SUPERGEN MARINE 7th DOCTORAL TRAINING PROGRAMME WORKSHOP

Control of Wave and Tidal Converters

22-26 February 2010, Lancaster University

CONTROL TECHNIQUES FOR WAVE ENERGY CONVERTERS

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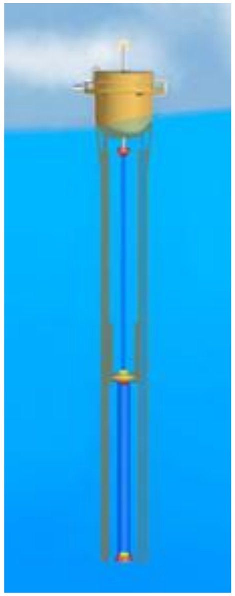
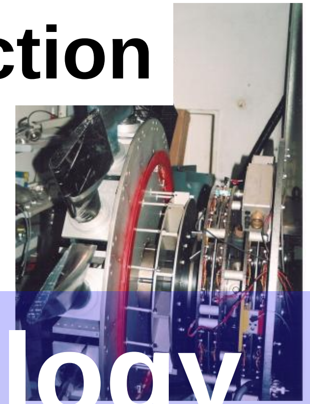
How far have we gone in 30+ yrs ? Some milestones:

Introduction





Technology challenge

THE DIVERSITY

Introduction

RESONANCE CONTROL

- **Oscillating Water Column** (with air turbine)
  - **Fixed structure**
    - **Isolated:** Pico, LIMPET, Oceanlinx
    - **In breakwater:** Sakata, Mutriku
  - **Floating:** Mighty Whale, BBDB
- **Oscillating body** (hydraulic motor, hydraulic turbine, linear electric generator)
  - **Floating**
    - **Heaving:** Aquabuoy, IPS Buoy, Wavebob, PowerBuoy, FO3
    - **Pitching:** Pelamis, PS Frog, Searev
  - **Submerged**
    - **Heaving:** AWS
    - **Bottom-hinged:** Oyster, Waveroller

RUN UP

- **Overtopping** (low head water turbine)
  - **Fixed structure**
    - **Shoreline (with concentration):** TAPCHAN
    - **In breakwater (without concentration):** SSG
  - **Floating structure (with concentration):** Wave Dragon

Introduction

The size

While, in other renewables, the power is more or less proportional to size/area, …

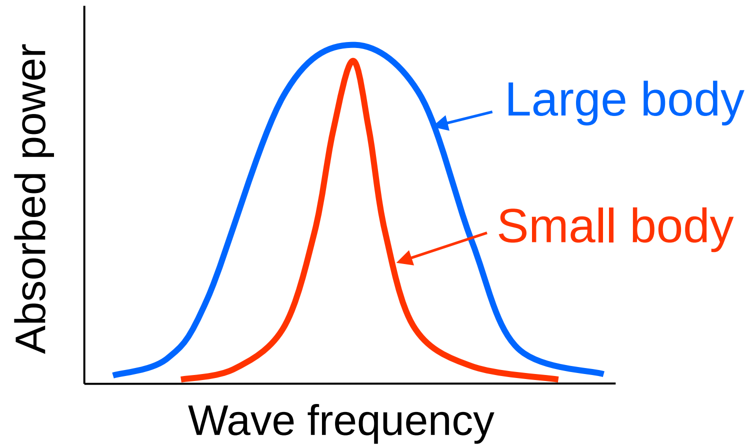
… the power-versus-size relationship is much more complex for wave energy converters.

The concept of "point absorber" was introduced in Scandinavia around 1980 to describe efficient wave-energy absorption by well-tuned small devices.

Theoretically (in linear wave theory), energy from a regular wave of given frequency can be absorbed by a large oscillating body as well as from a small one, provided both are tuned.

The oscillation amplitude is larger for the smaller body.

Introduction

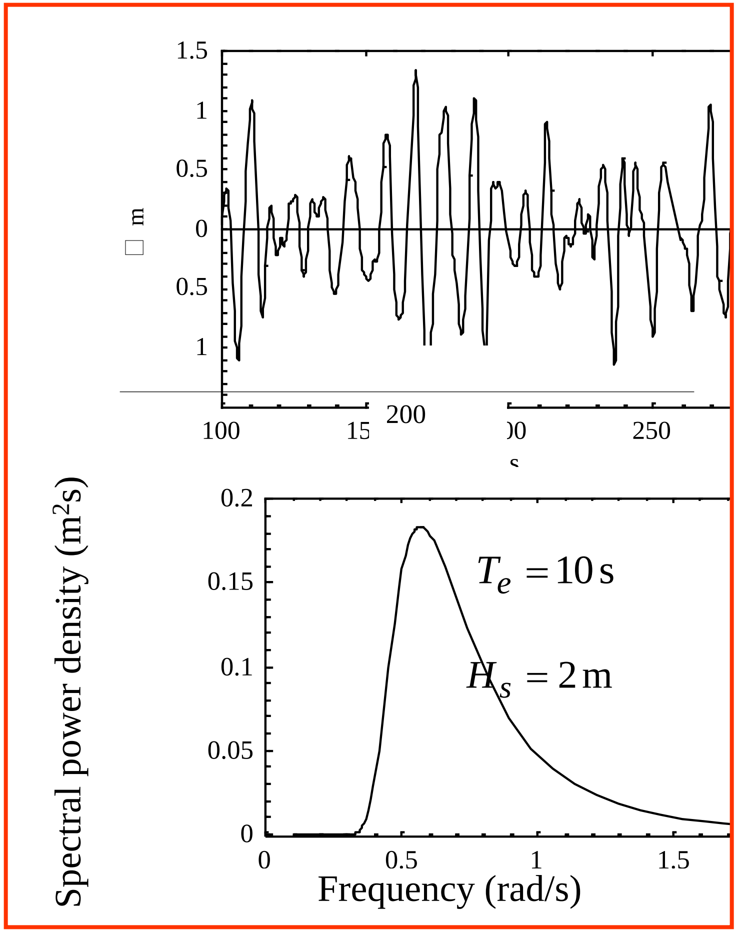


Wave energy absorption is wider-banded for a large body than for a "point-absorber".

This is relevant for **real polychromatic multi-frequency waves**.

Here smaller oscillating-bodies are less efficient than larger ones.

This can be (partially) overcome by **control (phase control)**.

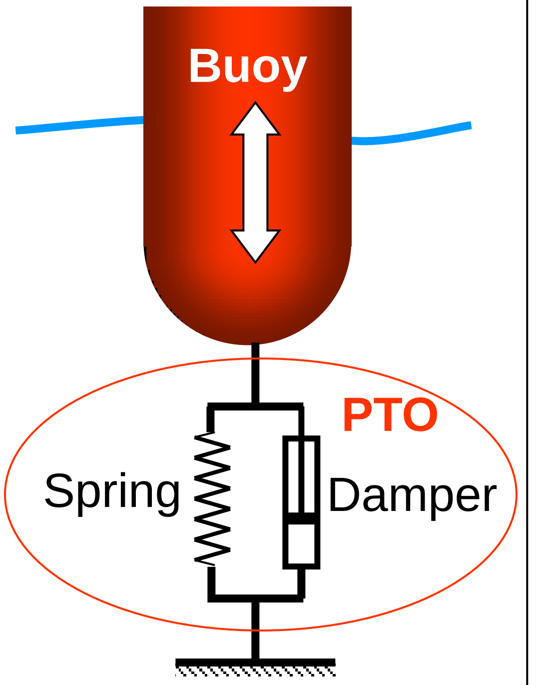


Oscillating-body dynamics

Most wave energy converters are complex (possibly multi-body) mechanical systems with several degrees of freedom.

We consider first the simplest case:

- **A single floating body.**
- **One degree of freedom: oscillation in heave (vertical oscilation).**



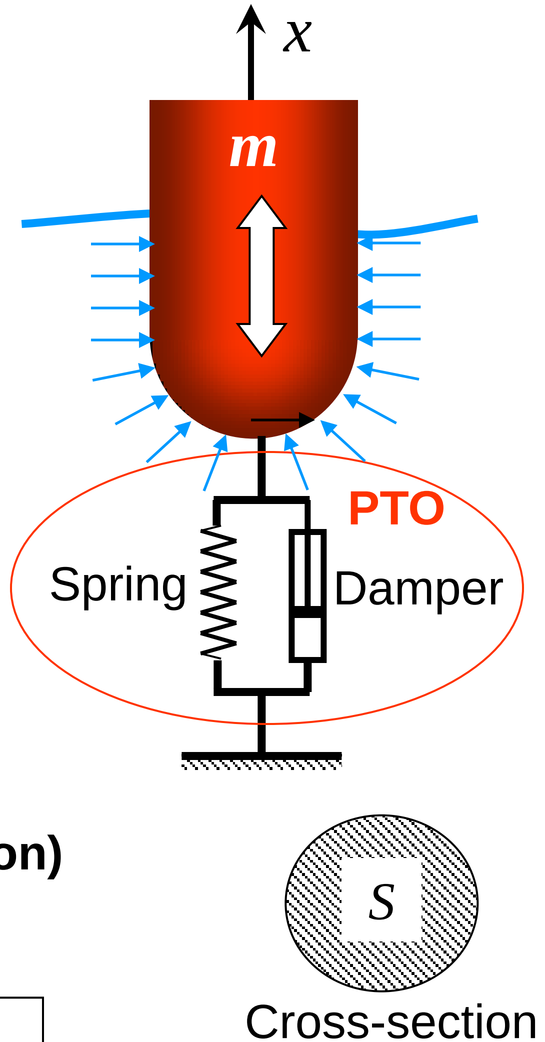
Oscillating-body dynamics

Basic equation (Newton):

$$m\ddot{x} = f_h(t) + f_m(t)$$

↑ **on wetted surface** (f_h) ↑ **PTO** (f_m)

$$f_h = \begin{cases} f_d = \text{excitation force (incident wave)} \\ f_r = \text{radiation force (body motion)} \\ f_{hs} = -\rho g S x = \text{hydrostatic force (body position)} \end{cases}$$



$$\boxed{m\ddot{x} = f_d + f_r - \rho g S x + f_m}$$

Oscillating-body dynamics

Frequency-domain analysis

- Sinusoidal monochromatic waves
- Linear system

Oscillating-body dynamics

$$m\ddot{x} = f_d + f_r - \rho g S x + f_m$$

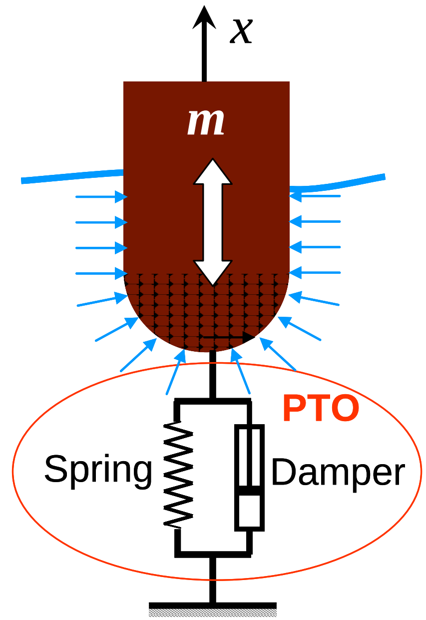
$$f_r = -A\ddot{x} - B\dot{x}$$

$B =$ radiation damping

$A =$ added mass

$$f_m = -Kx - C\dot{x}$$

Linear spring — Linear damper



A **and** ***B*** **to be computed (commercial codes WAMIT, AQUADYN, ...) for given** $\boldsymbol{\omega}$ **and body geometry.**

$$(m + A)\ddot{x} + (B + C)\dot{x} + (\rho g S + K)x = f_d$$

mass — added mass — radiation damping — PTO damping — buoyancy — PTO spring — Excitation force

Oscillating-body dynamics

$$(m+A)\ddot{x}+(B+C)\dot{x}+(\rho g S+K)x = f_d$$

Method of solution: $(e^{i\omega t} = \cos\omega t + i\sin\omega t)$

- **Regular waves**
- **Linear system**

$\Rightarrow$ $x(t) = \mathrm{Re}\{X_0 e^{i\omega t}\}, \quad f_d = \mathrm{Re}\{F_d e^{i\omega t}\}$

or simply $x(t) = X_0 e^{i\omega t}, \quad f_d = F_d e^{i\omega t}$

Note : X_0, F_d are in general complex amplitudes

$$\frac{|F_d|}{\text{wave amplitude}} = \Gamma(\omega) \Rightarrow \text{ to be computed for given } \omega \text{ and body geometry}$$

$$X_0 = \frac{F_d}{-\omega^2(m+A)+i\omega(B+C)+\rho g S+K}$$

Oscillating-body dynamics

$$X_0 = \frac{F_d}{-\omega^2(m+A) + i\omega(B+C) + \rho g S + K}$$

Power = force × velocity

Time-averaged power absorbed from the waves :

$$\overline{P} = \frac{1}{8B}|F_d|^2 - \frac{B}{2}\left|i\omega X_0 - \frac{F_d}{2B}\right|^2$$

= 0

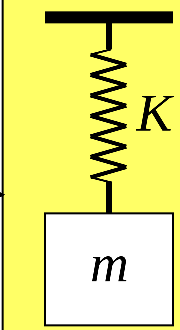
Note: for given body and given wave amplitude and frequency ω, B and F_d are fixed.

Then, the absorbed power $\overline{P}$ will be maximum when :

$$i\omega X_0 = \frac{F_d}{2B} \quad \begin{cases} \omega = \sqrt{\dfrac{\rho g S + K}{m + A}} \\ B = C \end{cases}$$

Resonance condition

Radiation damping = PTO damping



$$\omega = \sqrt{\frac{K}{m}}$$

Oscillating-body dynamics

Capture width L : measures the power absorbing capability of device (like power coefficient of wind turbines)

$$L = \frac{\overline{P}}{E} \quad \begin{cases} \overline{P} = \text{absorbed power} \\ E = \text{energy flux of incident wave per unit crest length} \end{cases}$$

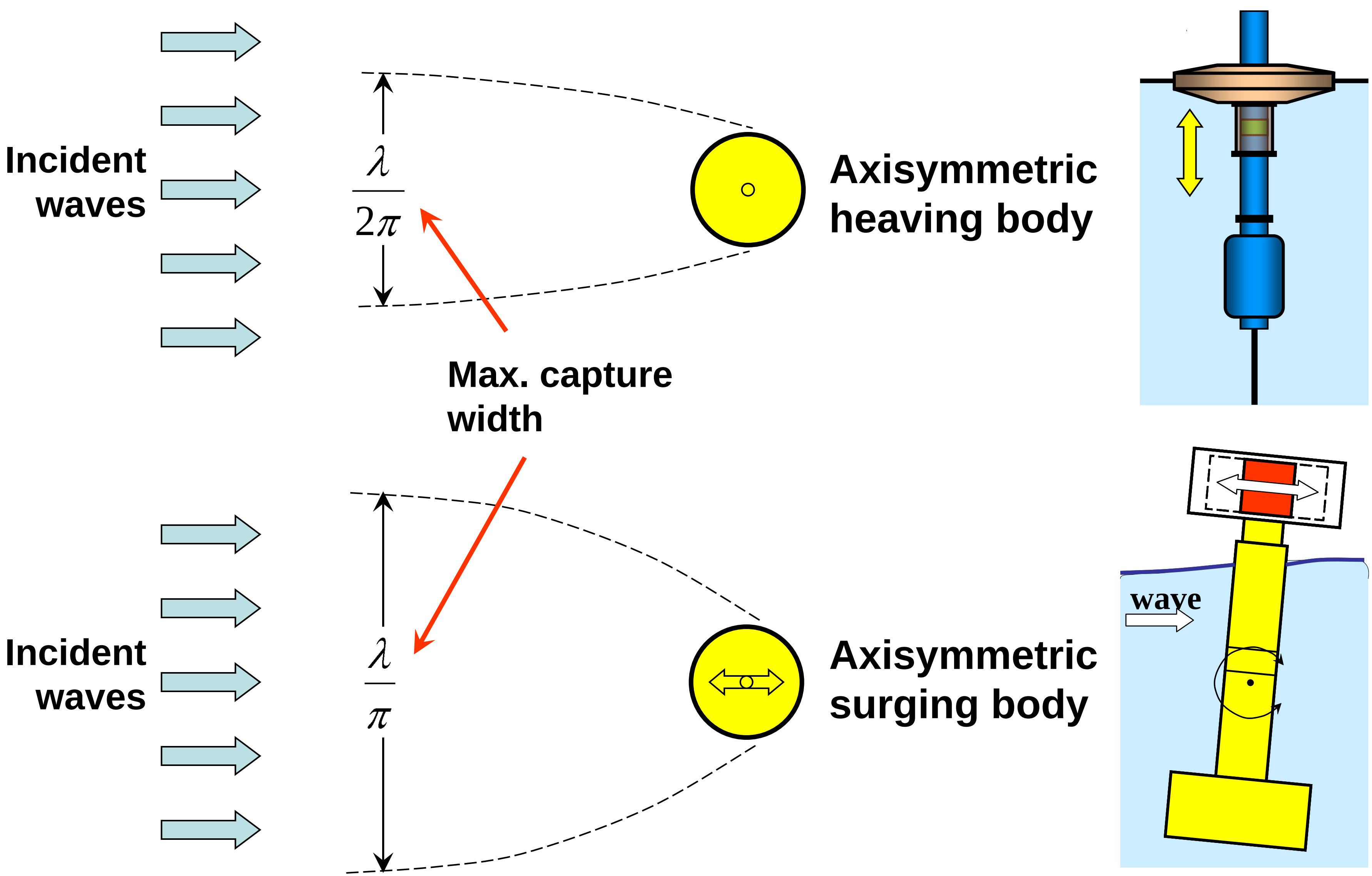
For an axisymmetric body oscillating in heave (vertical oscillations), it can be shown (1976) that

$$\overline{P}_{\max} = \frac{E\lambda}{2\pi} \quad \text{or} \quad L_{\max} = \frac{\lambda}{2\pi}$$

Note: $L_{\max}$ may be larger than width of body

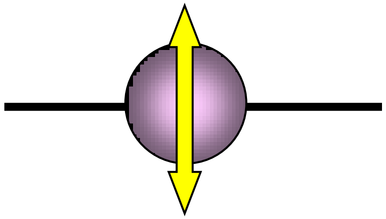
For wind turbines, Betz's limit is $C_P = 0.593$

Oscillating-body dynamics



Oscillating-body dynamics

Example: hemi-spherical heaving buoy of radius a



No spring, no reactive control, $K = 0$

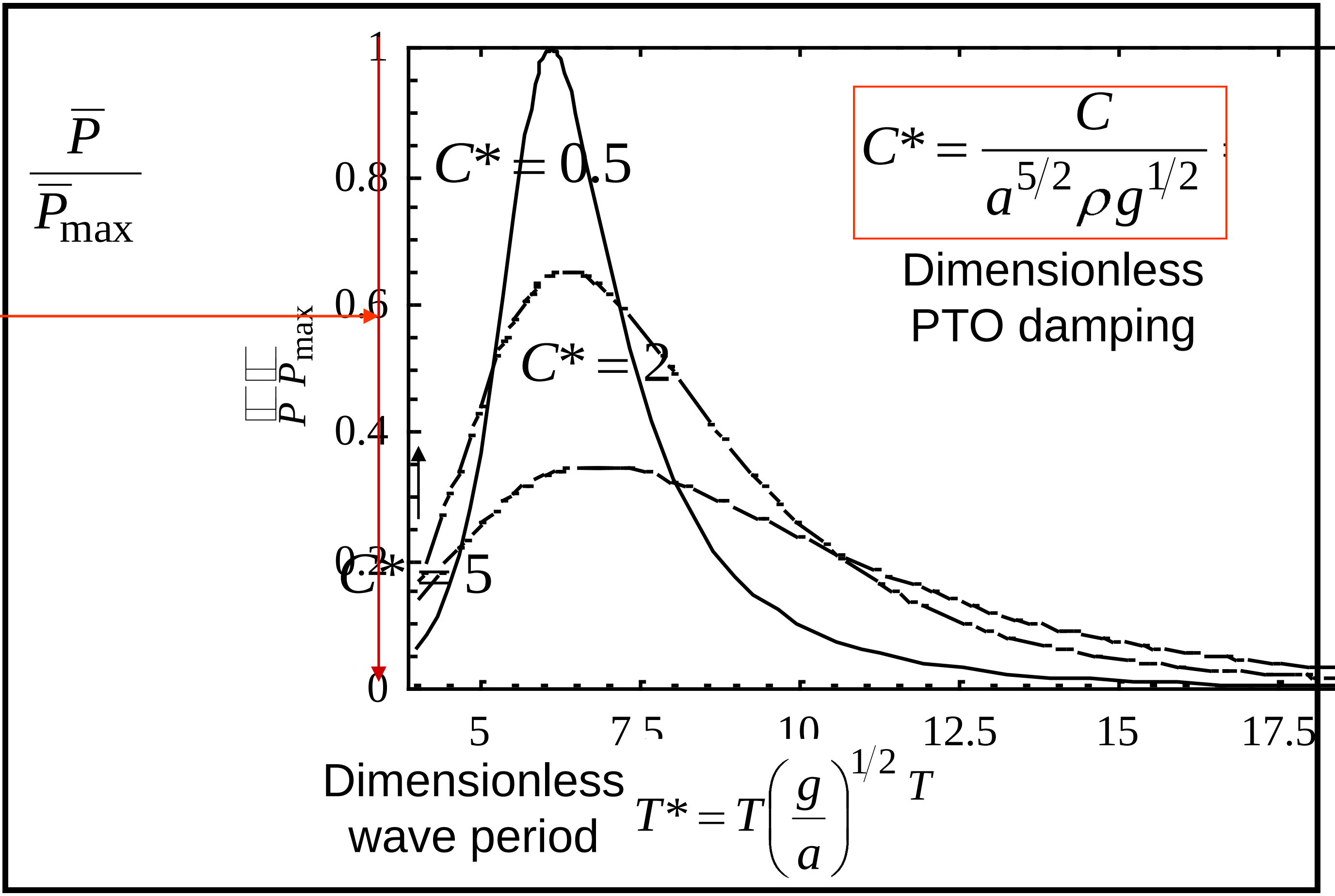
$\overline{P} = P_{max}$ for

$$T^* = T\left(\frac{g}{a}\right)^{1/2} = 6$$

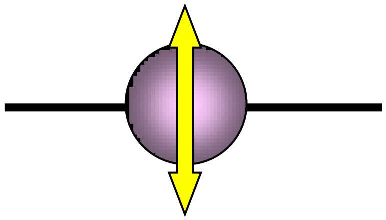
If $T = 9$ s

$$a_{opt} = 22\,\mathrm{m}$$

Too large !

