. Under load condition motor takes a current of 30A. Find the motor speed under load condition.

A dc shunt motor takes a current of 3A at no load with terminal voltage 240V & runs at 1200rpm. The armature and Field resistances are 0.25Ω & 135Ω. Under load condition motor takes a current of 45 A. Find the motor speed under load condition.

A dc series motor takes a current of 56A from a 230V supply and runs at a speed of 900rpm. The armature and Field resistances are 0.2Ω and 0.25Ω respectively. Find the speed of the motor when the line current is 15A. Consider the flux for 15A current is 40 percent of the flux for the line current of 56A.

A 220V dc series motor takes a current of 120A and runs at a speed of 1000rpm. The armature and Field resistances are 0.15Ω and 0.25Ω respectively. Find the speed of the motor when the line current is 35A. Consider the flux for 35A current is 70 percent of the flux for the line current of 120A.
**Speed Regulation**

When increases $I_L$, $I_a$ is also increases. As a result $E_b$ decreases and the speed of the motor decreases slightly. The speed regulation of a dc motor is the change in speed from no-load to full-load and it is express as,

$$\% \text{ Speed Regulation} (N_R) = \left(\frac{N_{nl}-N_{fl}}{N_{fl}}\right) \times 100$$

**Efficiency of a DC Motor**

The efficiency of motor is,

$$\eta = \frac{P_o}{P_{in}} \times 100, \quad P_{in} = P_o + \text{losses},$$

The efficiency of a dc motor will be maximum when the

**Variable losses = Constant losses**

Variable losses = armature cu losses

Constant losses = iron and friction losses + shunt cu losses
Losses in a DC Motor

A small amount of power will be lost due to $I^2R_a$ for the armature and the $I^2R_{sh}$ and rest is converted into mechanical power $E_bI_a$ that becomes available in armature.

A small amount of the developed power in armature will be lost due to rotational losses. The remainder becomes available at shaft as useful work.
**Practice Problems**

At no-load condition, a shunt motor takes 5A current from a 230V supply. The resistances of the armature & field circuit are 0.25Ω & 115Ω respectively. Under load condition the motor can carry 40A current. Calculate the iron and friction losses and efficiency.

A shunt motor takes a current of 80A from 220V supply and runs at 800rpm. The shunt field and armature resistances are 50Ω & 0.1Ω respectively. The iron & friction losses are 1600W. Determine the (i) cu losses (ii) armature torque (iii) shaft torque and (iv) efficiency.

At no load condition, a shunt motor takes 4A current from a 220V supply. The resistances of the armature & field circuit are 0.15Ω & 155Ω respectively. Under load condition the motor can carry 36A. Calculate the iron & friction losses and efficiency.

A shunt motor takes a current of 75 A from 220V supply & runs at 1100rpm. The shunt field and armature resistances are 65Ω & 0.2Ω respectively. The iron & friction losses are 900 W. Find the (i) cu losses (ii) armature torque (iii) shaft torque (iv) efficiency.
DC Motor Characteristics

The x-tics of dc motors used to identify the their performance. Three importance x-tics of dc motor are $T & I_a$, $N & I_a$ and $N & T$.

**Torque & Armature Current Characteristic**

The general torque equation of shunt motor is, $T \alpha \phi I_a$, since the field winding $R_{sh}$ is connected in parallel with the armature circuit. Therefore, the flux is constant, $T \alpha I_a$
The speed of the shunt motor can be expressed as,

$$N\alpha \frac{E_b}{\phi} \rightarrow N\alpha(V_t - I_aR_a)$$

Let the shunt motor is operated under no-load condition at rated speed. Under this stage, $\phi$ & $E_b$ are constant. Therefore, $N$ is constant with the $I_a$.

Under loaded condition, $I_a$ will increase and $E_b$ will decrease. As a result, the $N$ of the motor will be decreased slightly (see fig).
Speed & Torque Characteristic

When the load current $I_\ell$ of the motor increases, as a result, armature current $I_a$ is also increased. As a result, the increases that decreases the $E_b$ decreases. Therefore, the speed of the motor decreases with the variation of torque (see fig)
Series Motor Characteristics

Torque Vs Armature Current characteristic

The torque equation of series motor is, $T \alpha \phi I_a$. As the $\phi$ is directly proportional to $I_a$. Therefore, the final expression of the torque is,

$$T \alpha I_a^2$$

The $T$ will be increased with the square of the $I_a$ up to saturation of magnetic material. After saturation, the flux remain constant & equation of torque is, $T \alpha I_a$
The basis speed equation of the motor is, $N\alpha E_b/\varphi$

For series motor, this equation becomes $\rightarrow N\alpha[V_t - I_a(R_{se} + R_a)]/\varphi$, since $\varphi$ is proportional to $I_a$, the speed equation becomes,

$$N\alpha [V_t - I_a (R_{se}+ R_a)] / I_a$$

Under no-load condition, the value of the $I_a$ is small and under loaded condition, the value of the $I_a$ is high. Therefore, the speed of the series motor decreases with the higher value of the $I_a$. 
\( I_a \) increase with the increases the \( I_L \). As a result, the \( T \) increases and also increases the \( I_a \). This increase in \( I_a \) decreases the speed of the motor (see figure)
Comparison of Generator and Motor

Generator is driven by mechanical m/c and generates voltage. As a result, a current will flow through the electrical circuit.

The motor is connected across the supply. The armature of the motor takes a current which in turn produces a force. This force produces a driving torque. As a result, the motor rotates.

In generator, the direction of induced voltage is find by the Fleming’s LHR. In motor, the direction of induced voltage is find by the Fleming’s RHR.

In generator, the $E_g$ and the $I_a$ are in the same direction. In dc motor, these are in opposite direction.

In the generator, the relationship b/w the induced voltage, armature current and speed are considered as main x-tics. Whereas in the motor, speed, torque & current are considered.
Applications of DC Motor

Shunt Motor

The characteristics of shunt motor is an approximately constant speed. It has medium starting torque. This motor is used in the lathe machine, drill machine, shaper machine, blower and fan, etc.

Series Motor

Is a variable speed motor and it has high starting torque. It is motor is used in the trolley car, crane, electric traction, elevators, air-compressors, vacuum cleaners, hair drier and conveyor machines etc.

Compound Motor

The differential compound motor has poor torque characteristics. As a result, this motor is rarely used. However, cumulative-compound motor has high starting torque and it is used in the printing presses, ice machines, air compressors and rolling mils etc.