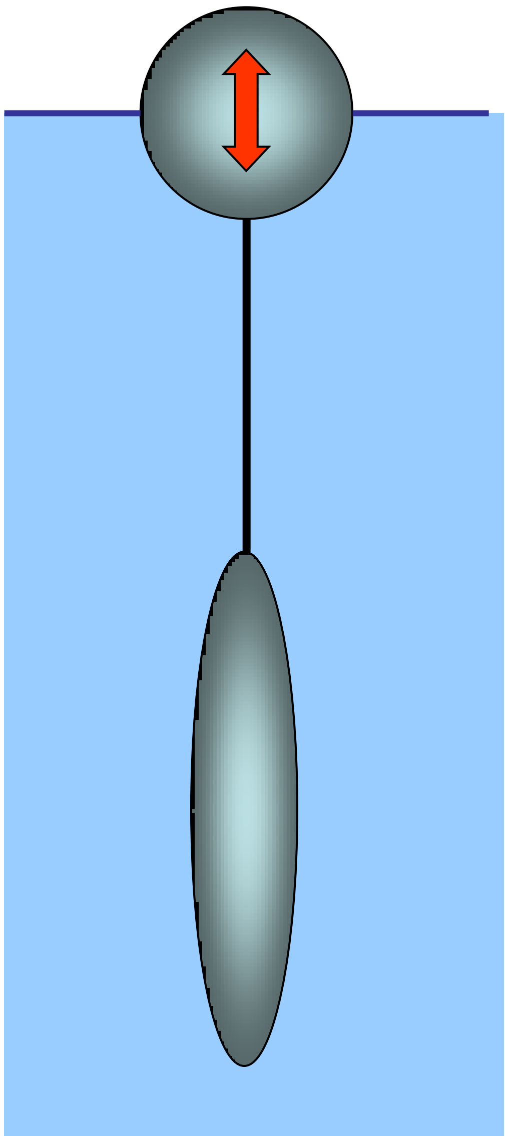


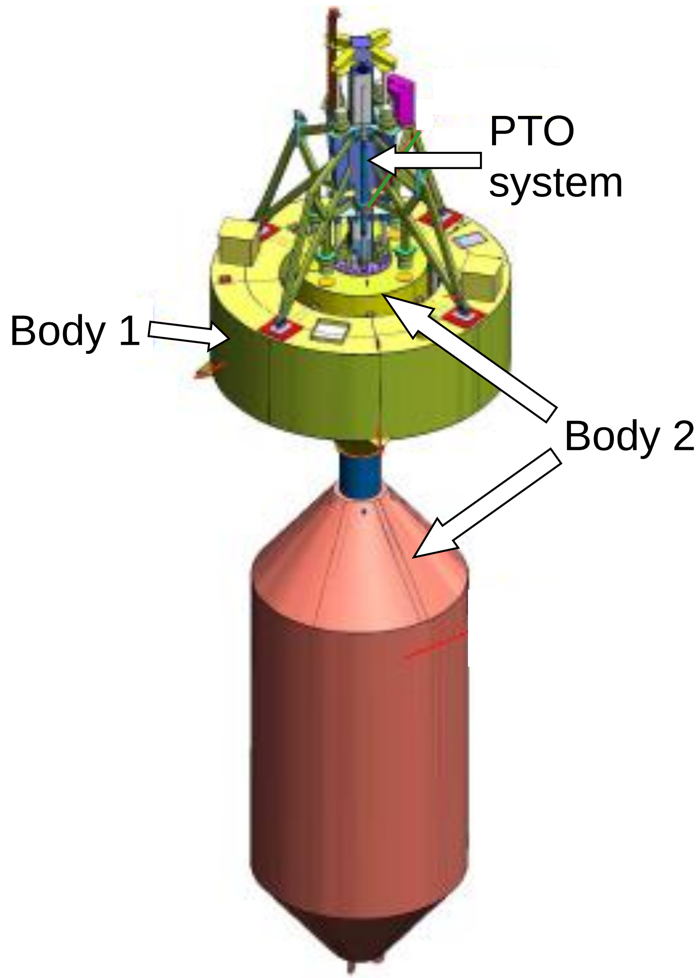
Oscillating-body dynamics



How to decrease the resonance frequency of a given floater, without affecting the excitation and radiation forces ?

$$\begin{cases} \omega = \sqrt{\dfrac{\rho g S + K}{m + A}} & \text{Resonance condition} \\ B = C & \text{Radiation damping = PTO damping} \end{cases}$$





WAVEBOB

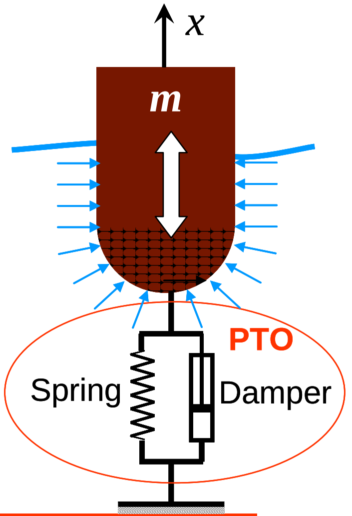
Oscillating-body dynamics

Time-domain analysis

- **Regular or irregular waves**
- **Linear or non-linear PTO**

- May require significant computing-time
- Yields time-series
- Essential for control studies

Oscillating-body dynamics



Time domain

From Fourier transform techniques:

$$(m + A_\infty)\,\ddot{x}(t) = f_d(t) - \rho g S\, x(t) - \int_{-\infty}^{t} L(t-\tau)\,\ddot{x}(\tau)\,d\tau + f_m(x, \dot{x}, t) \quad (1)$$

added mass — **excitation** — **hydrostatic** — **radiation** — **PTO**

forces

$$L(t) = \frac{1}{2\pi} \int_0^\infty \frac{B(\omega)}{\omega} \sin \omega t \, d\omega$$ **memory function**

$$f_d(t) = \sum_n f_{d,n}(t)$$ **from $\Gamma(\omega)$ and spectral distribution (Pierson-Moskowitz, …)**

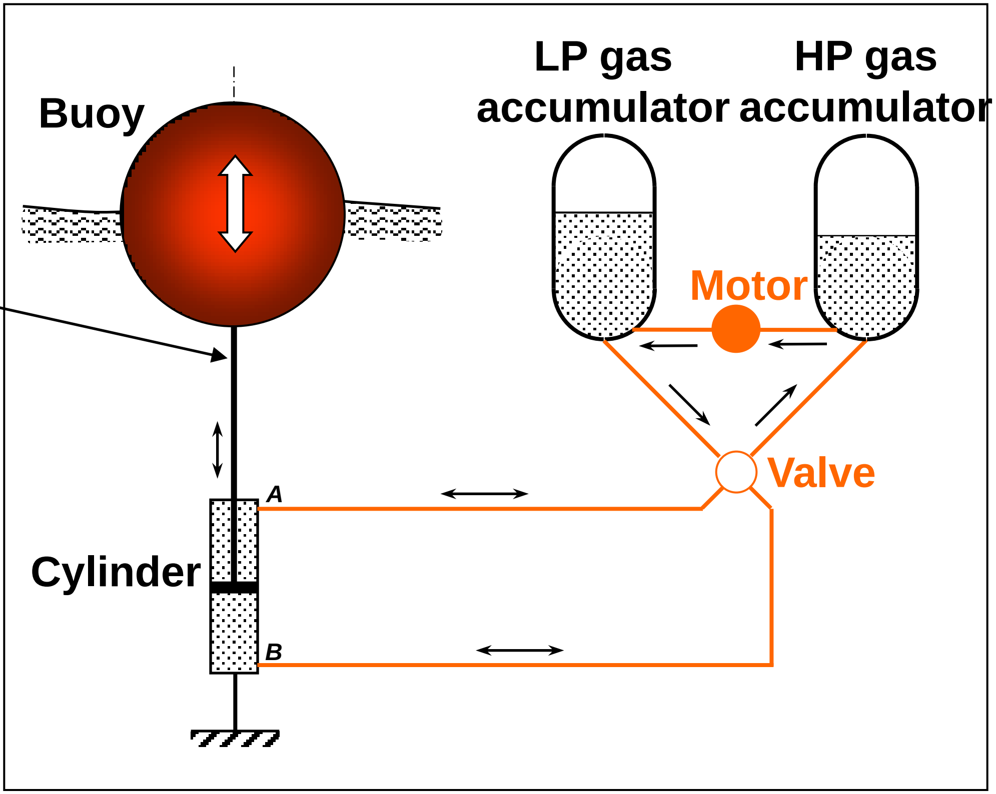
Equation (1) to be numerically integrated

Oscillating-body dynamics

Example: Heaving buoy with hydraulic PTO (oil)

- Hydraulic cylinder (ram)
- HP and LP gas accumulator
- Hydraulic motor

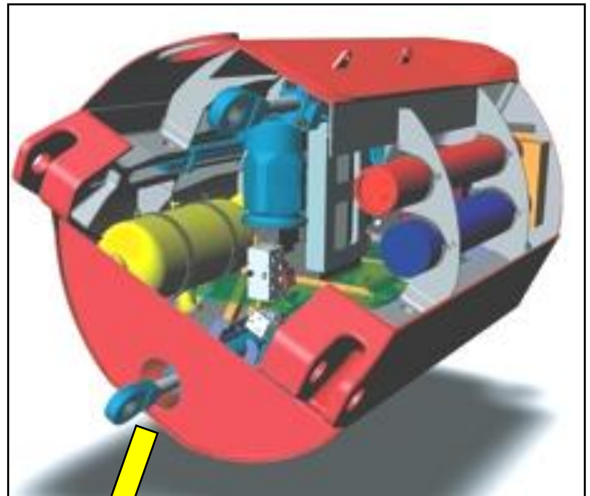
PTO force:
Coulomb type (imposed by pressure in accumulator, piston area and rectifying valve system)



PTO Equipment

High-pressure-oil PTO

Pelamis



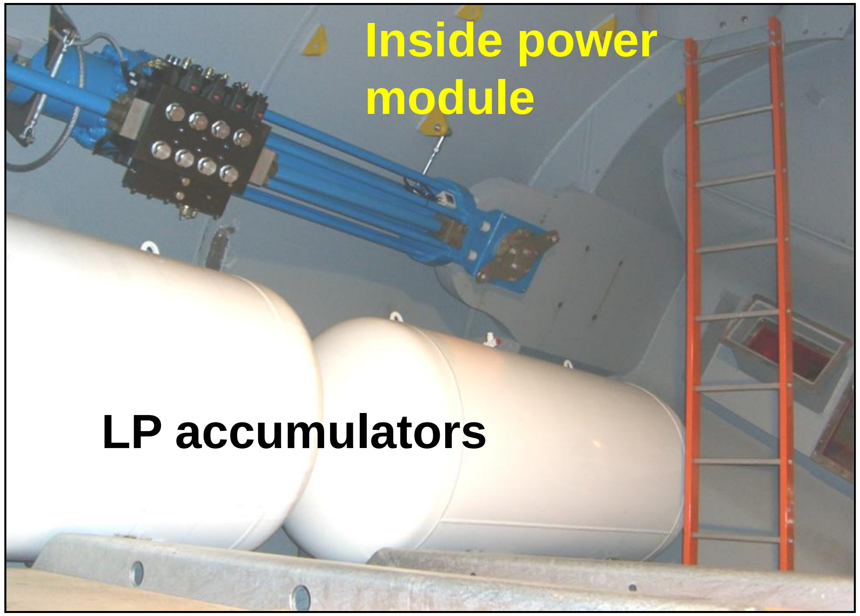
One of the three power modules of a Pelamis

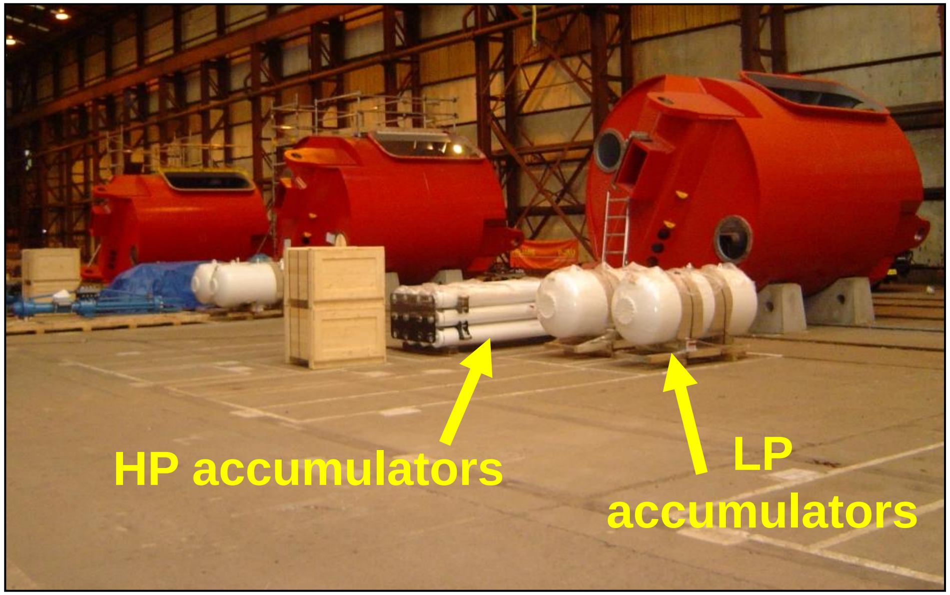
Peniche shipyard, Portugal, 2006

PTO Equipment

High-pressure-oil PTO

Pelamis

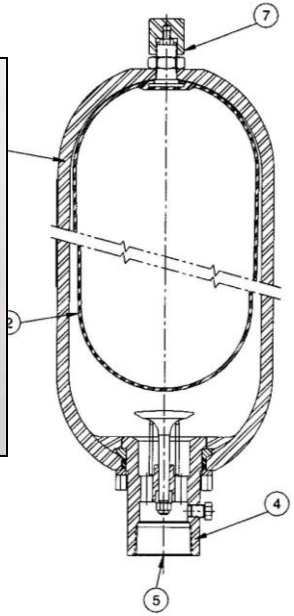




PTO Equipment

High-pressure-oil PTO

High-pressure accumulators



- Commercially available
- Bladder or piston types
- Gas: Nitrogen
- Max. working pressure up to ~ 500 bar
- Banks of unit required for full-sized WECs

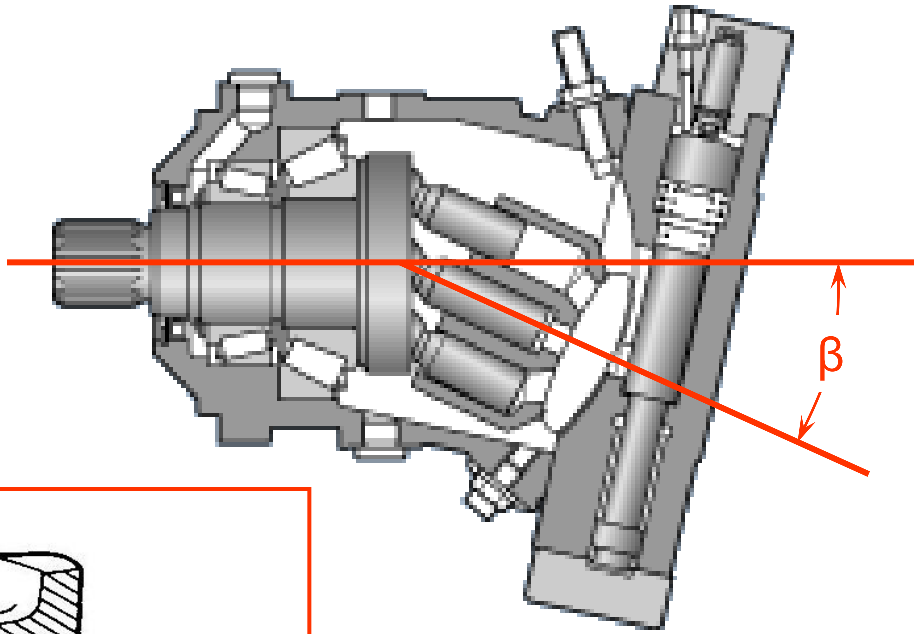
Thermodynamics of gas in accumulator (isentropic process):

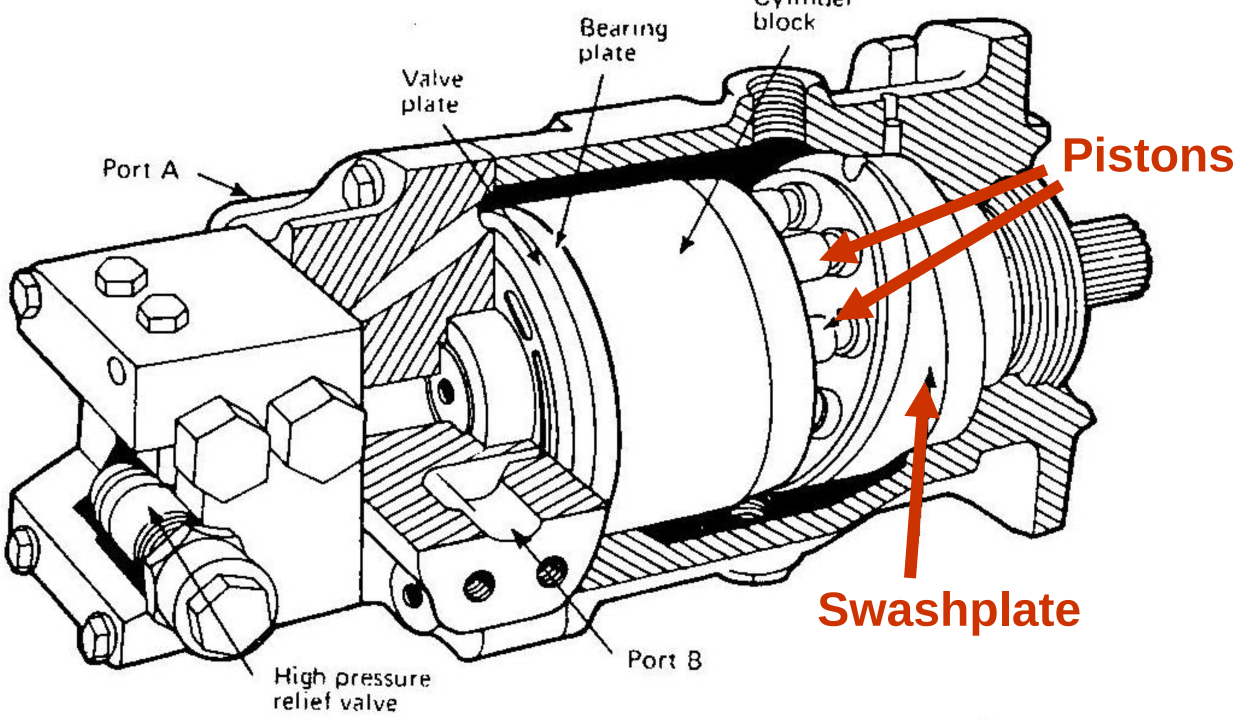
- **pressure-volume** $pV^{\gamma} = \text{constant}$ $\gamma = 1.4$ **for air and Nitrogen**
- **pressure-temperature** $p = \text{constant} \times T^{\gamma/(\gamma-1)}$
- **<u>energy storage</u> (internal energy)** $\Delta U = C_v \Delta T$

PTO Equipment

High-pressure-oil PTO

Hydraulic motor





Bent axis, variable displacement

PTO Equipment

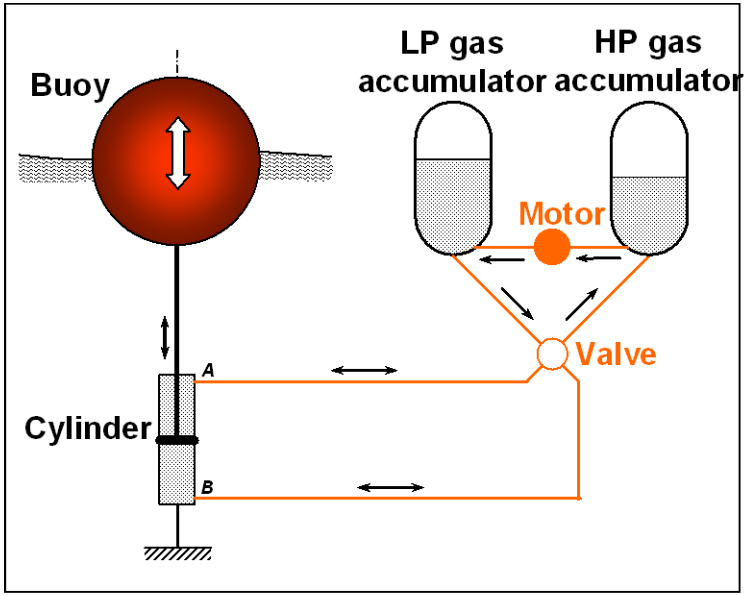
High-pressure-oil PTO

Hydraulic motor

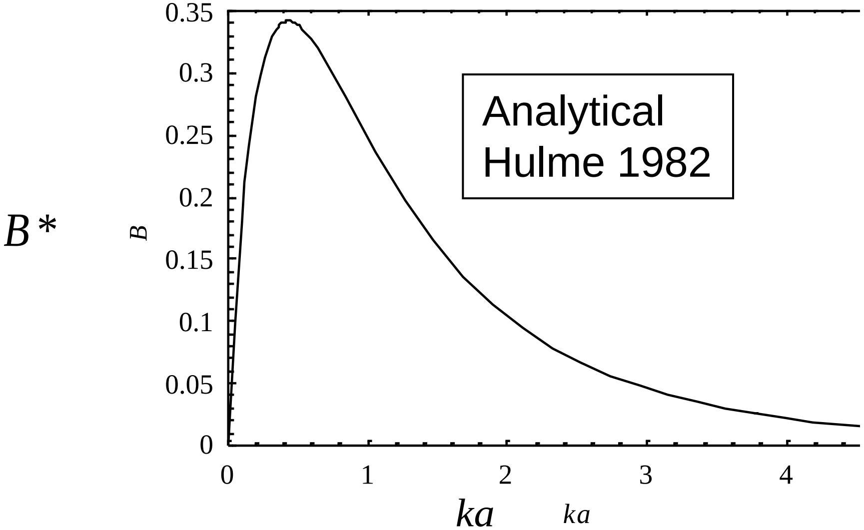
- Positive displacement machine.
- Max. power up to ~ 300 – 500 kW at > 1000 rpm.
- Direct drive of electric generator.
- Relatively compact.
- Variable displacement (double flow control capability).
- Fairly good efficiency at maximum flow.
- Reversible (as pump).
- Available from a few manufacturers.
- Not “too expensive”.

Oscillating-body dynamics

Example:
- **Hemispherical buoy, radius $= a$**

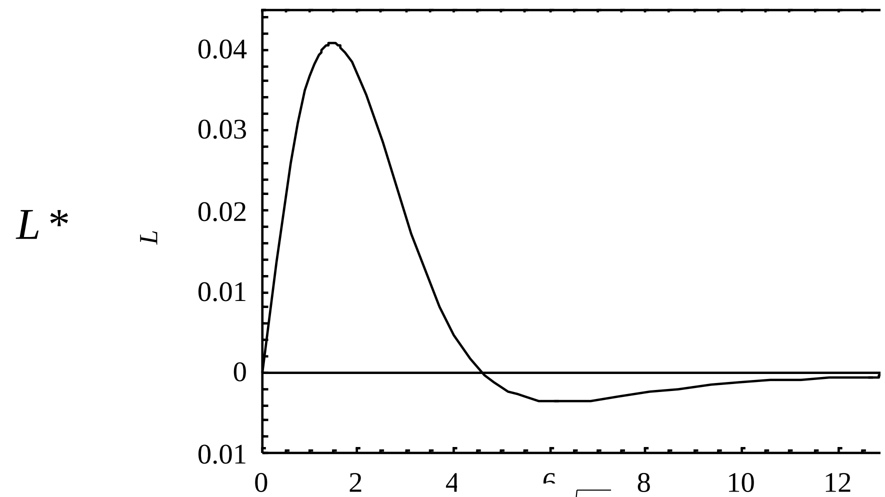


Dimensionless radiation damping coefficient



Dimensionless radius

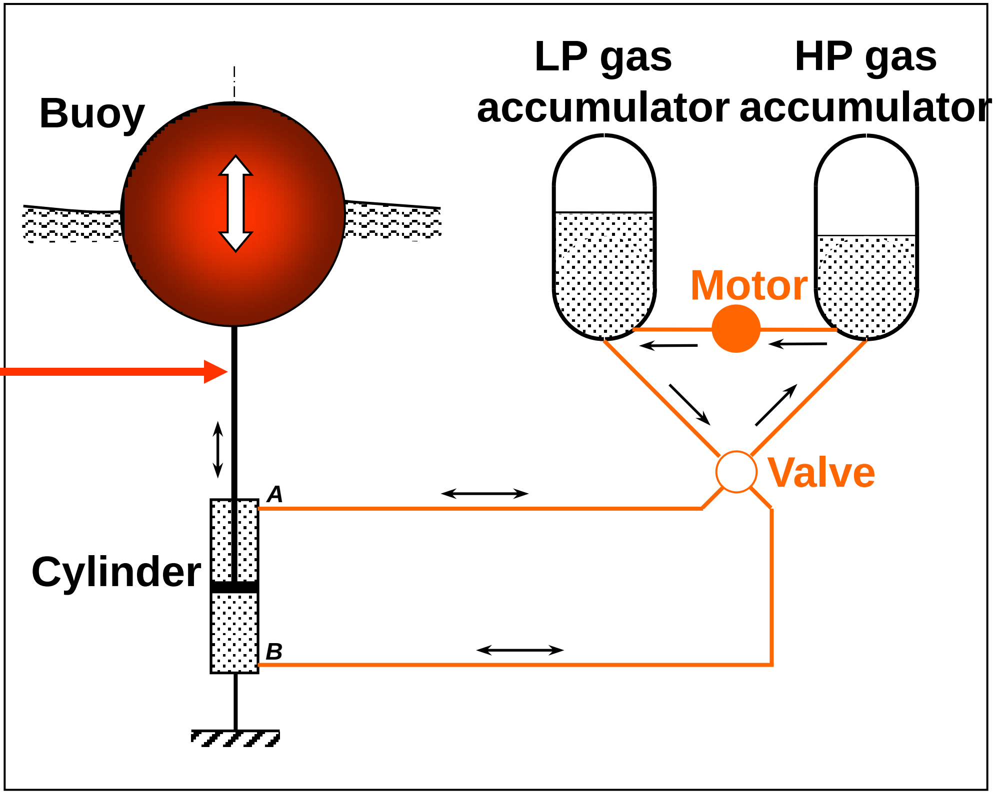
Dimensionless memory function



Dimensionless time $t^* = t\sqrt{\frac{g}{a}}$

Oscillating-body dynamics

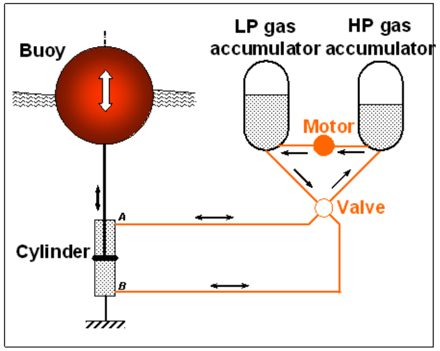
External PTO force:
Coulomb type (imposed by pressure in accumulator, piston area and rectifying valve system)



Irregular waves with H_s, T_e and Pierson-Moskowitz spectral distribution

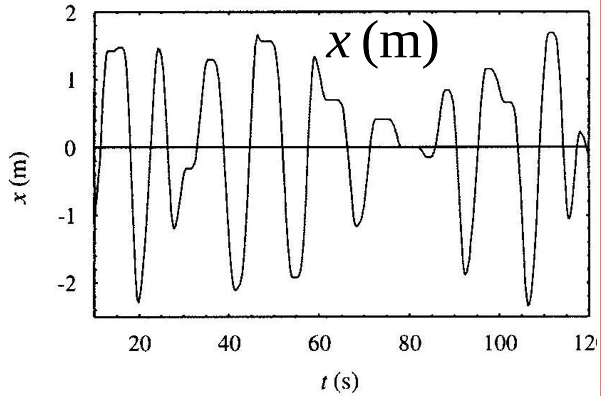
$$S_{\zeta}(\omega) = 263 H_s^2 T_e^{-4} \omega^{-5} \exp(-1054 T_e^{-4} \omega^{-4})$$

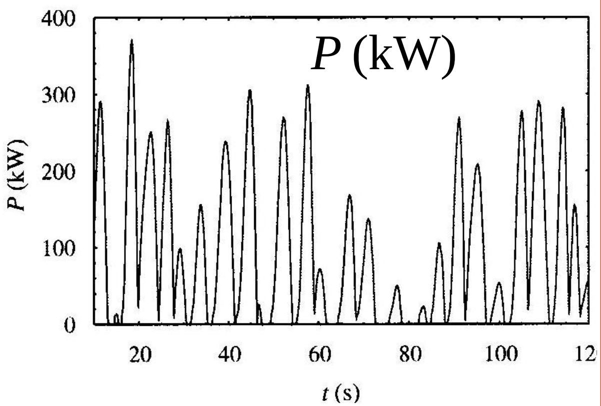
Oscillating-body dynamics



Sphere radius $a = 5\,\mathrm{m}$

Sea state $H_s = 3\,\mathrm{m},\ T_e = 11\,\mathrm{s}$

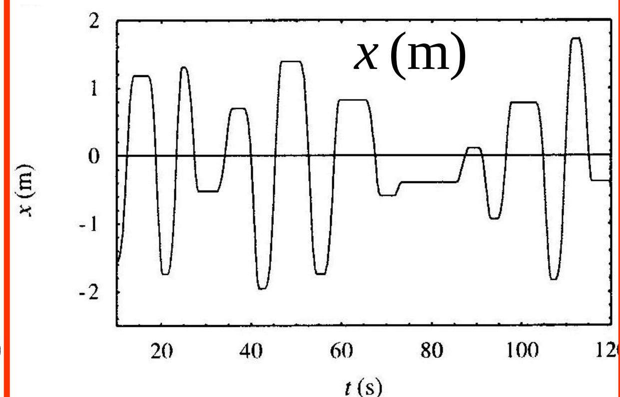


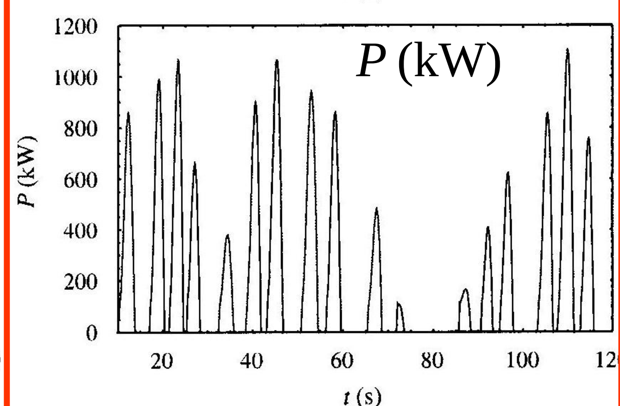


Under damped

External force 200 kN

$\bar{P} = 83.1\,\mathrm{kW}$

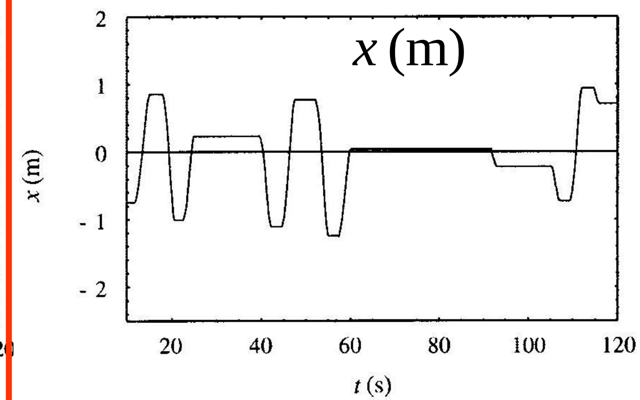


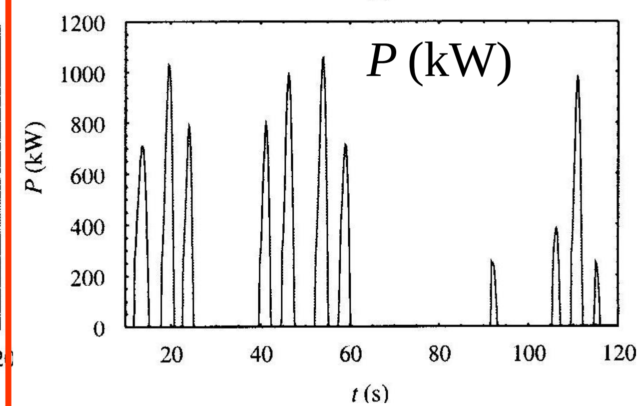


Optimally damped

External force 647 kN

$\bar{P} = 178.4\,\mathrm{kW}$





Over damped

External force 1000 kN

$\bar{P} = 97.0\,\mathrm{kW}$

Avoid overdamping and underdamping. Recall that accumulator size is finite.

How to control the damping level (PTO force or accumulator pressure) to the current sea state (or wave group) ?

Answer: Control the oil flow rate *q* through hydraulic motor as function of pressure difference Δ*p*

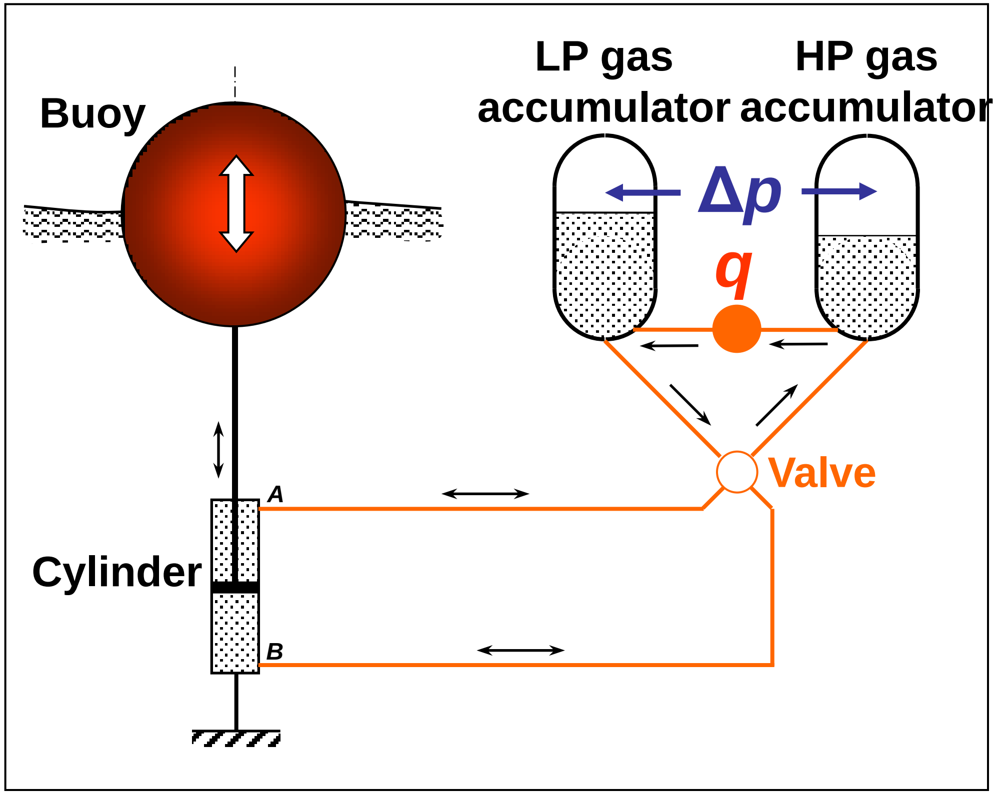
Algorithm:

$$q = \Delta p \times \text{constant}$$

$$q = G S_c^2 \Delta p$$

control parameter (G)

piston area (S_c)

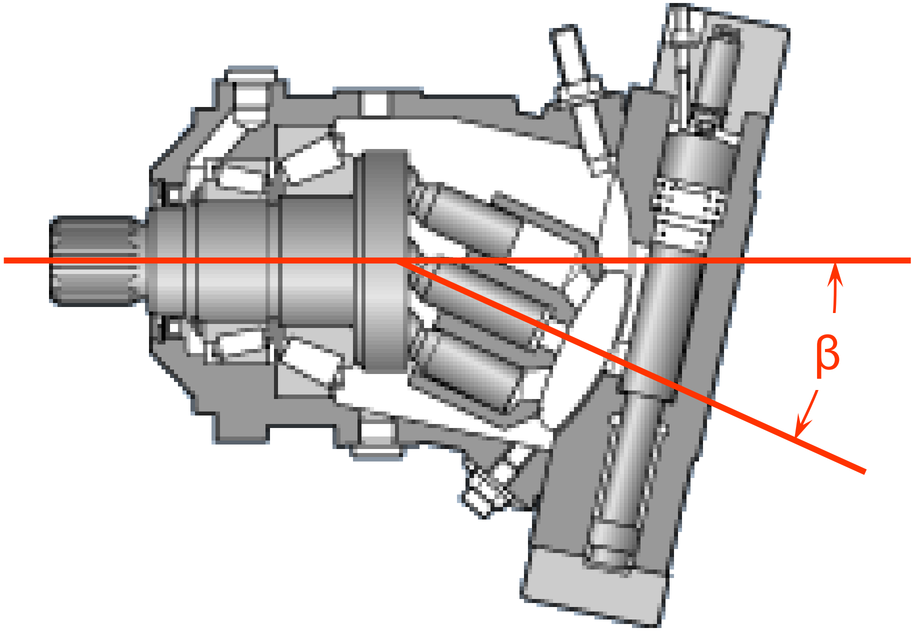


How to control the instantaneous flow rate of oil?

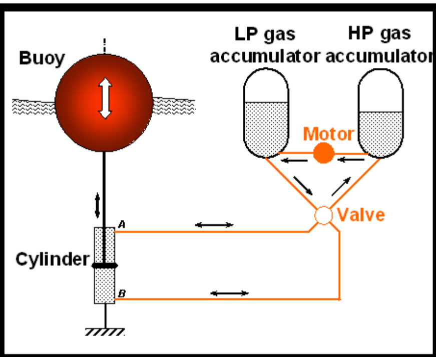
- **Control the rotational speed**

and/or

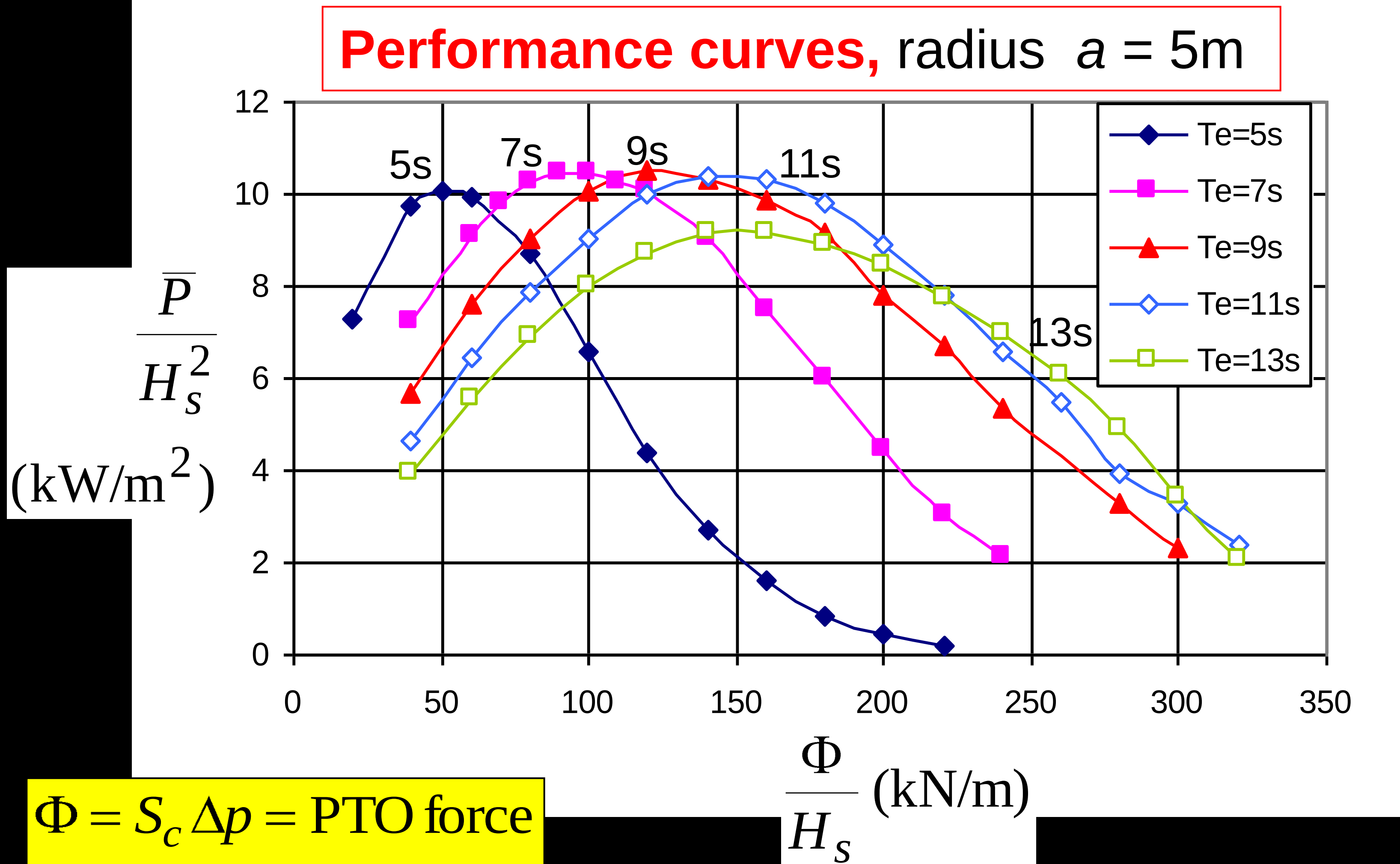
- **control the angle β (displacement)**



CONTROL OF WAVE ENERGY CONVERTER

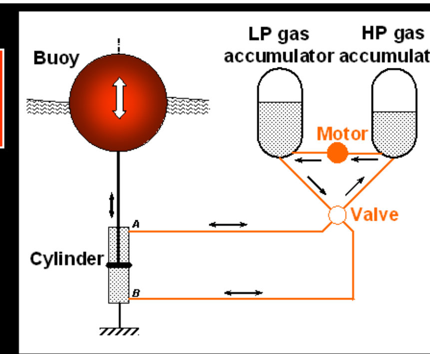


Note: hydrodynamically the system is linear



$\Phi = S_c \Delta p = \text{PTO force}$

CONTROL OF WAVE ENERGY CONVERTER



Control algorithm

$$q(t) = S_c^2 G \Delta p$$

piston area

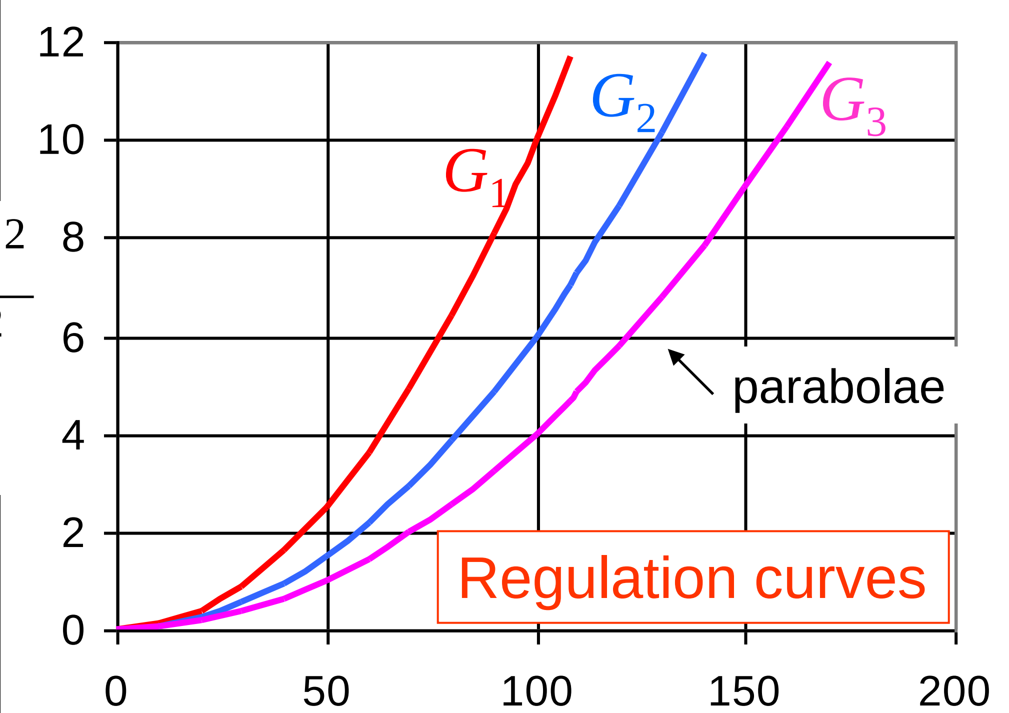
Control parameter

$$\Phi = S_c \Delta p = \text{piston force}$$

$$P_{\text{motor}} = q\,\Delta p = G\,\Phi^2$$

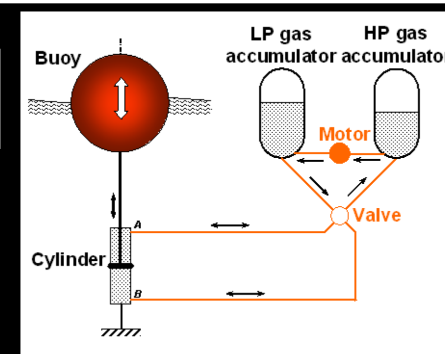
$$\frac{P_{\text{motor}}}{H_s^2} = \frac{G\,\Phi^2}{H_s^2}$$

(kW/m^2)



$$\frac{\Phi}{H_s}\ (\text{kN/m})$$

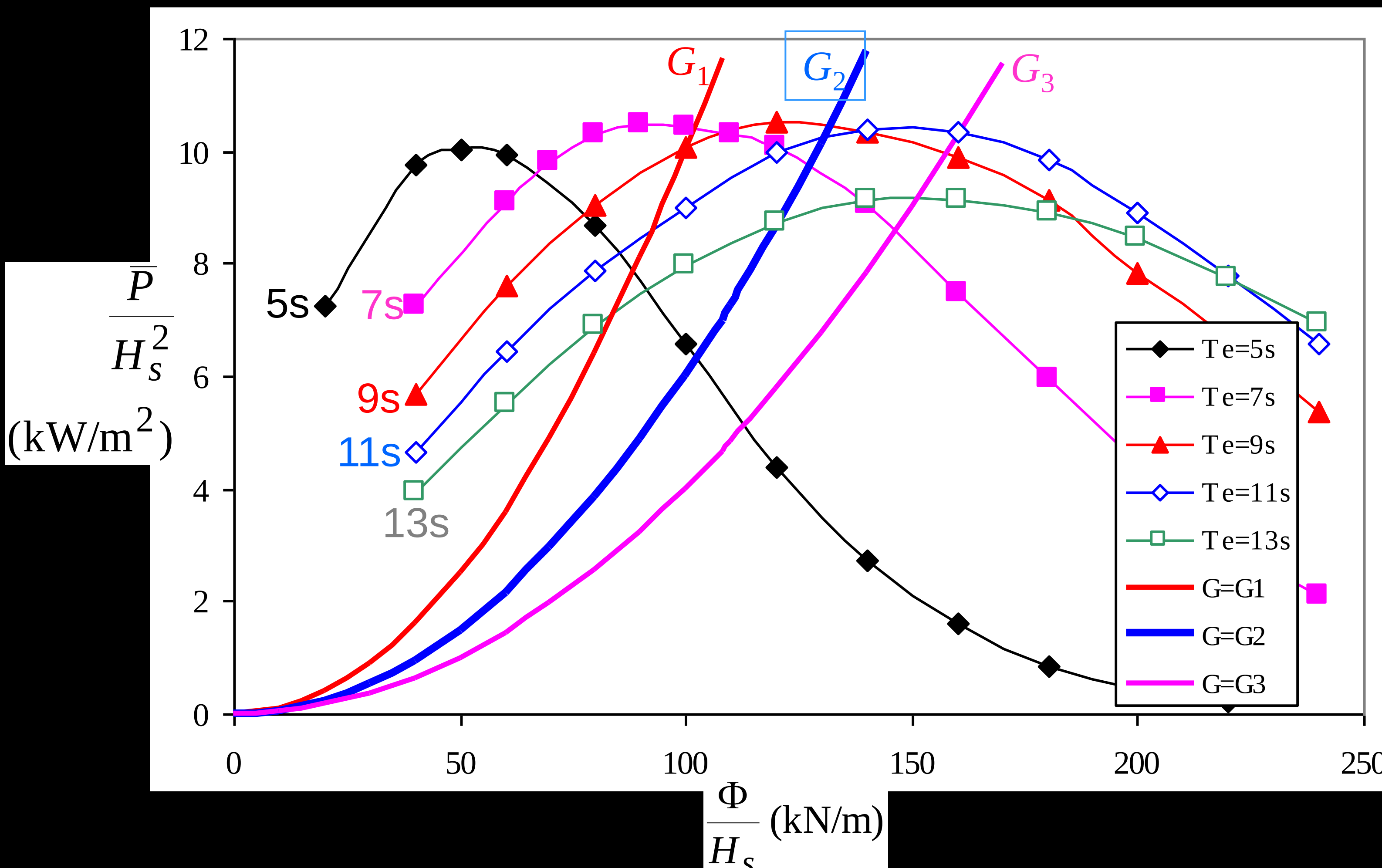
CONTROL OF WAVE ENERGY CONVERTER



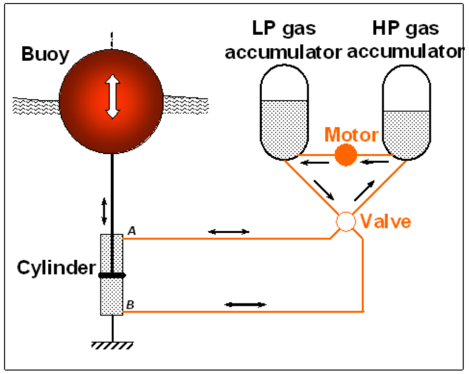
Control algorithm $q_m(t) = S_c G \Delta p$

piston area (S_c)

control parameter (G)

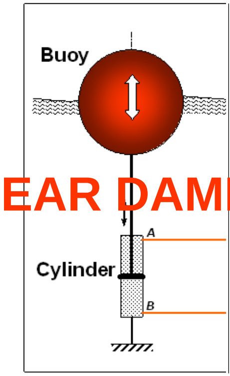


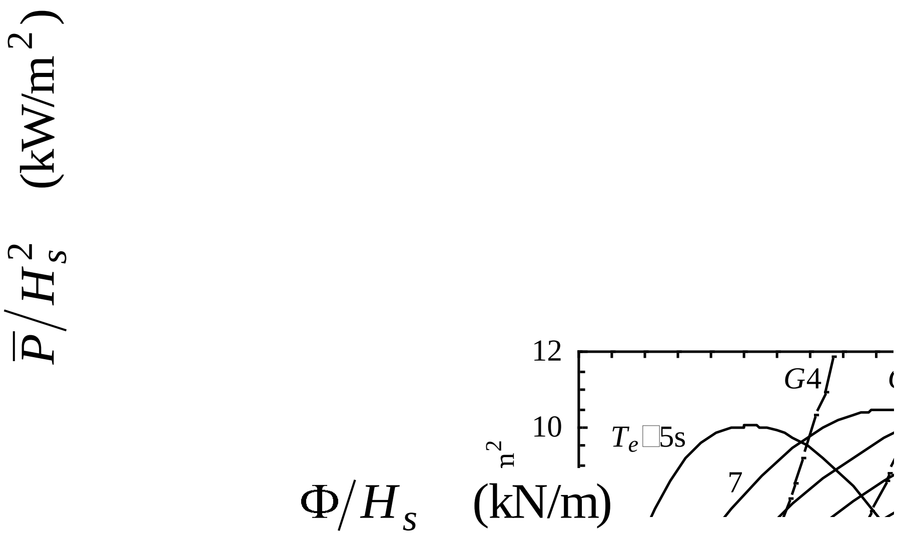
Oscillating-body dynamics

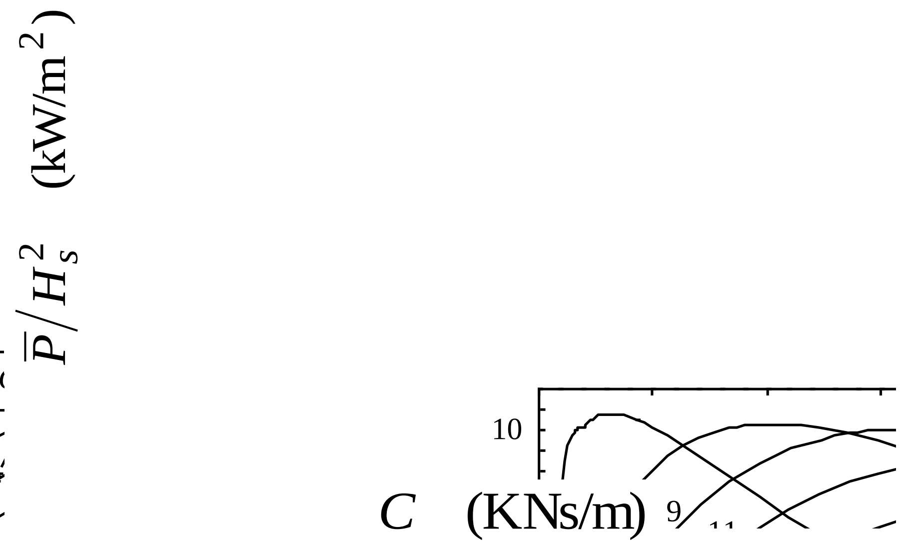


Buoy radius 5m

LINEAR DAMPER







A hydraulic PTO and a linear damper may be almost equally effective in irregular waves (NO PHASE CONTROL).

Oscillating-body dynamics

For **point absorbers** (relatively small bodies) the resonance frequency of the body is in general much larger than the typical wave frequency of sea waves:

- No resonance can be achieved.
- Poor energy absorption.

How to increase energy absorption?

Phase control !

Oscillating-body dynamics

Phase control, i.e. wave-to-wave control in radom waves, is one of the main issues in wave energy conversion.

Optimal control is a difficult theoretical control problem, that has been under investigation since the late 1970s.

Control is made difficult by the randomness of the waves and by the wave-device interaction being a process with memory.

The difficulty increases for multi-mode oscillations and for multi-body systems.

Control should be regarded as an open problem and a major challenge in the development of wave energy conversion.

Oscillating-body dynamics

Phase-control by **latching**

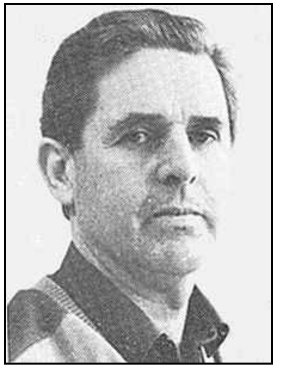
Whenever the body velocity comes down to zero, keep the body fixed for an appropriate perid of time.

This is an artificial way of reducing the frequency of the body free-oscillations, and achieving resonance.

Phase-control by latching was introduced by Falnes and Budal

J. Falnes, K. Budal, Wave-power conversion by power absorbers. *Norwegian Maritime Research*, vol. 6, p. 2-11, 1978.

Johannes Falnes



Kjell Budall (1933-89)

Optimal phase control in random waves requires the prediction of incoming wave and heavy computing.

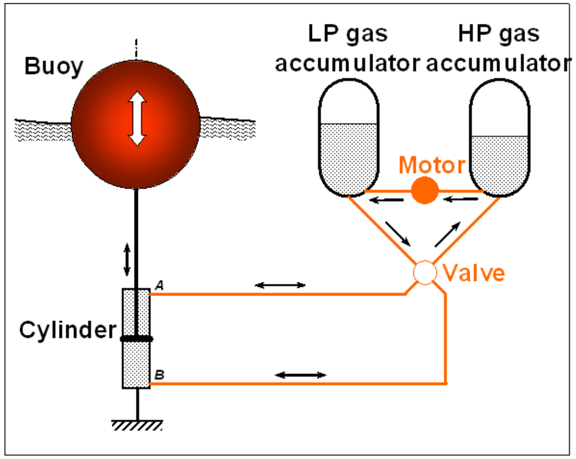
Sub-optimal control strategies by latching were devised by several teams.

Usually, control algorithm determines the **duration of time** the oscillator is kept fixed (latched) in each wave cycle.

Alternative strategy is in terms of **load** (not time duration):

Opposing force to be overcome before the body is released.

Numerical simulations of phase control



Sphere radius 5 m

Gas (Nitrogen):
- accumulator: 100 kg
- turbine casing: 20 kg

$$q_m(t) = G S_c \Delta p$$

$$S_c = 0.0314 \text{ m}^2 (\text{diameter } 200 \text{ mm})$$

Phase-control by latching: body is released when

hydrodynamic force on body exceeds $R(S_c \Delta p) \quad (R > 1)$

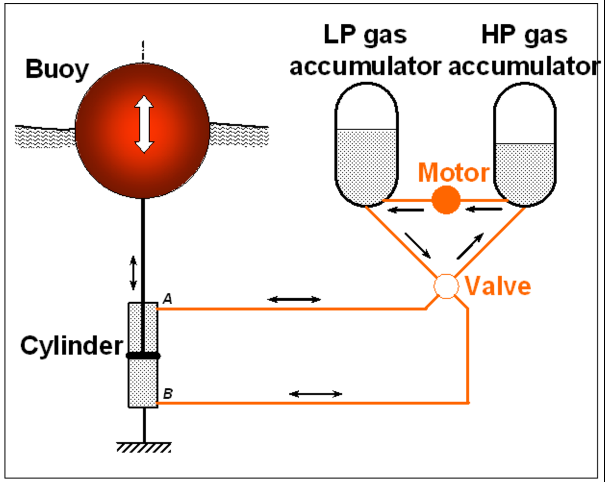
Control parameters:

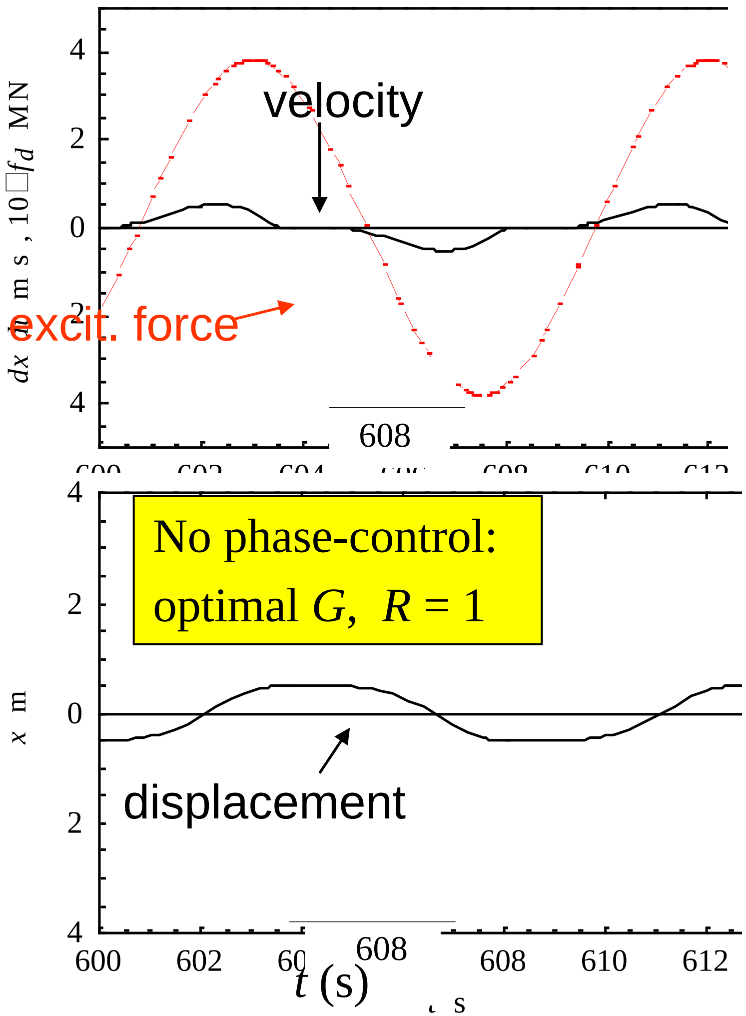
G controls oil flow rate through hydraulic motor

R controls latching (release of body)

PHASE CONTROL

How to achieve **phase-control by latching** in a floating body with a hydraulic power-take-off mechanism?





Introduce a delay in the release of the latched body.

How?

Increase the resisting force the hydrodynamic forces have to overcome to restart the body motion.

REGULAR WAVES

Period ***T*** **= 9 s**
Amplitude 0,667 m

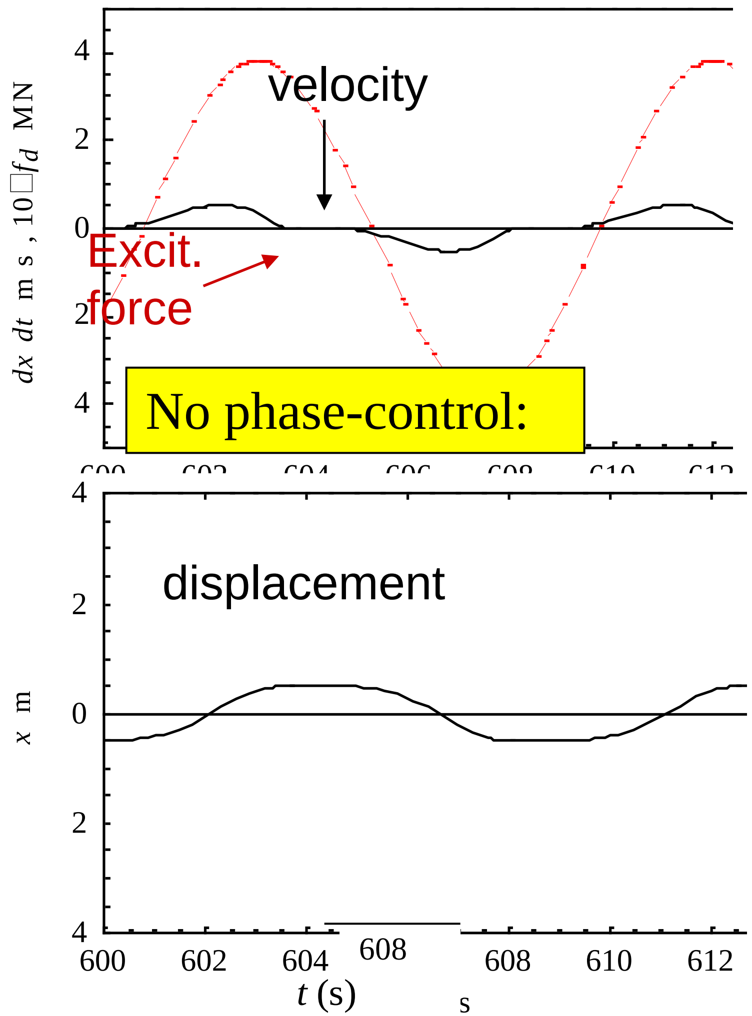
Regular waves: T = 9 s, amplitude 0.67 m

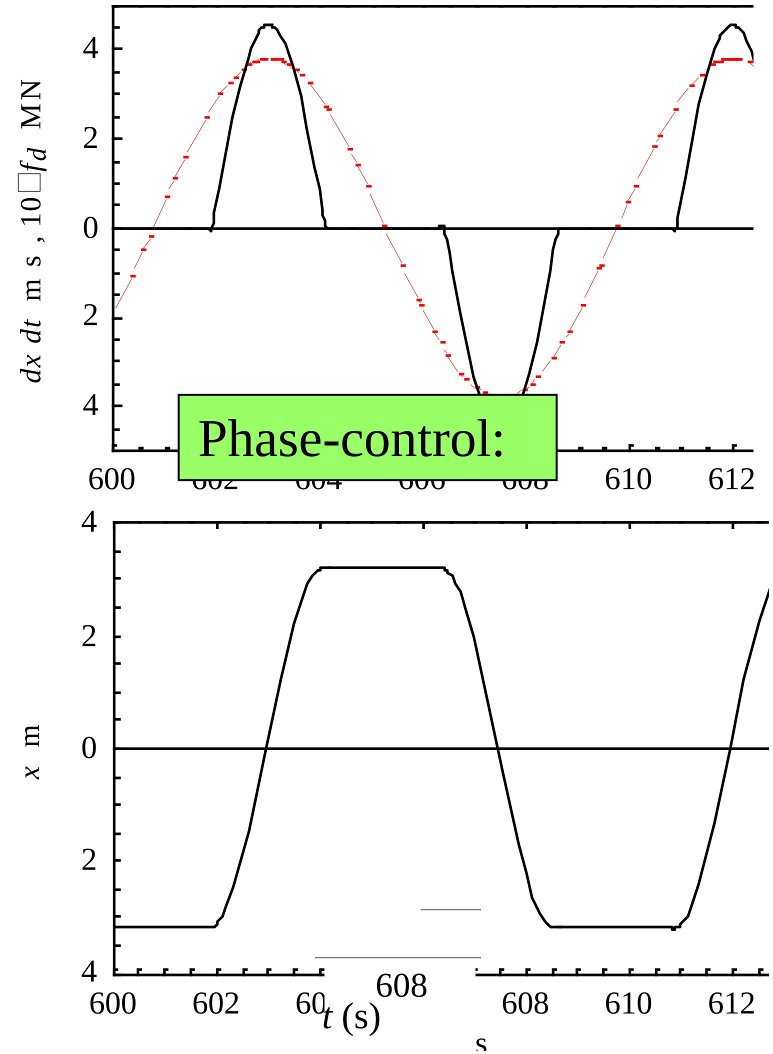
$G = 0.86 \times 10^{-6}$ s/kg

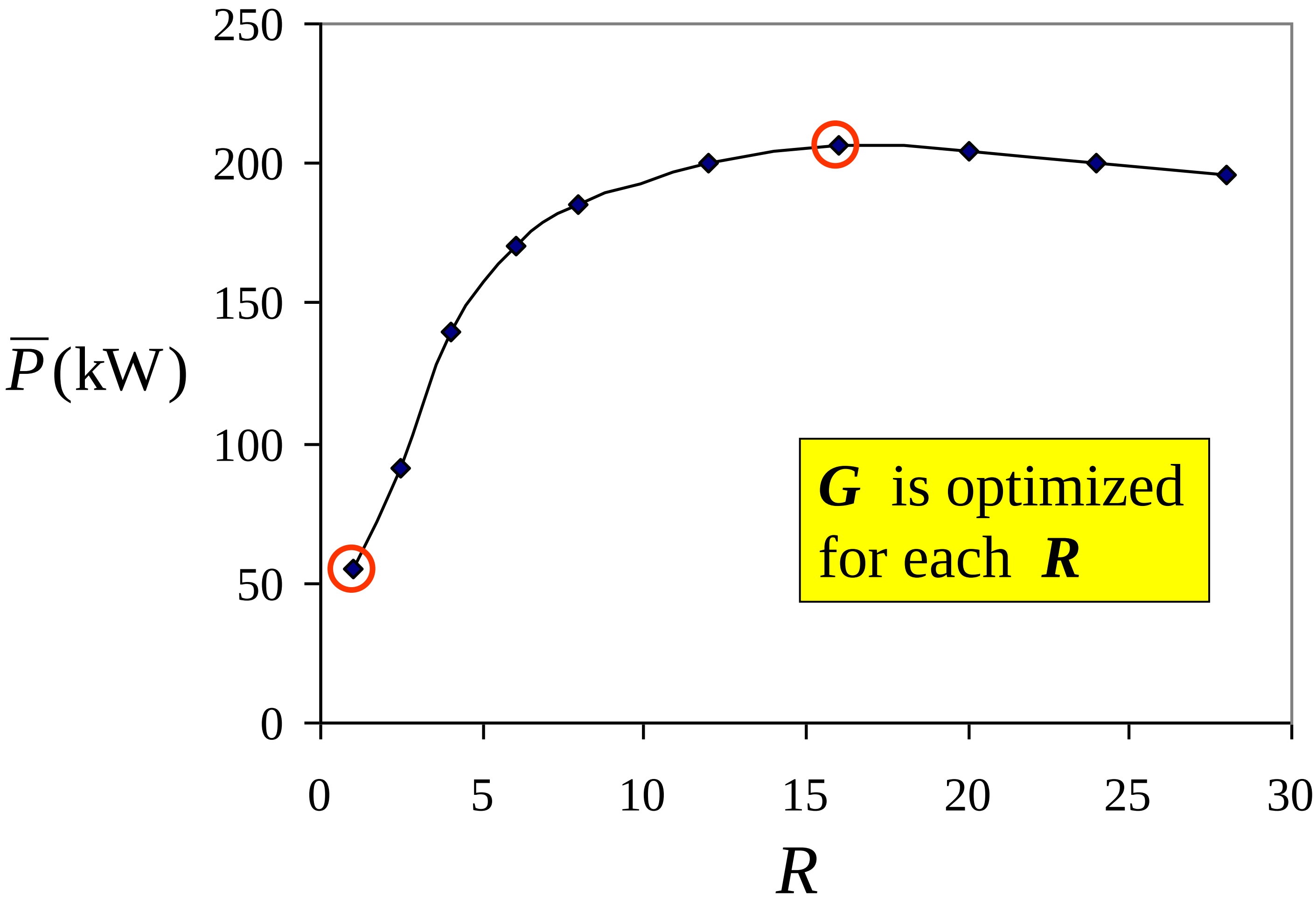
$R = 1 \qquad \overline{P} = 55.0$ kW

$G = 7.7 \times 10^{-6}$ s/kg

$R = 16 \qquad \overline{P} = 206.1$ kW



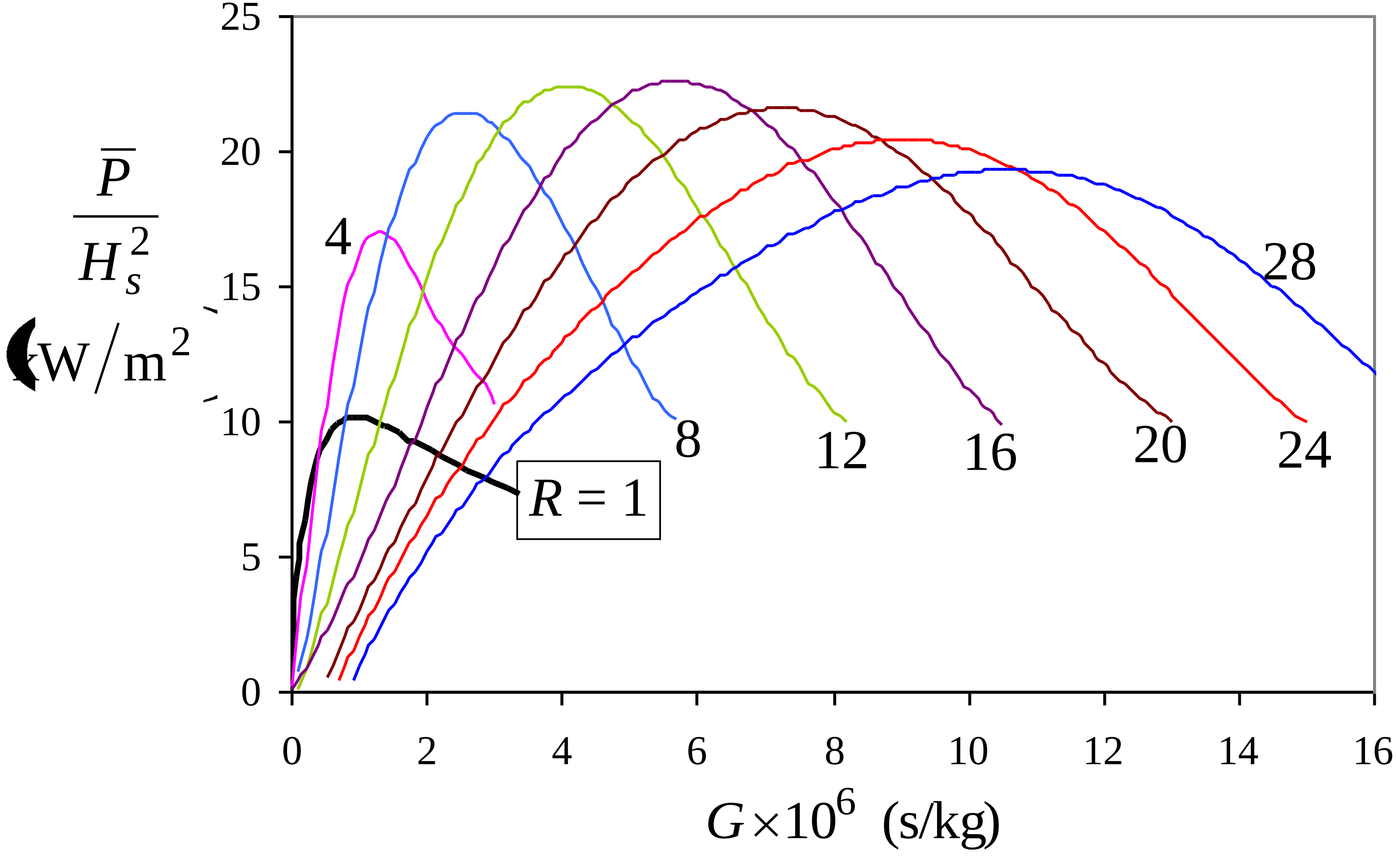



250
200
150
100
50
0
$\overline{P}$ (kW)
0
5
10
15
20
25
30
R
G is optimized for each R

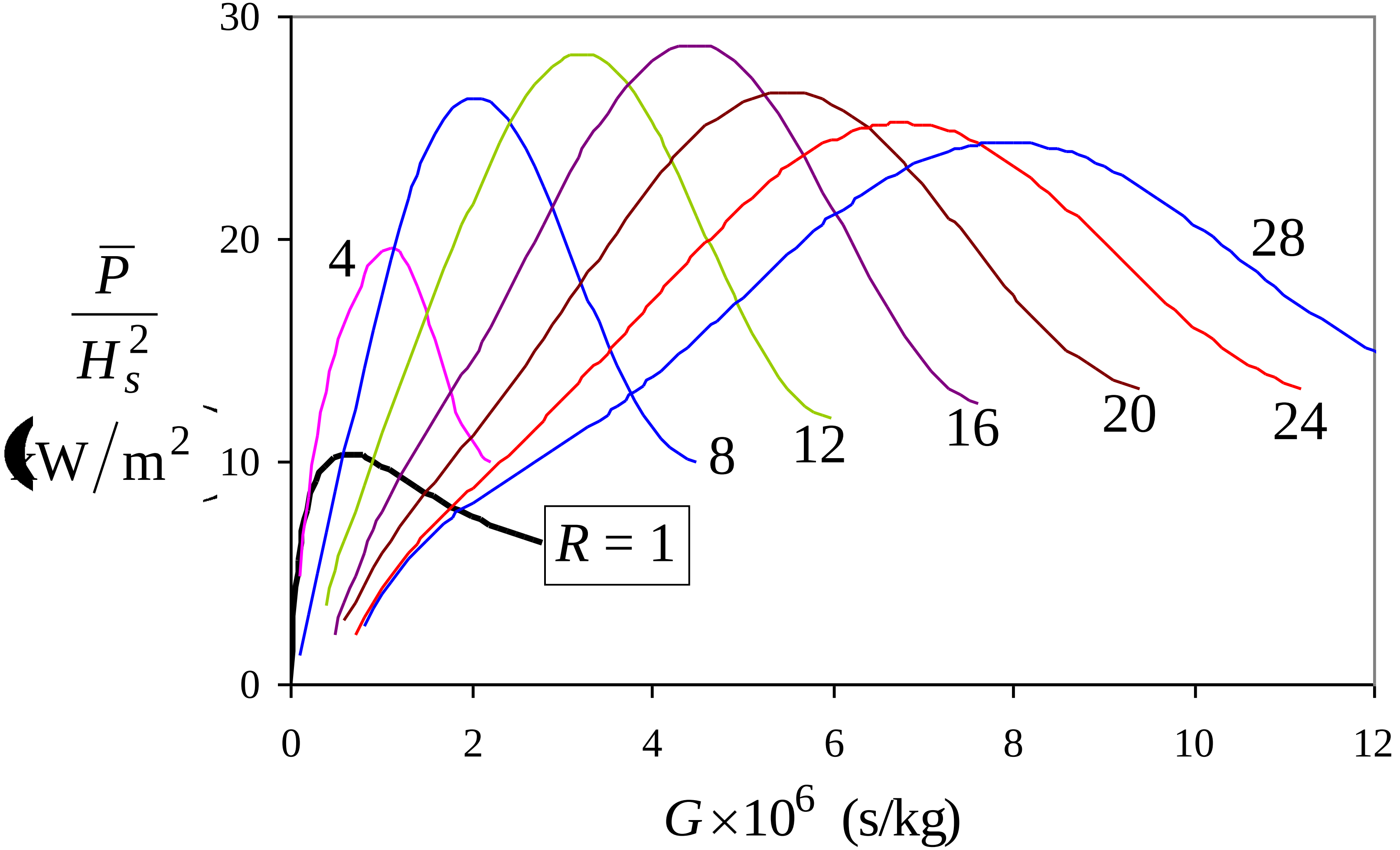
IRREGULAR WAVES

Period T_e **= 7, 9, 11 s**
Height H_s **= 2 m**

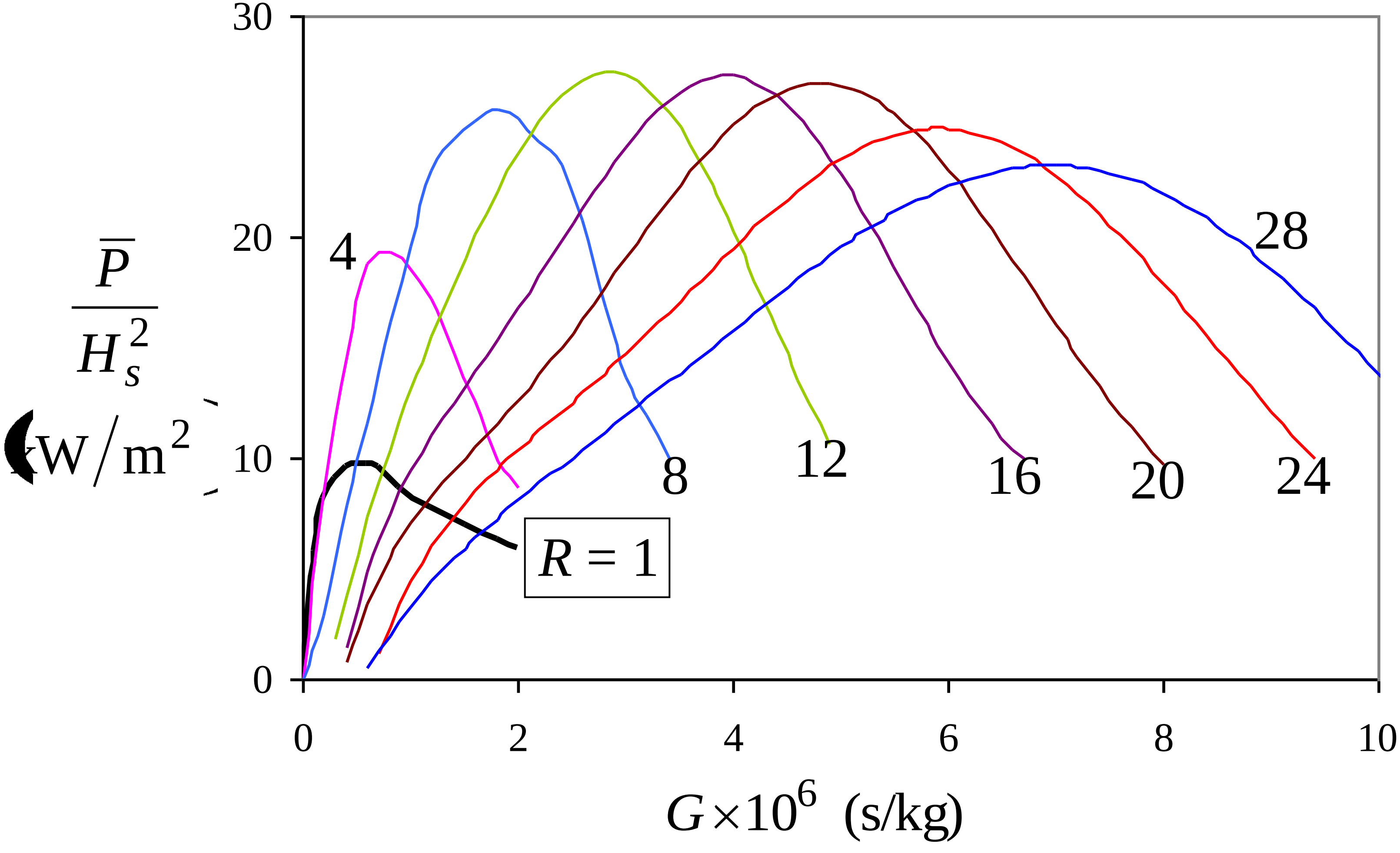
$T_e = 7\,\mathrm{s}, \quad H_s = 2\,\mathrm{m}$



$T_e = 9\,\mathrm{s}, \quad H_s = 2\,\mathrm{m}$



$T_e = 11\,\text{s}, \quad H_s = 2\,\text{m}$



Detailed analysis

$T_e = 9$ s

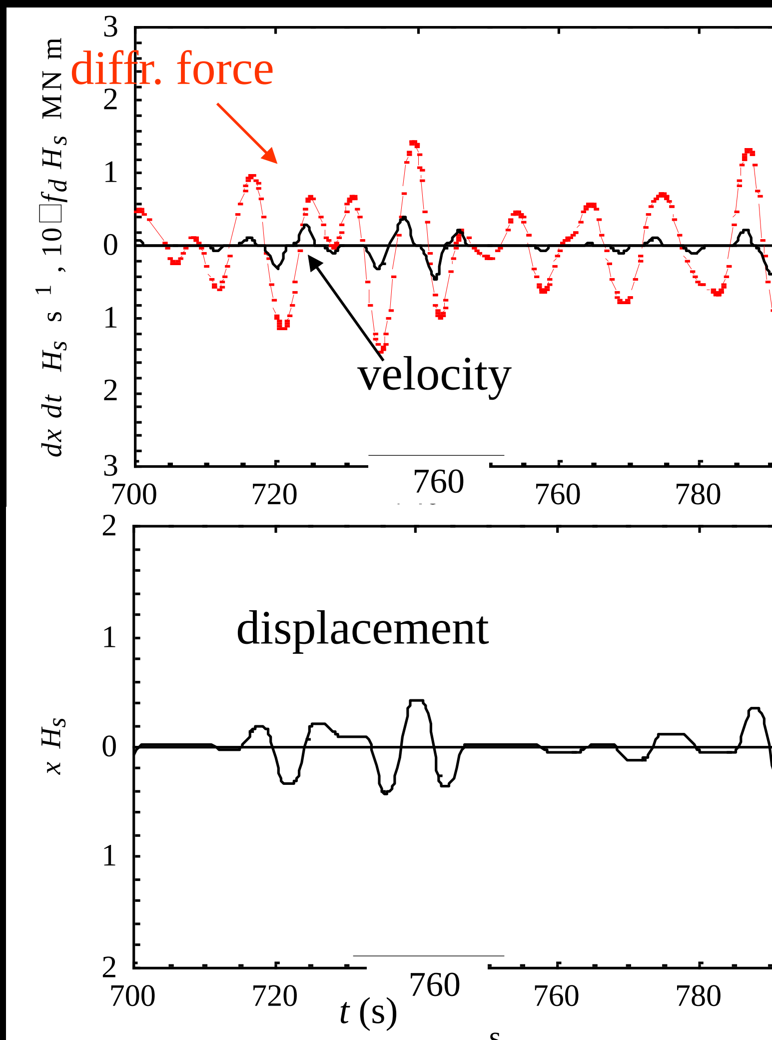
$H_s = 2$ m

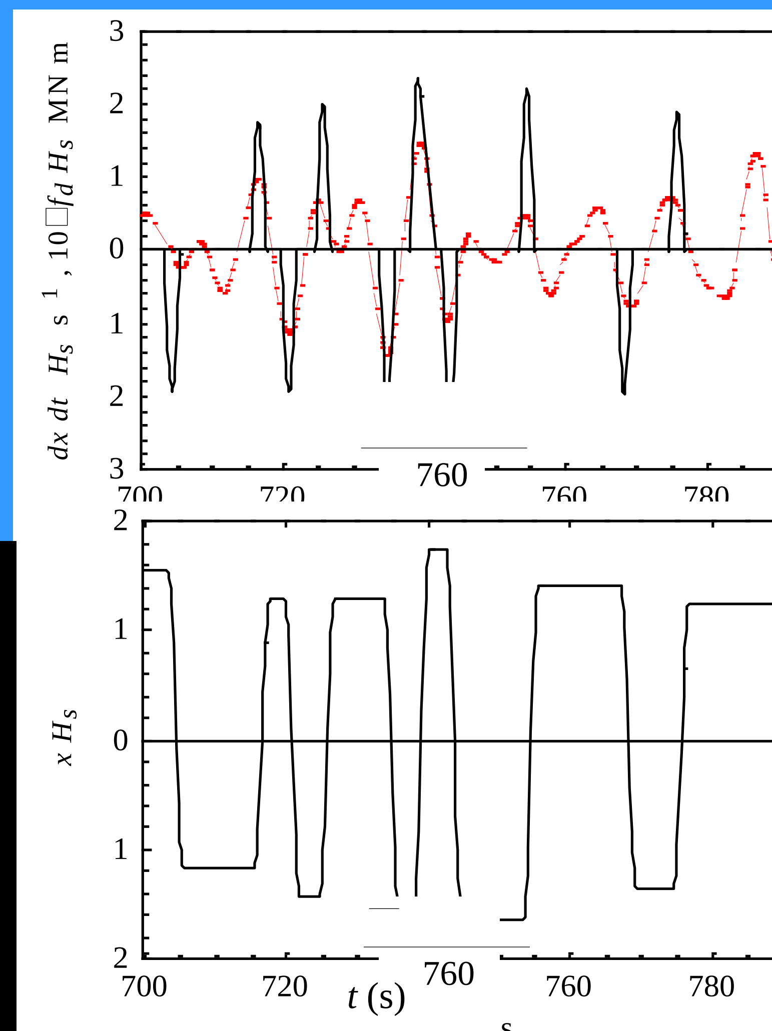
$G = 0.7\times10^{-6}$ s/kg

$R = 1 \quad \bar{P}/H_s^2 = 10.3\,\mathrm{kW/m^2}$

$G = 4.2\times10^{-6}$ s/kg

$R = 16 \quad \bar{P}/H_s^2 = 28.5\,\mathrm{kW/m^2}$

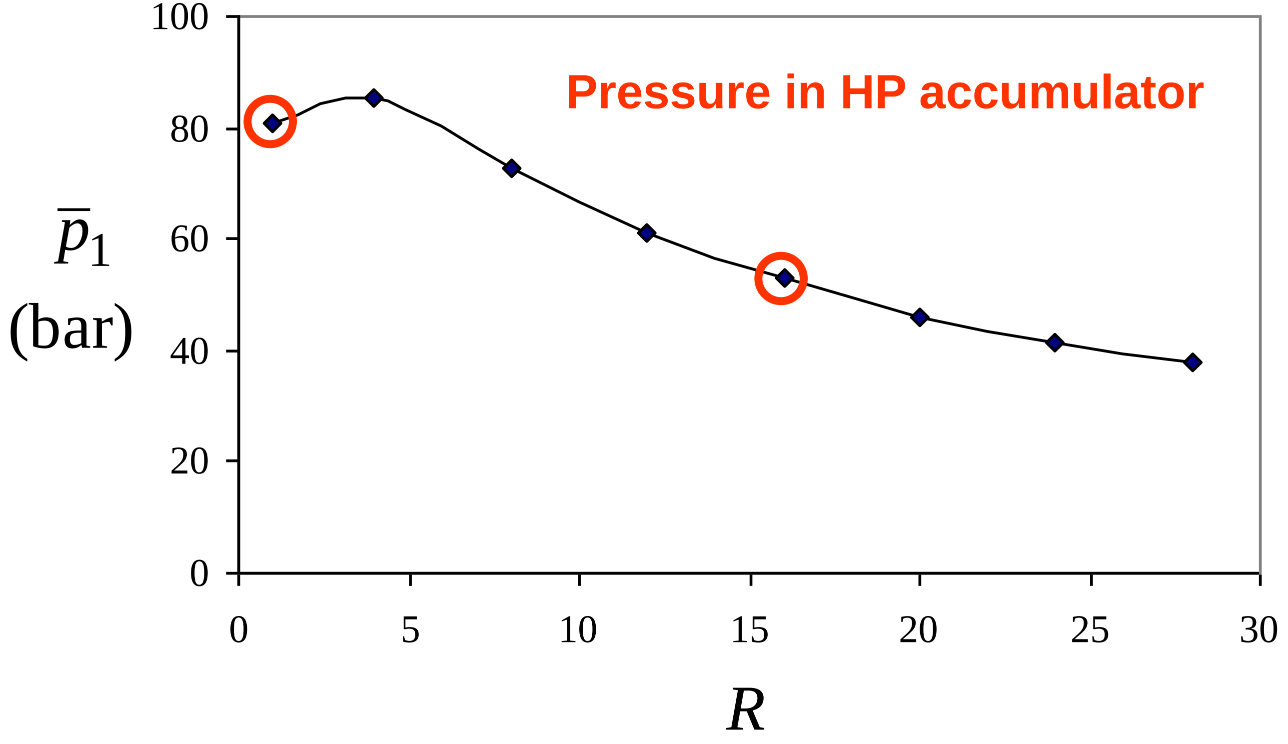




The large increase in time-averaged power output results:

• from a large increase in oil flow rate (increase in control parameter *G*), and hence in motor size;

• not from an increase in hydraulic circuit pressure.



Oscillating-body dynamics

Phase control by latching may significantly increase the amount of absorbed energy by point absorbers.

Control of load prior to release is an alternative to latch duration control.

Control parameters (*G*, *R*) are practically independent of wave height and weakly dependent on wave period.

Problems with latching phase control:

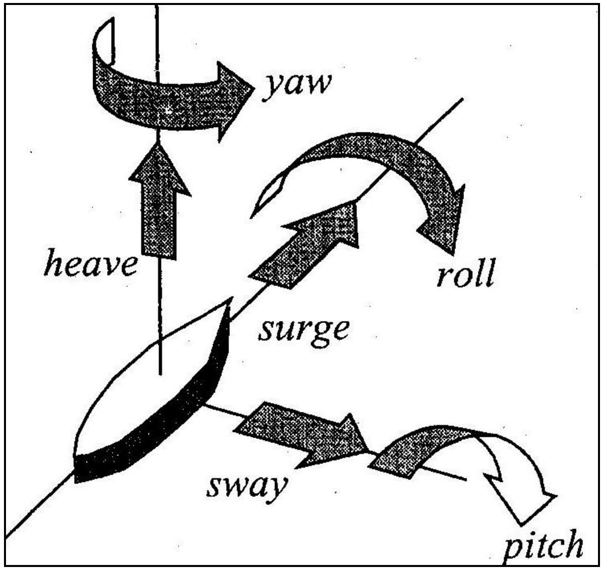
- Latching forces may be very large.
- Latching control is less effective in two-body WECs.

Apart from latching, there are forms of phase control (reactive, uncluching, …).

Oscillating-body dynamics

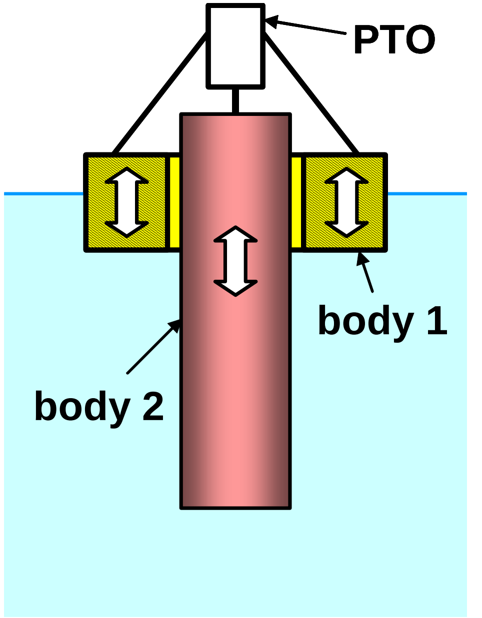
Several degrees of freedom

- **Each body has 6 degrees of freedom**
- **A WEC may consist of *n* bodies (*n* >1)**



All these modes of oscillation interact with each other through the wave fields they generate.

Number of dynamic equations = 6*n*



The interference between modes affects:

- **added masses**
- **radiation damping coefficients**

Hydrodynamic coefficients A_{ij}, B_{ij} **are defined accordingly.**

They can be computed with commercial software (WAMIT, …).

WAVE ENERGY TECHNOLOGIES

Oscillating water column
(with air turbine)

- Fixed structure
  - Isolated: **Pico, LIMPET**
  - In breakwater: **Sakata, Douro river**
- Floating structure: **Mighty Whale, BBDB, Oceanlinx**

Oscillating b
(with hydraulic
hydraulic turbir
electrical gene

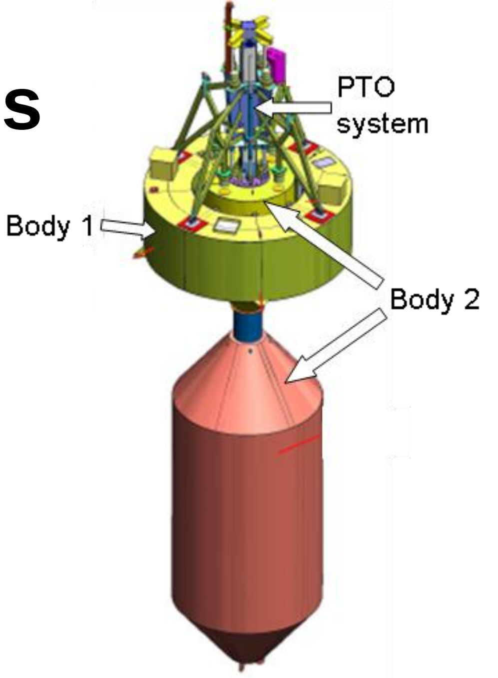
- Floating
  - Essencially translation (heave): **IPS Buoy, WaveBob, PowerBuoy**
  - Essencially rotat n: **Pelamis, PS og, SEAREV**
- erged
  - Essencially tr slation (heave): **WS**
  - **Ov**
  - nt
  - t c

Run up
(with low-head
turbine)

- ture
- Floating st **Wa**

TWO-BODY POINT ABSORBERS

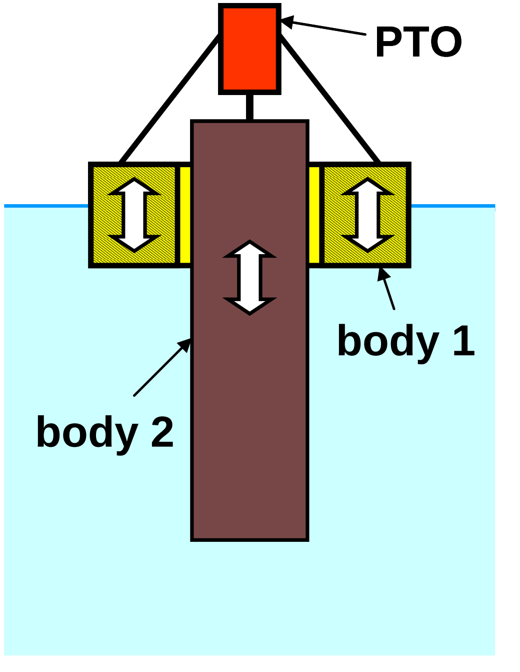
Oscillating-body dynamics



Several degrees of freedom

Example: heaving bodies 1 and 2 reacting against each other.

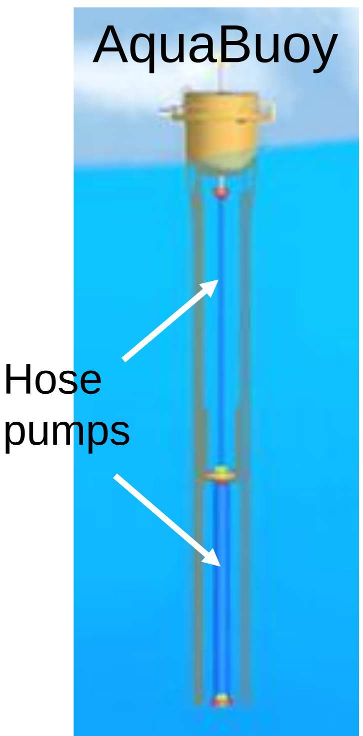
$$\begin{cases} (m_1 + A_1)\ddot{x}_1 + B_1\dot{x}_1 + \rho g S_1 x_1 + C(\dot{x}_1 - \dot{x}_2) + K(x_1 - x_2) \\ \quad + A_{12}\ddot{x}_2 + B_{12}\dot{x}_2 = f_{d1} \\ (m_2 + A_2)\ddot{x}_2 + B_2\dot{x}_2 + \rho g S_2 x_2 - C(\dot{x}_1 - \dot{x}_2) - K(x_1 - x_2) \\ \quad + A_{12}\ddot{x}_1 + B_{12}\dot{x}_1 = f_{d2} \end{cases}$$

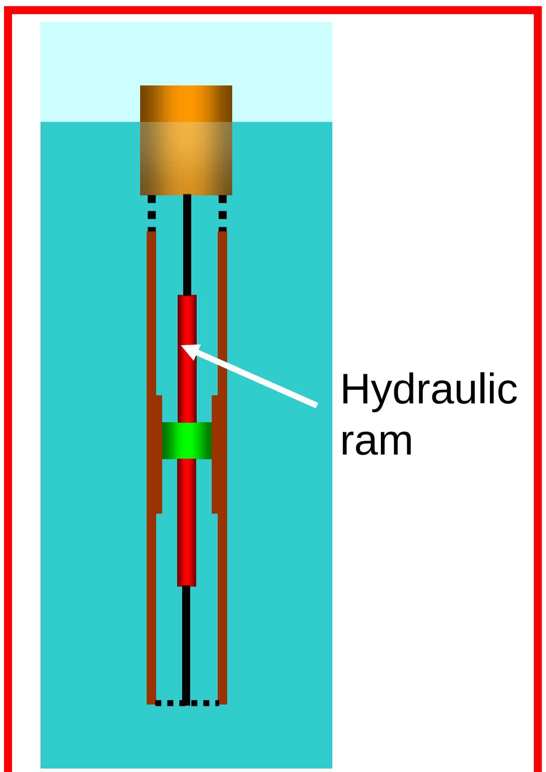


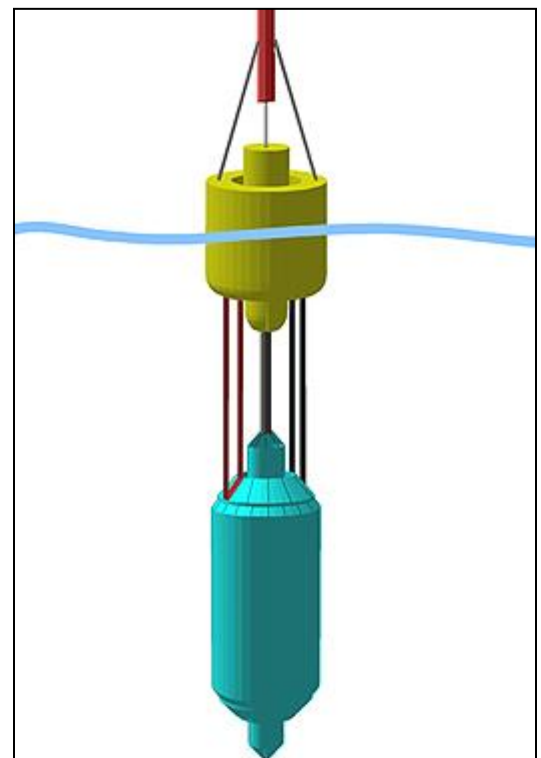
Note: $A_{12} = A_{21}, \quad B_{12} = B_{21}$

IPS Buoy

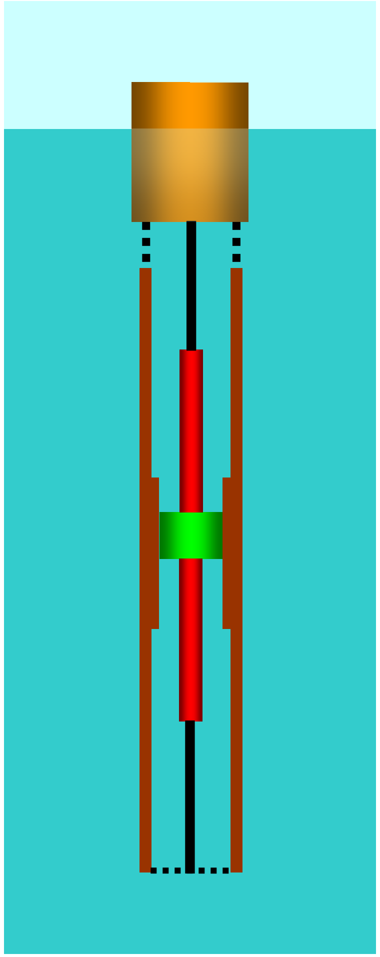
Wave Bob







Simplifying assumptions for optimization and control



Buoy represented by body 1a (hemispherical buoy)

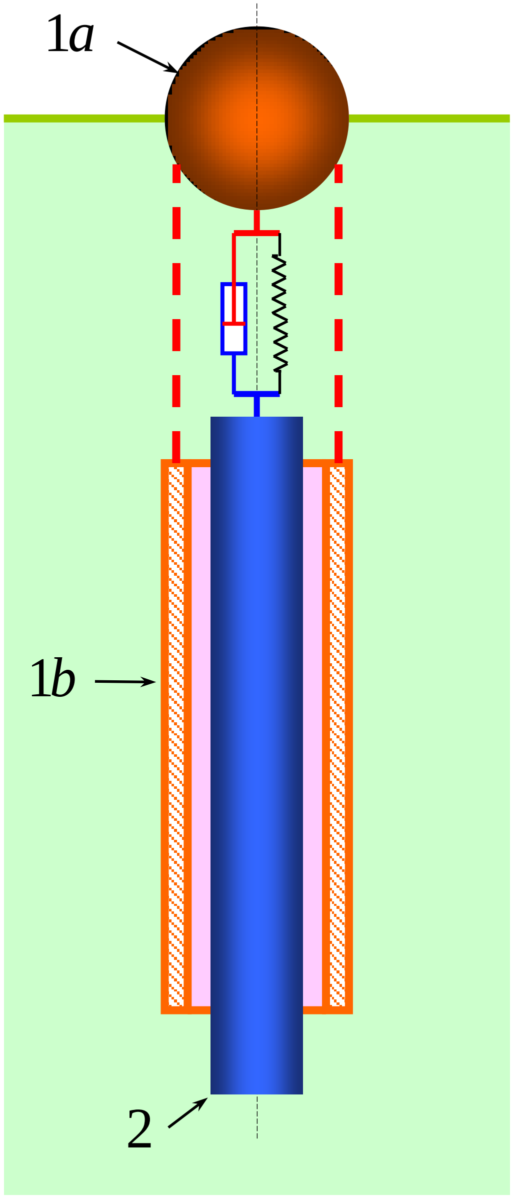
Acceleration tube represented by body 1b

Inertia of piston and enclosed water represented by body 2

Bodies 1b and 2 are "deeply" submerged:

- Wave excitation forces neglected
- Radiation forces neglected

Hydrodynamic interference between bodies 1a, 1b and 2 neglected



Two-body motion, linear PTO

Coordinates:

x : body 1 (1a+1b)

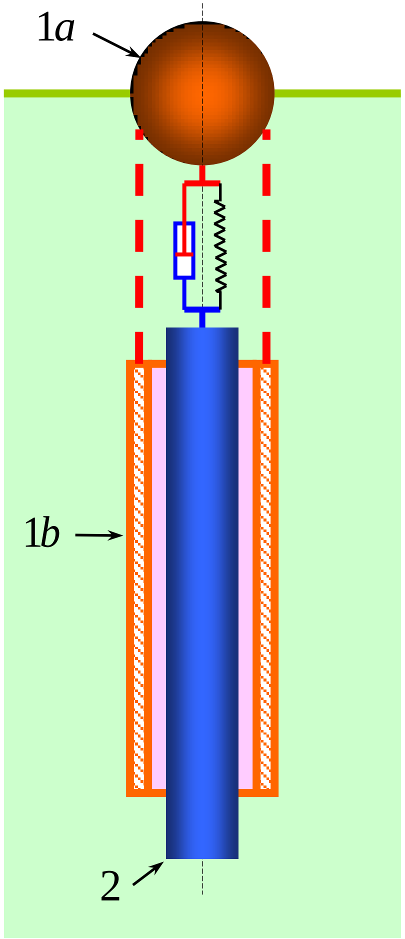
y : body 2

$M_{1b},\ M_2$ include added mass

$$(m_{1a}+A_{1a}+M_{1b})\ddot{x}+B_1\dot{x}+\rho gSx+C(\dot{x}-\dot{y})+K(x-y)=f_{d1}$$

damper spring

$$M_2\ddot{y}-C(\dot{x}-\dot{y})-K(x-y)=0$$



Regular waves, frequency domain

$$x(t) = X_0 e^{i\omega t}, \quad y(t) = Y_0 e^{i\omega t} \qquad f_{dj}(t) = A_w \Gamma_j(\omega) e^{i\omega t}$$

$$X_0 \left(-\omega^2 (m_{1a} + A_{1a}(\omega) + M_{1b}) + i\omega(B + C) + \rho g S \right)$$
$$- Y_0 (i\omega C + K) = A_w \Gamma_{1a} \,,$$

$$X_0(-i\omega C - K) + Y_0(-\omega^2 M_2 + i\omega C + K) = 0 \,.$$

Time-averaged absorbed power

$$\overline{P} = \frac{1}{2}\omega^2 C \left| X_0 - Y_0 \right|^2$$

Theoretical max power (axisymmetric body, heave motion):

$$\overline{P}_{\max} = \frac{g^3 \rho A_w^2}{4\omega^3}$$

Radius of buoy = 7.5 m

Mass of buoy $m_{1a} = 905.7\times10^{3}\,\mathrm{kg}$

Hydrostatic restoring force coeff. $\rho g S = 1.776\ \mathrm{M\,Nm^{-1}}$

Dimensionless values

Motion amplitude $X^* = X_0 / A_w$

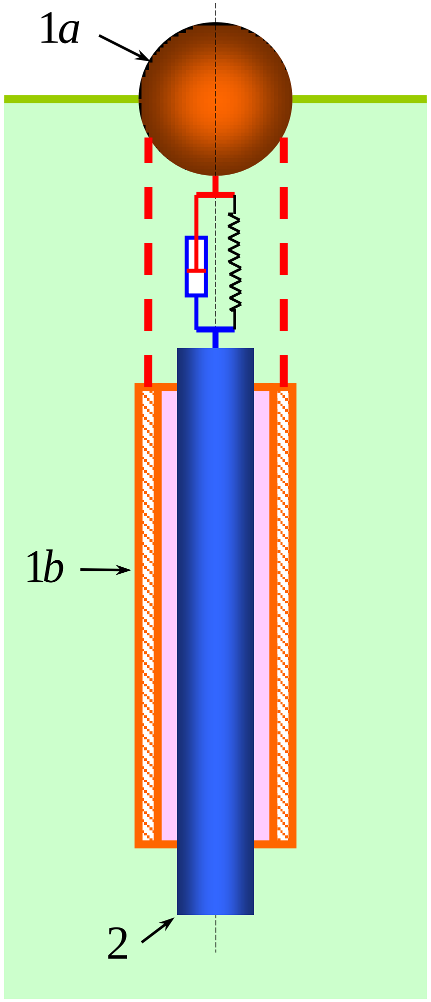
Mass of body 1b $M_{1b}^* = M_{1b} / m_{1a}$

Mass of body 2 $M_2^* = M_2 / m_{1a}$

Damping coefficient $C^* = \dfrac{C}{B(\omega_8)}$

Spring stiffness $K^* = \dfrac{K}{\rho g S}$

Power $\bar{P}^* = \dfrac{\bar{P}}{\bar{P}_{\max}}$



Results from optimization

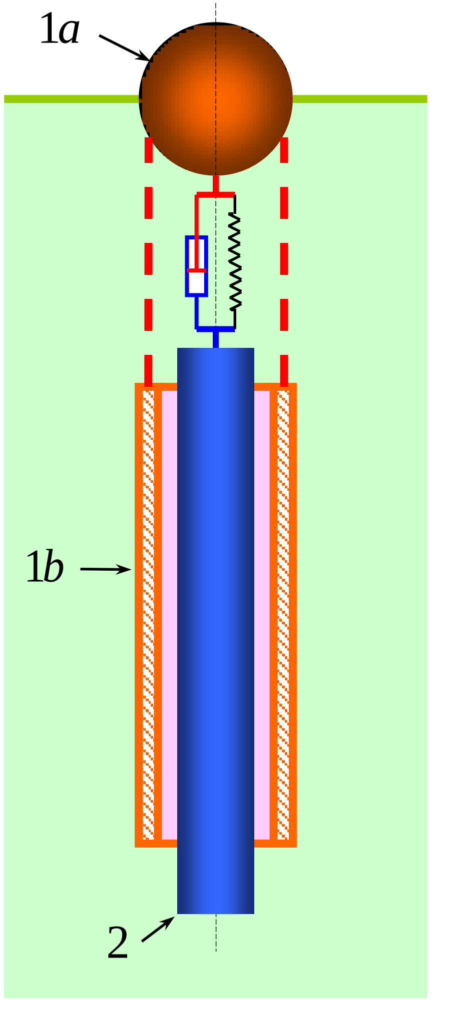
Regular waves

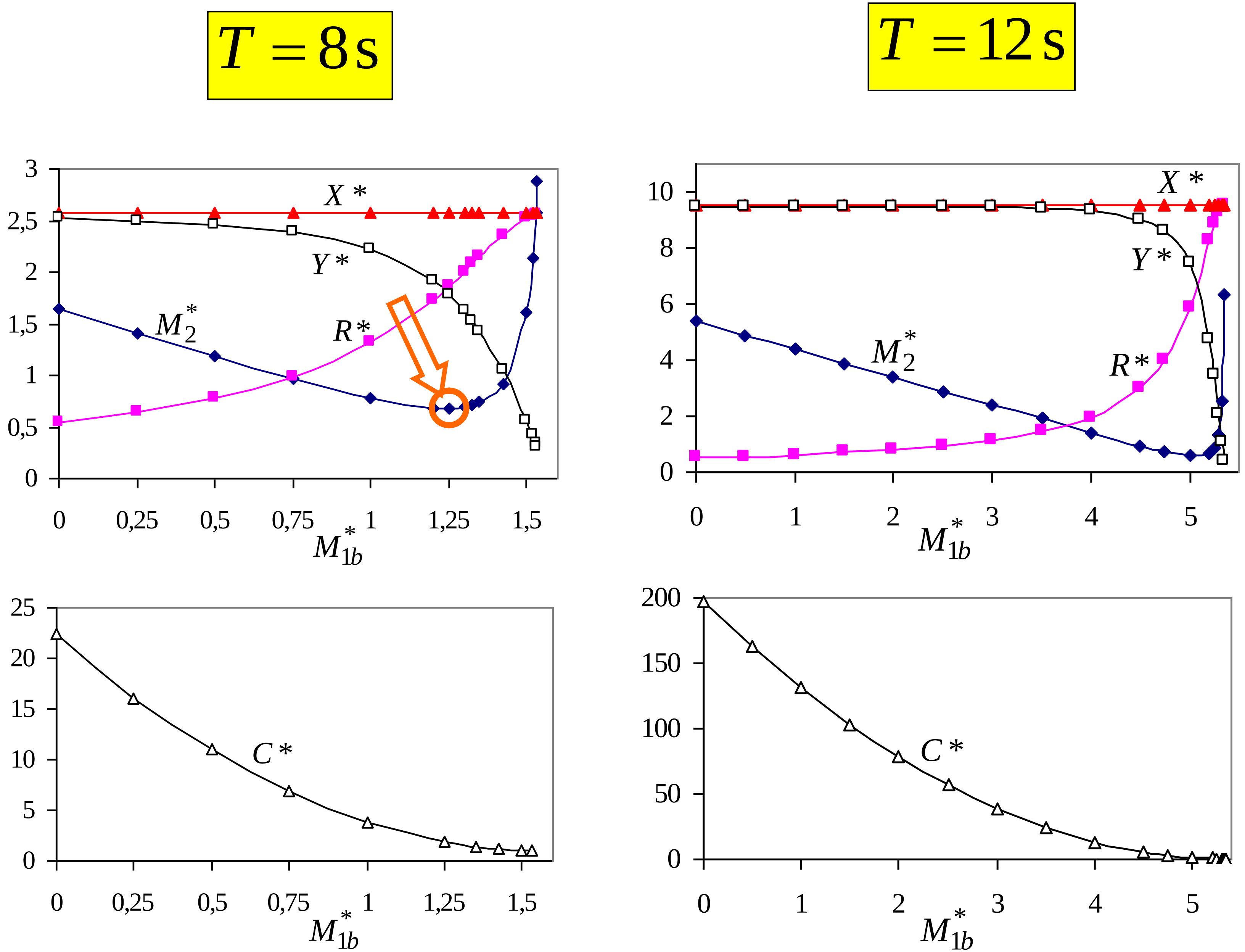
Linear PTO

No spring $K = 0$

$$\bar{P}^* = \frac{\bar{P}}{\bar{P}_{max}} = 1$$

Motion amplitude (dimensionless) $\begin{cases} X^* \\ Y^* \\ R^* \text{ (relative)} \end{cases}$





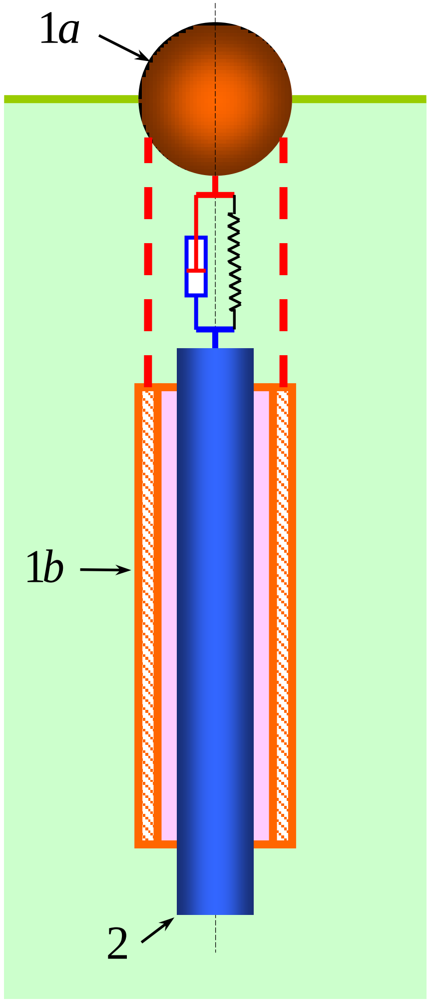
T = 8 s
T = 12 s
X *
Y *
R *
M_2^*
C *
M_{1b}^*

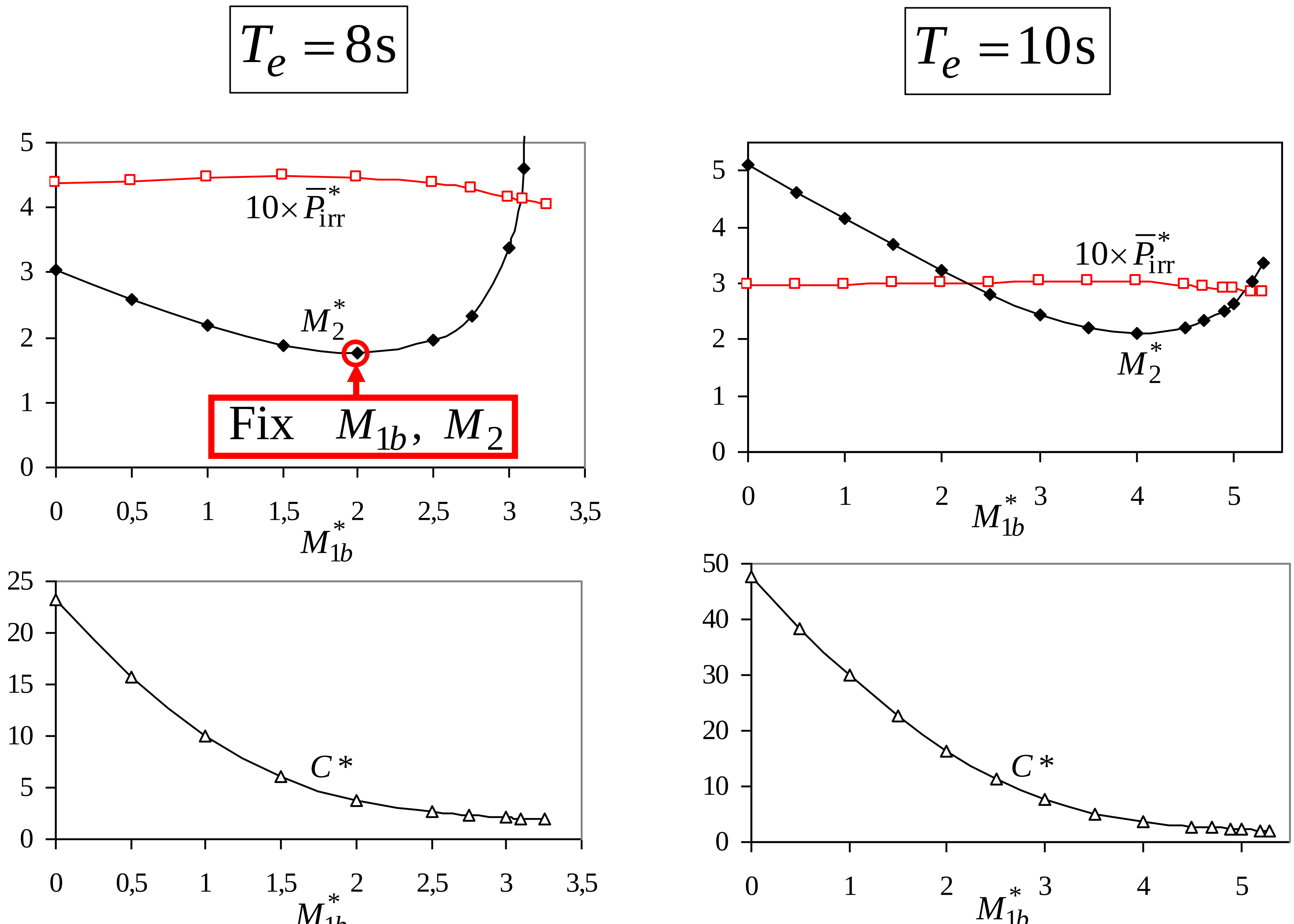
Results from optimization

Irregular waves
(Pierson-Moskowitz)

Linear PTO

No spring $K = 0$





$T_e = 8\text{s}$
$T_e = 10\text{s}$
$10\times \overline{P}^*_{\text{irr}}$
M_2^*
Fix M_{1b}, M_2
M_{1b}^*
C^*
0
0,5
1
1,5
2
2,5
3
3,5
4
5
10
15
20
25
30
40
50

Results from optimization

Reactive phase control

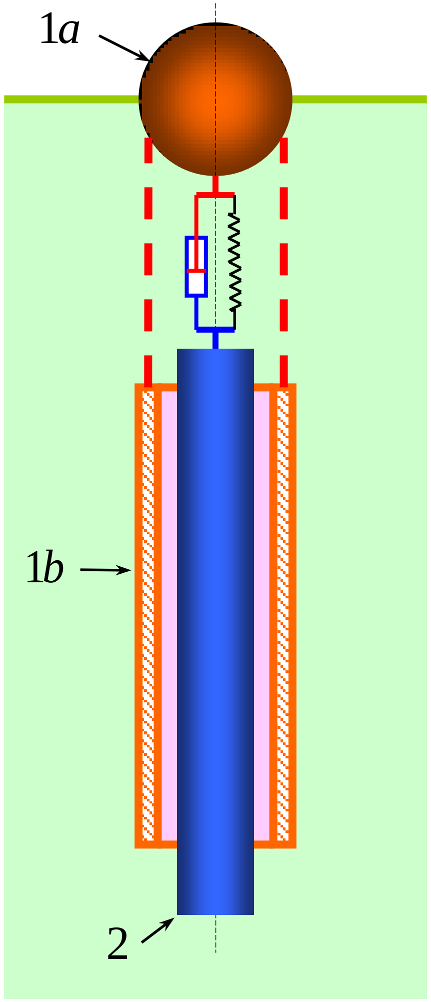
Irregular waves
(Pierson-Moskowitz)

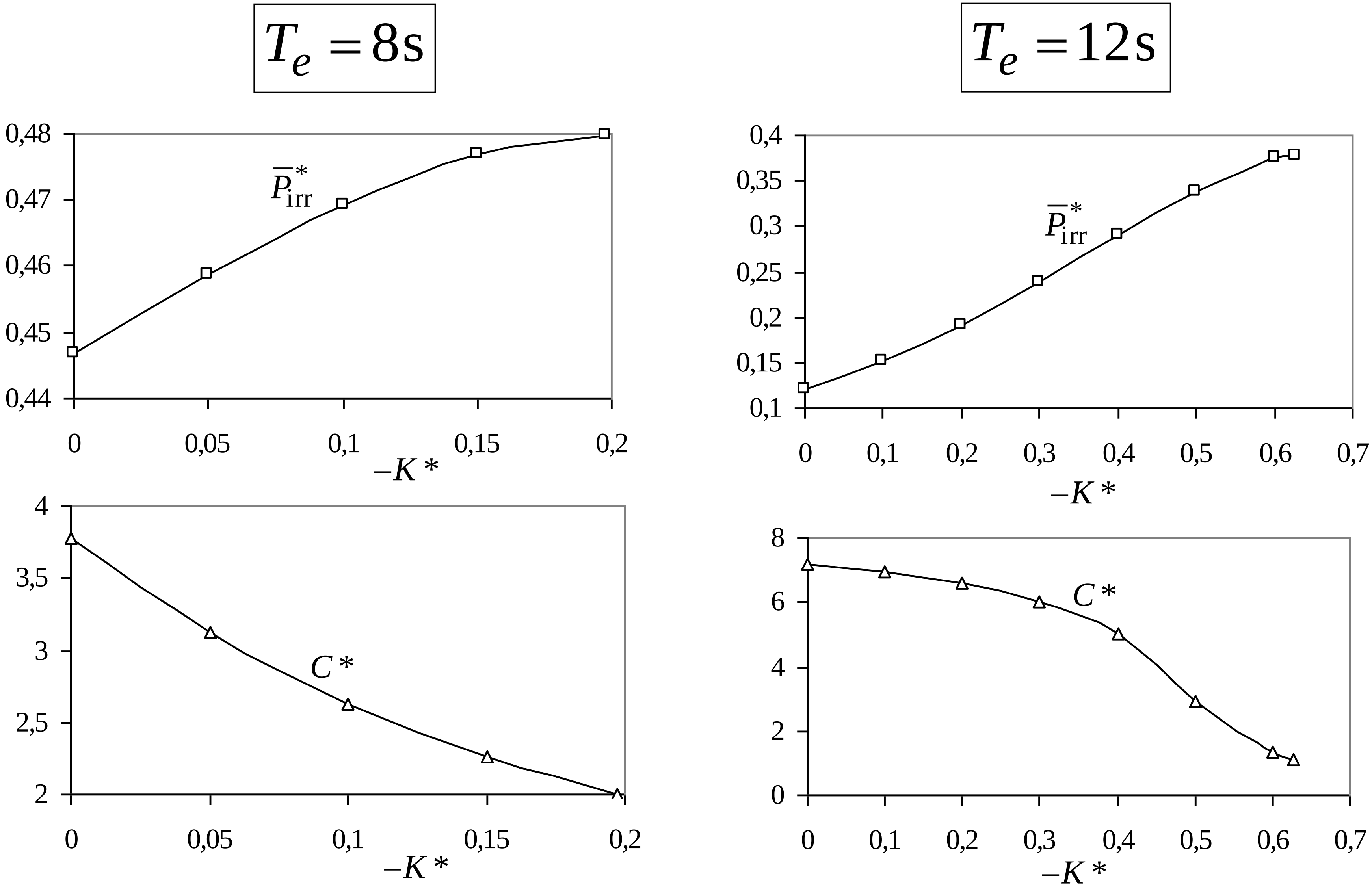
$$M_{1b}^{*} = 2$$

$$M_{2}^{*} = 1.76$$

Linear PTO

Spring $K \leq 0$





$T_e = 8s$
$T_e = 12s$
$\overline{P}^*_{irr}$
C^*
$-K^*$
0,48
0,47
0,46
0,45
0,44
0
0,05
0,1
0,15
0,2
4
3,5
3
2,5
2
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,5
0,6
0,7
8
6
4
2

Phase control by latching

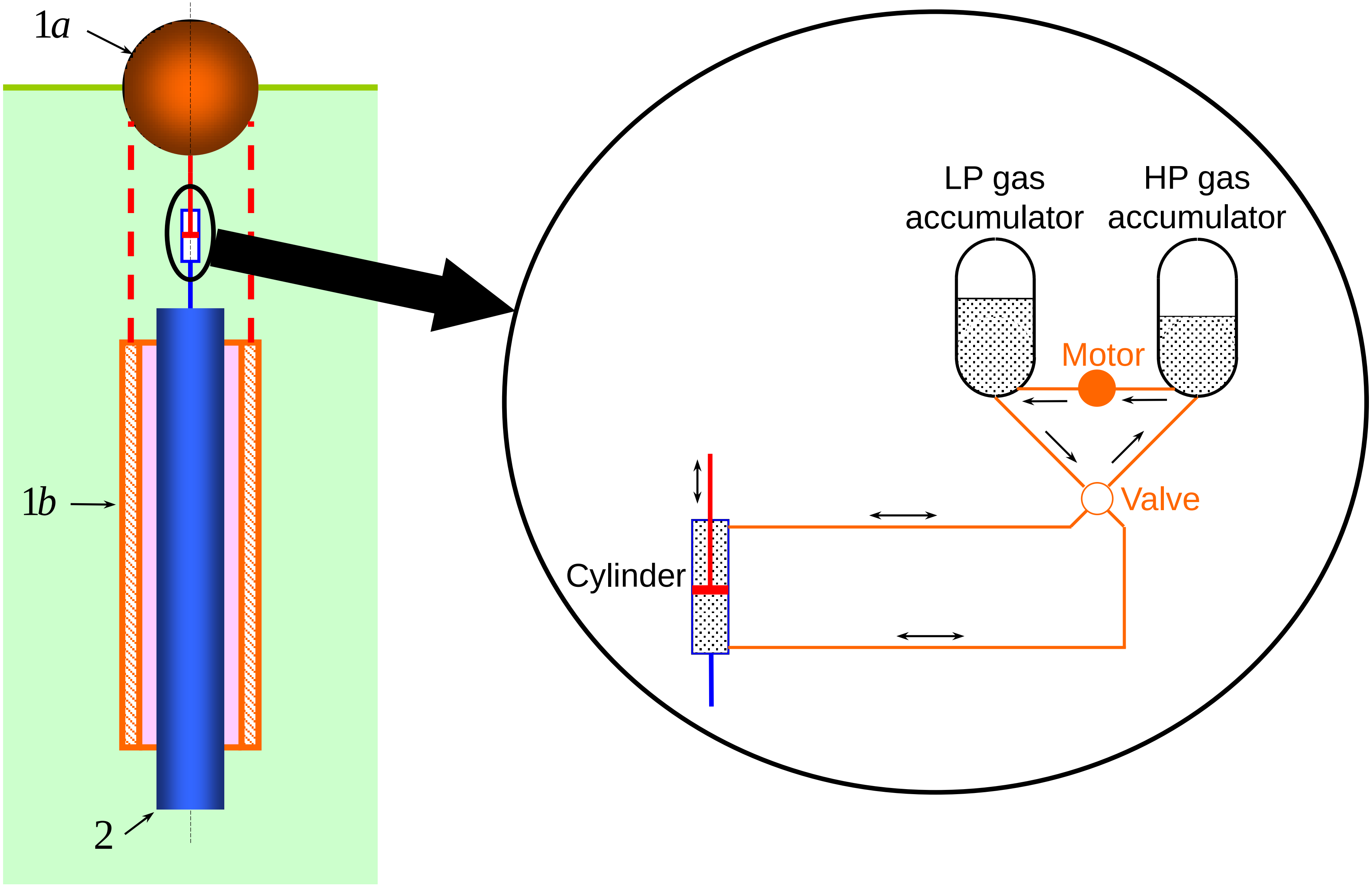
PTO: high pressure oil circuit

Hydraulic ram

Gas accumulator

Wavebob (Ireland)

Hydraulic motor



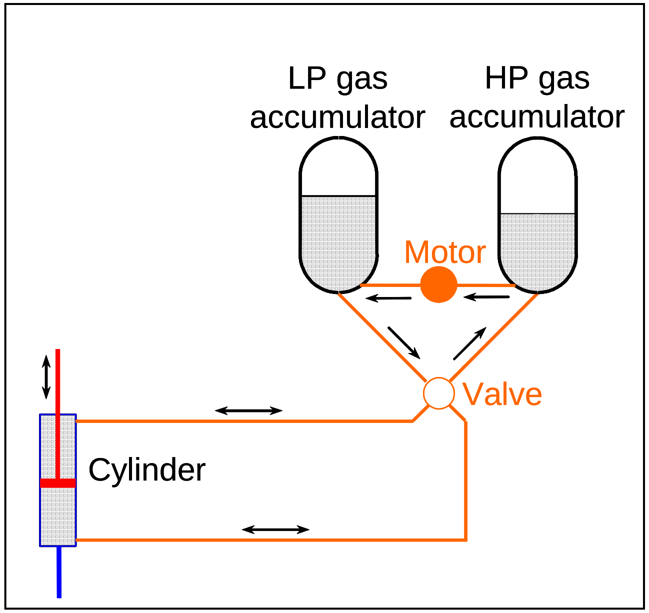
1a
1b
2
LP gas accumulator
HP gas accumulator
Motor
Valve
Cylinder

Flow rate control through motor

$$q_m(t) = G S_c^2 \Delta p$$

$$S_c = \text{piston area}$$

$$\Delta p = \text{pressure difference}$$



Phase-control by latching: body is released when

hydrodynamic force on body exceeds $R(S_c \Delta p) \quad (R > 1)$

Control parameters:

G controls oil flow rate through hydraulic motor

R controls latching (release of body)

Non-linear PTO: time-domain analysis

$$(m_{1a} + A_{1a}(\infty) + M_{1b})\ddot{x}(t) + \rho g S x(t)$$
$$+ \int_{-\infty}^{t} L(t-\tau)\ddot{x}(t)\,d\tau - f_{d1}(t) = f_m,$$

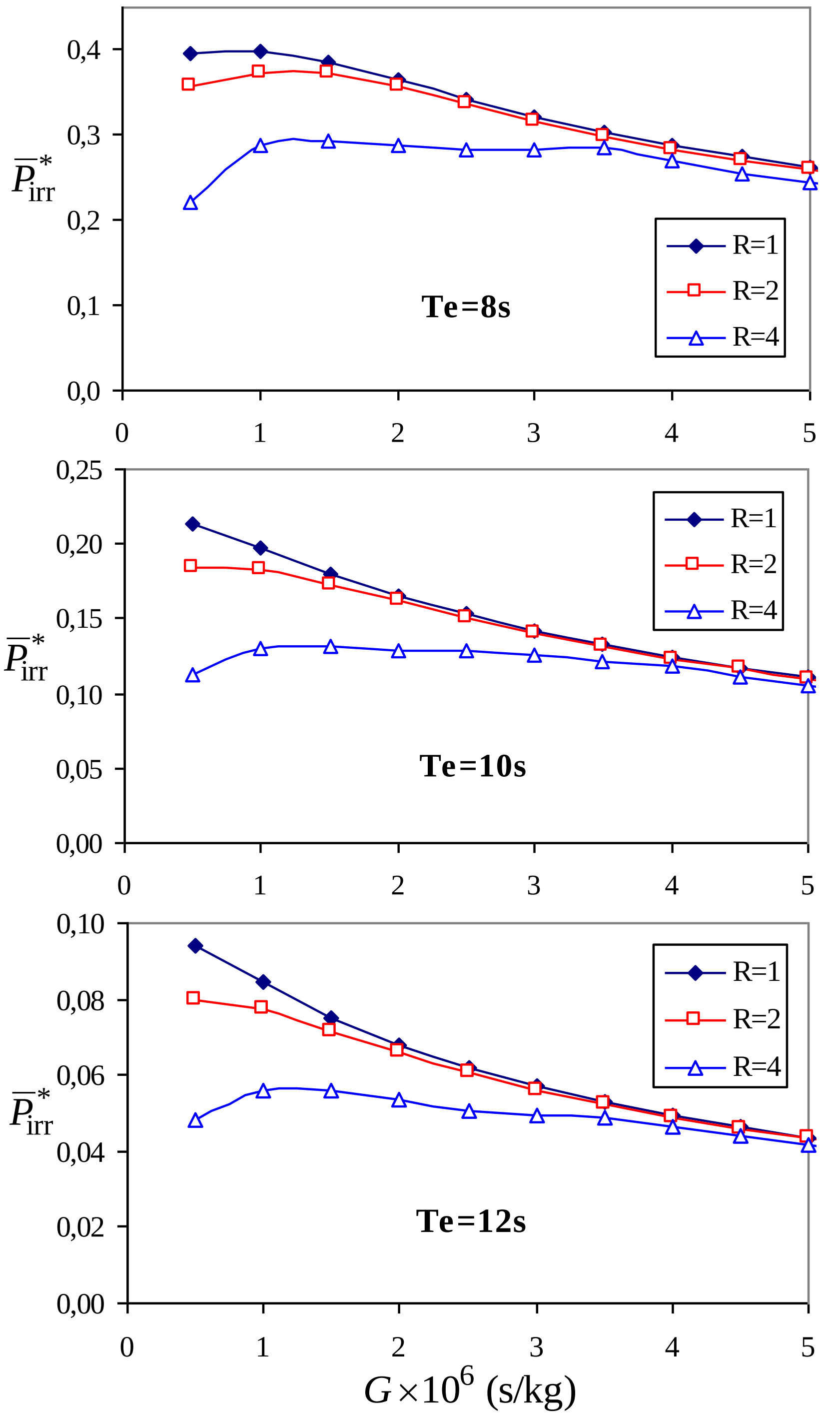
$$M_2 \ddot{y}(t) = -f_m.$$

- Continuity equation for oil-flow
- Accumulator gas thermodynamics

$a = 7.5\,\mathrm{m}$

$M_{1b}^{*} = 2$

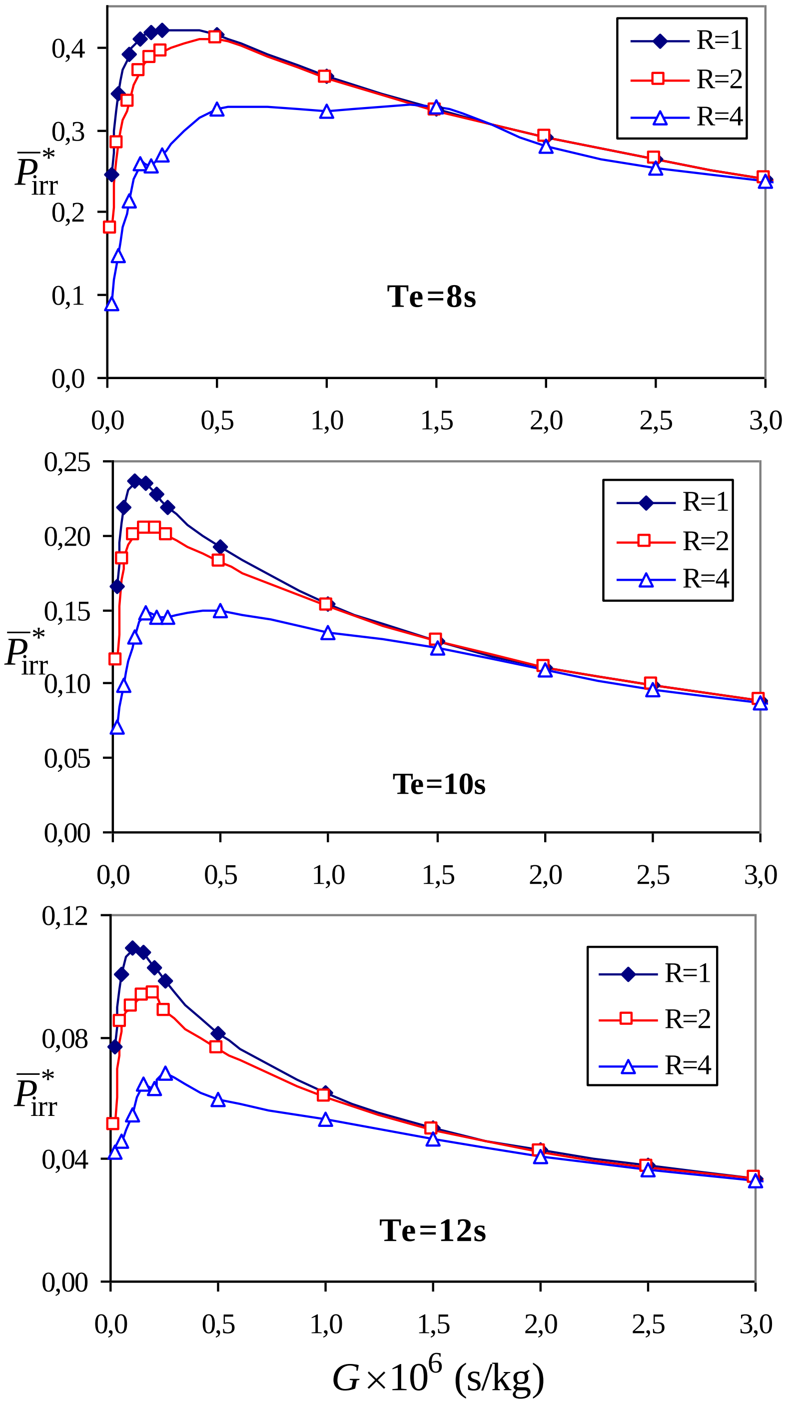
$M_{2}^{*} = 1.76$



$$a = 7.5\,\mathrm{m}$$

$$M_{1b}^{*} = 1$$

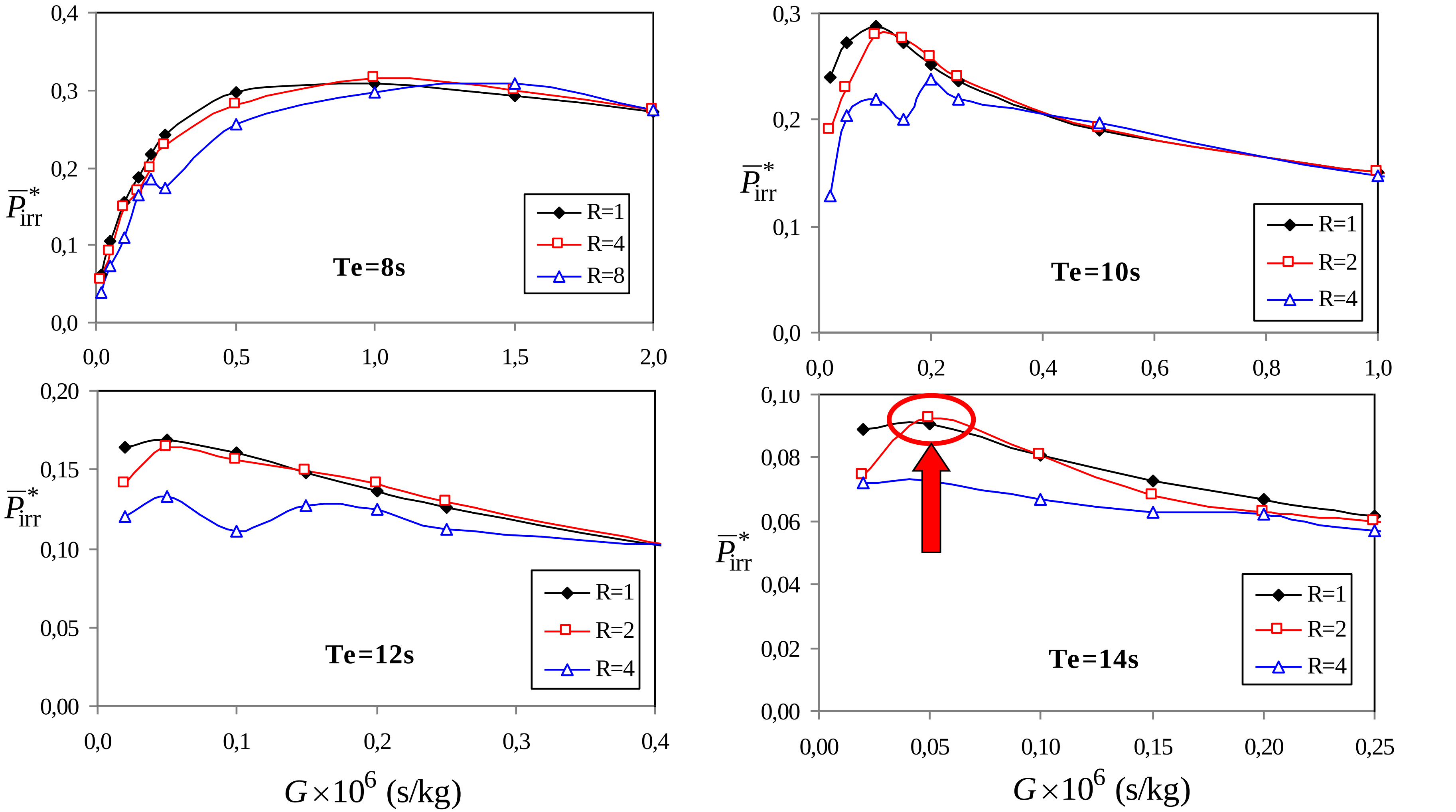
$$M_{2}^{*} = 3$$



$$a = 7.5\,\text{m}$$

$$M_{1b}^{*} = 1$$

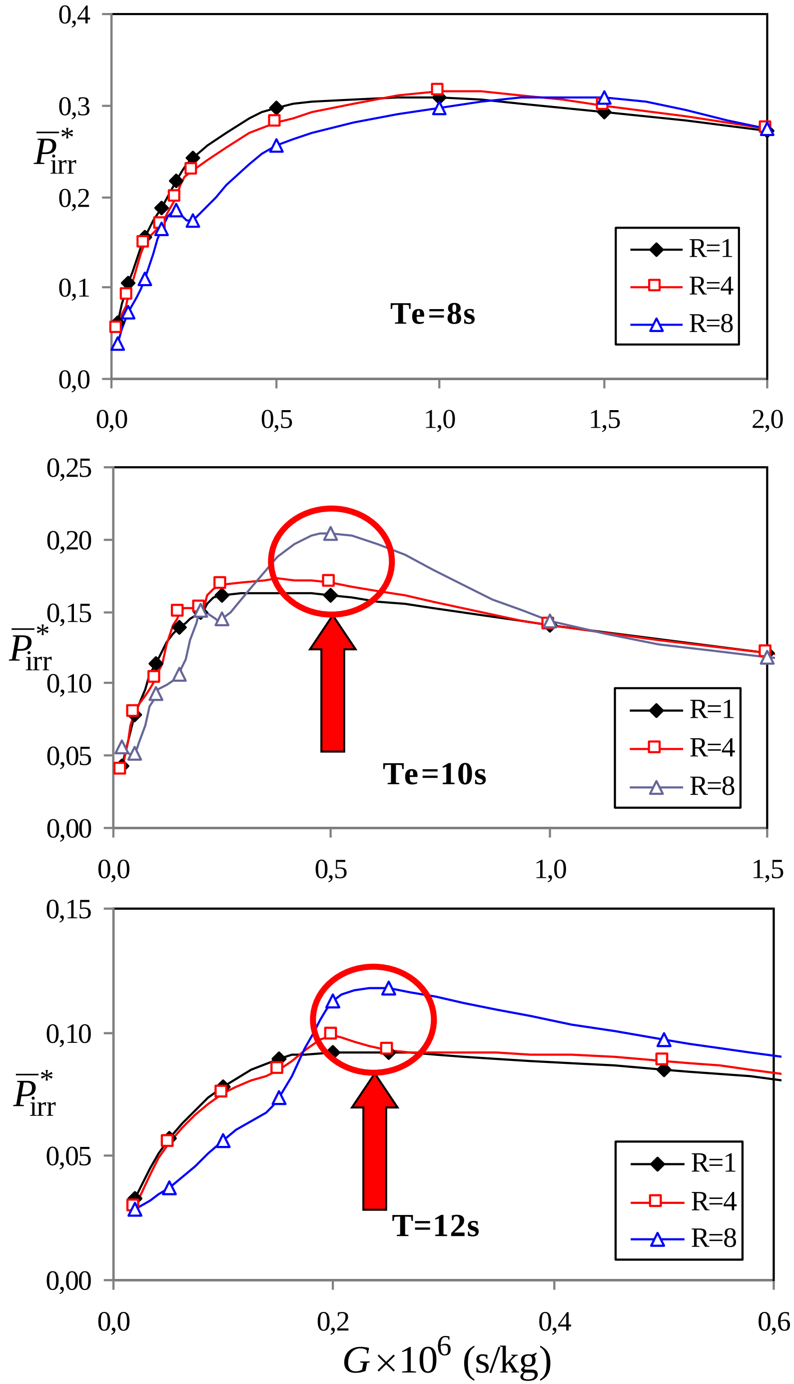
$$M_{2}^{*} = 6$$



$$a = 7.5\,\mathrm{m}$$

$$M_{1b}^{*} = 1$$

$$M_{2}^{*} = \infty$$

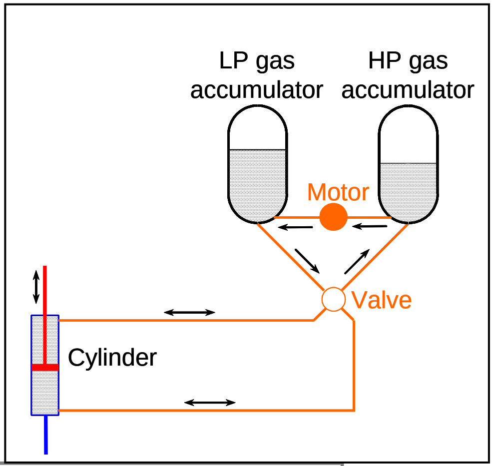


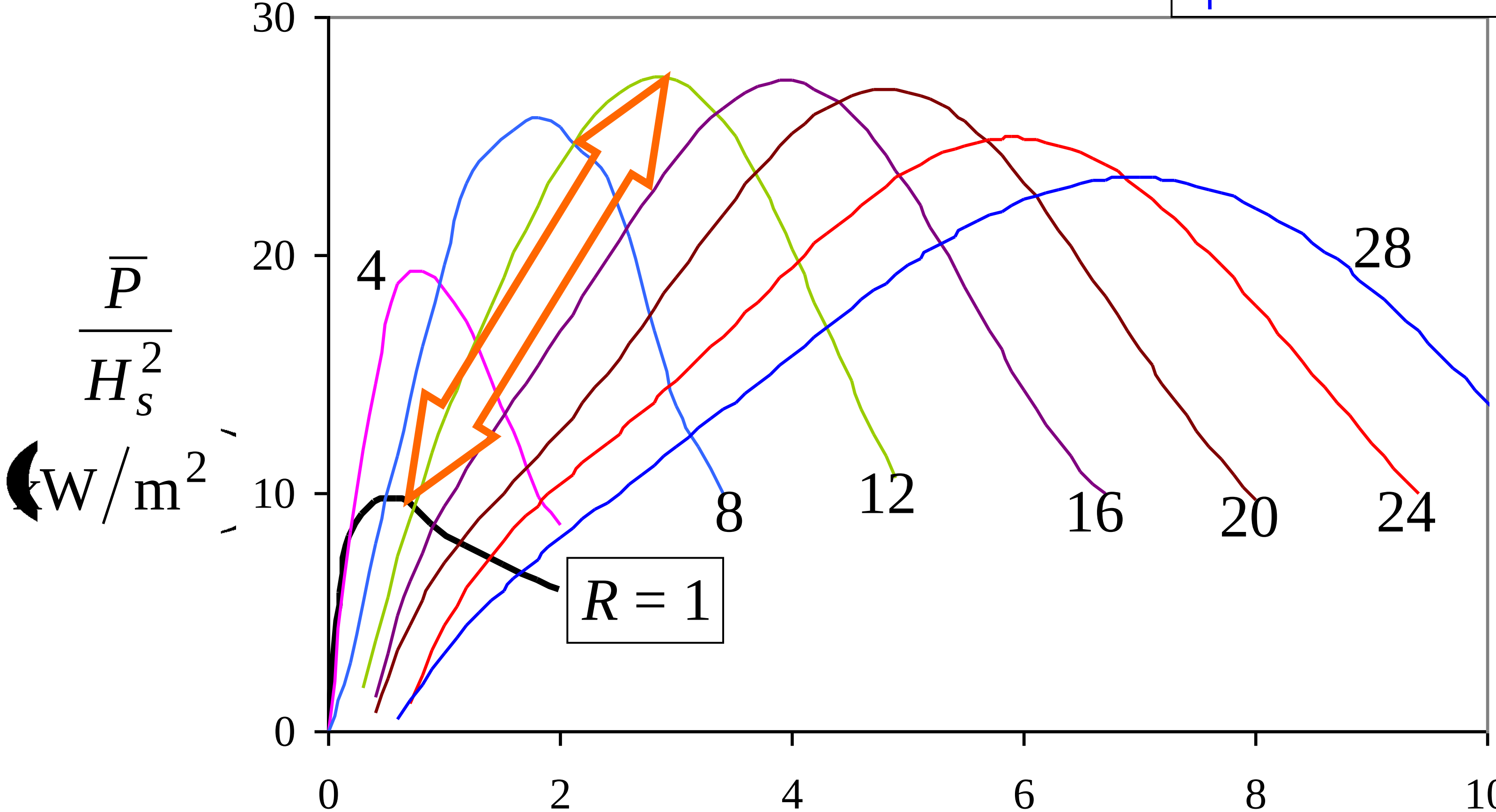
We are far from:

$$a = 5\,\text{m}$$

$$M_{1b}^{*} = 0$$

$$M_{2}^{*} = \infty$$





A. Falcão, *Ocean Engineering*, **35**, 358-366, 2008.

CONCLUSIONS

• A two-body system with a **linear damper** can be optimized to absorb theoretical maximum energy from **regular waves**: $\overline{P}* = 1$.

• This drops to typically less than 50% in **irregular waves**.

• For fixed masses, a linear PTO with a "**negative spring**" **(reactive control)** can significantly increase the energy absorbed from irregular waves.

• Simulations were made for **high-pressure-oil PTO**.

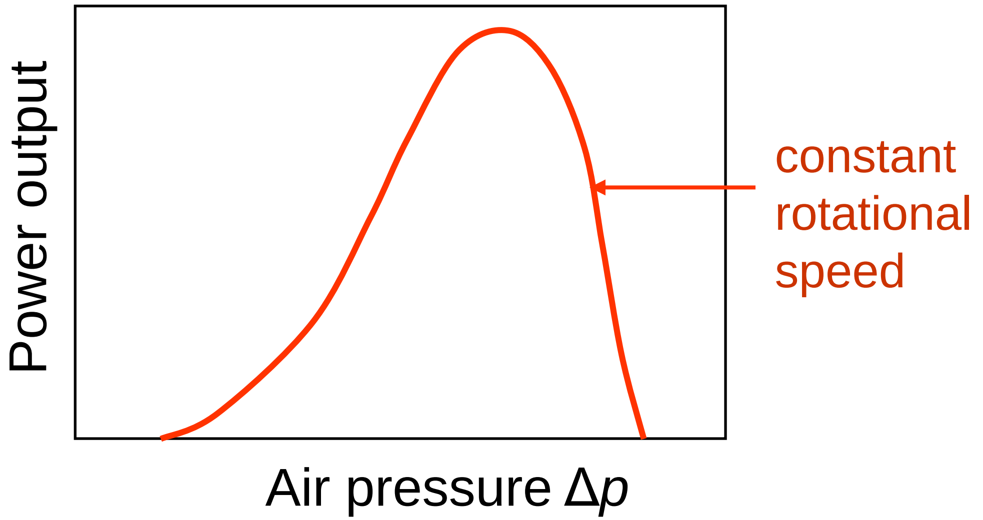
• The performance is slightly poorer than with a linear damper.

• In the simulated situations, **latching was unable to improve the performance**, except if mass of body 2 is very large (approaching a single-body system).

OSCILLATING WATER COLUMNS

The problem:

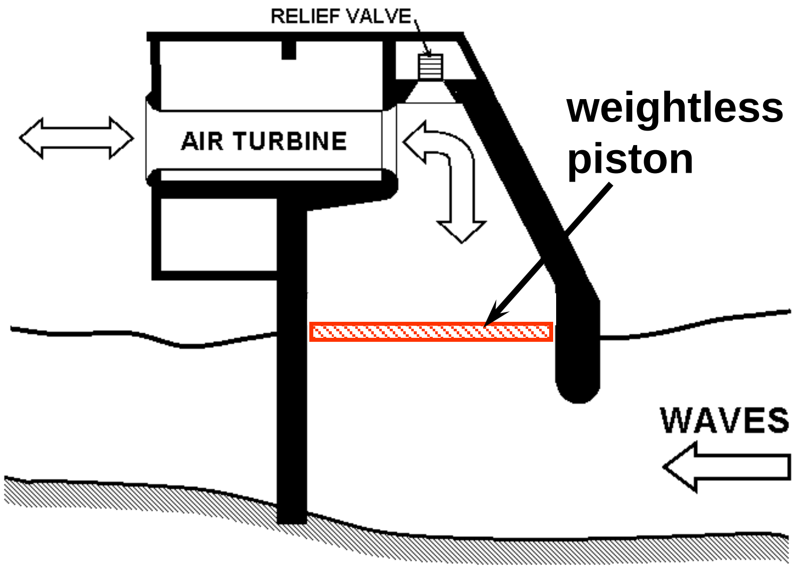
- **The performance of self-rectifying air turbines (Wells, impulse, …) is strongly dependent on pressure (or on flow rate) and on rotational speed.**



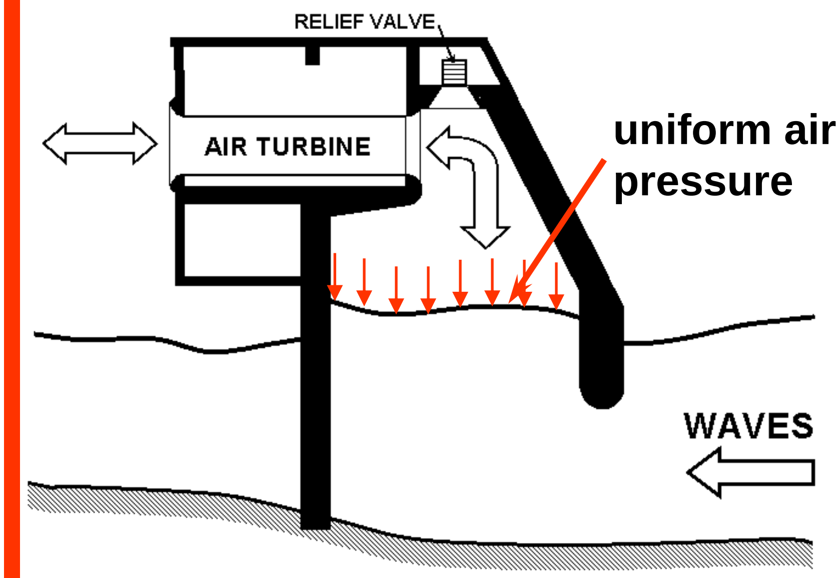
- **How to control the turbine (instantaneous rotational speed) to achieve maximum energy production ?**

OWC Dynamics

Two different approaches to modelling:

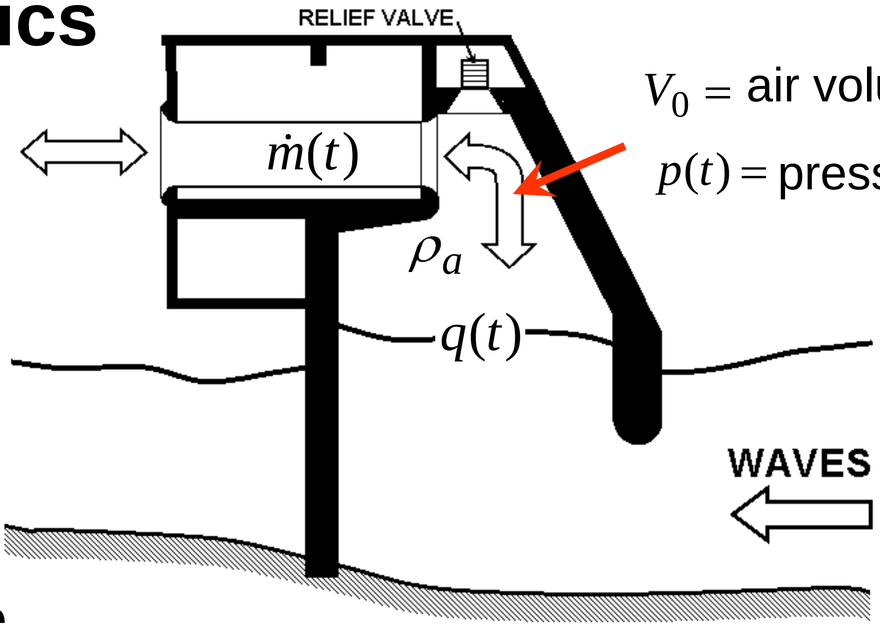


Oscillating body (piston) model
(rigid free surface)



Uniform pressure model
(deformable free surface)

OWC Dynamics



$V_0 =$ air volume

$p(t) =$ pressure

$q(t) =$ **volume-flow rate displace by free-surface**

$\dot{m}(t) =$ **mass-flow rate of air through turbine**

$\rho_a =$ **air density** $p(t) =$ **air pressure**

Conservation of air mass (linearized) ⇨

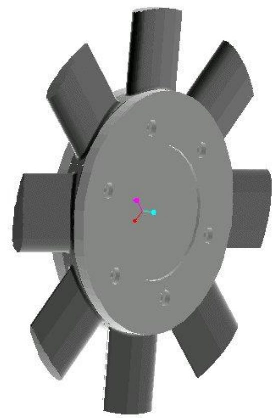
$$\frac{\dot{m}(t)}{\rho_a} = q(t) - \frac{V_0}{\rho_a c_a^2} \frac{dp(t)}{dt}$$

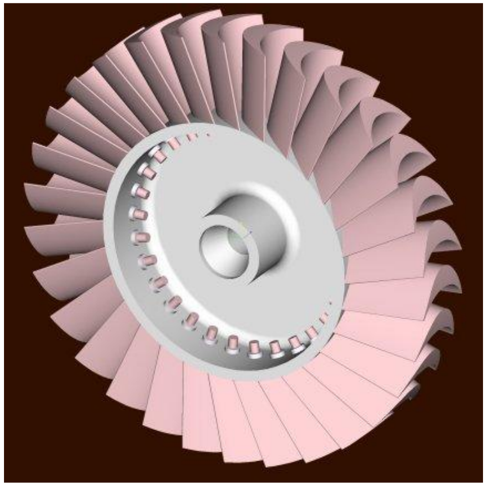
$$\text{flow rate } \; q = \begin{cases} q_{\text{exc}} & \text{excitation} \\ q_r & \text{radiation} \end{cases}$$

Effect of air compressibility

OWC Dynamics

Air turbine $\begin{cases} N = \text{rotational speed} \\ D = \text{rotor diameter} \\ P_t = \text{power output} \\ p = \text{pressure head} \end{cases}$





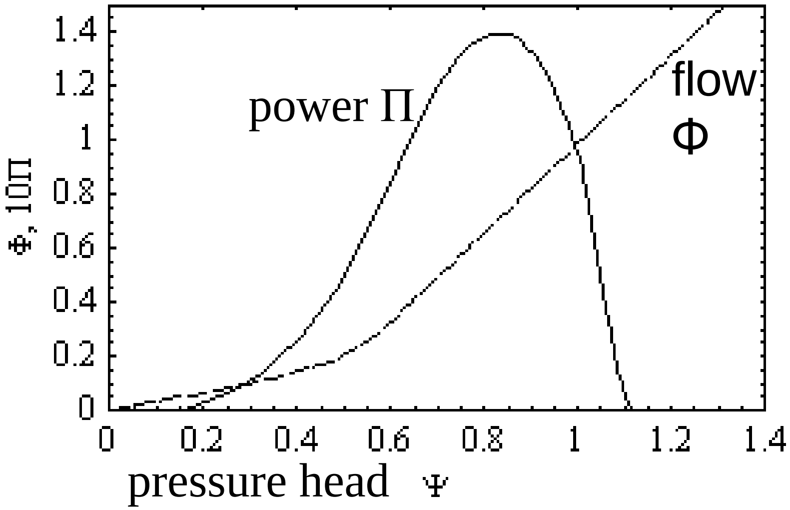
In dimensionless form:

$$\Phi = \frac{\dot{m}}{\rho_a N D^3} \qquad \Psi = \frac{p}{\rho_a N^2 D^2} \qquad \Pi = \frac{P_t}{\rho_a N^3 D^5}$$

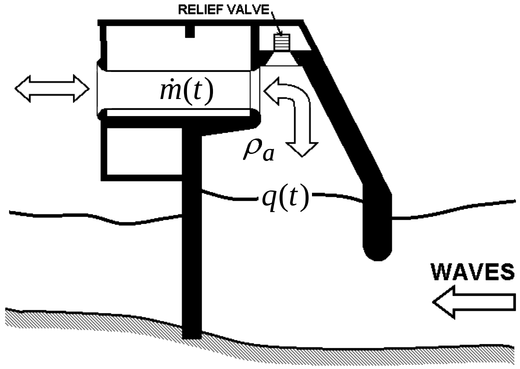
flow — pressure head — power

Performance curves of turbine (dimensionless form):

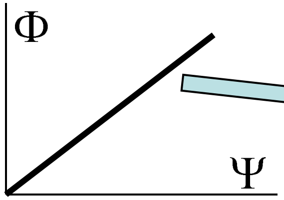
$$\Phi = f_w(\Psi), \qquad \Pi = f_P(\Psi)$$



OWC Dynamics



Frequency domain



Linear air turbine $\Phi = K\Psi$

$$\{p(t), \dot{m}, q(t), q_r(t), q_{exc}(t)\} = \{P, \dot{M}, Q, Q_r, Q_{exc}\}\, e^{i\omega t}$$

$$\frac{Q_r(\omega)}{P(\omega)} = -B(\omega) - iC(\omega) \begin{cases} B = \text{radiation conductance} \\ C = \text{radiation susceptance} \end{cases}$$

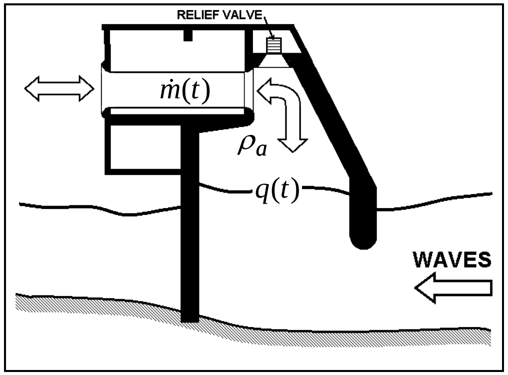
$$|Q_{exc}(\omega)| = \Gamma(\omega)\, A_w$$

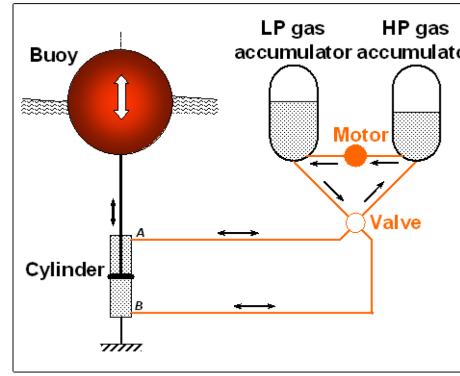
excitation coeff. (Γ)

wave ampl. (A_w)

$$P = \frac{Q_{exc}}{\dfrac{KD}{\rho_a N} + B + i\left(C + \dfrac{\omega V_0}{\rho_a c_a^2}\right)}$$

OWC Dynamics



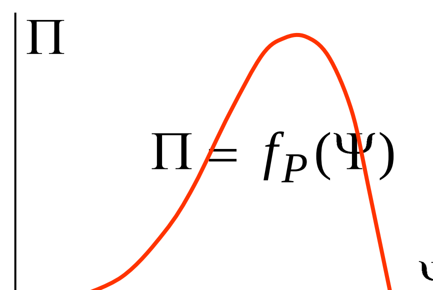


$$P = \frac{Q_{\text{exc}}}{\frac{KD}{\rho_a N} + B + i\left(C + \frac{\omega V_0}{\rho_a c_a^2}\right)}$$

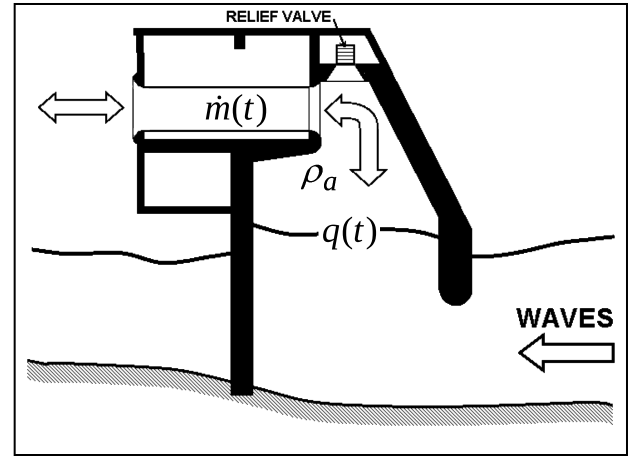
$$X_0 = \frac{F_d}{-\omega^2(m+A) + i\omega(B+C) + \rho g S + K}$$

$$p(t) = \mathrm{Re}\left(P e^{i\omega t} \right)$$

power output : $$\Pi(t) = \frac{P_t(t)}{\rho_a N^3 D^5} = f_P\left(\overbrace{\frac{p(t)}{\rho_a N^2 D^2}}^{\Psi}\right)$$



OWC Dynamics

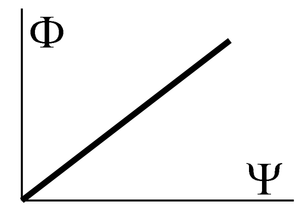


Time domain:

- **Linear or non-linear turbine**

$$\frac{V_0}{\rho_a c_a^2}\frac{dp(t)}{dt} + \frac{\dot{m}(t)}{\rho_a} - \int_{-\infty}^{t} g_r(t-\tau)\, p(\tau)\, d\tau = q_{\text{exc}}(t)$$

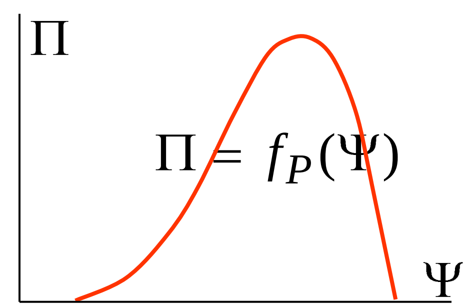
turbine flow vs pressure curve $\dot{m} = \rho_a N D^3 f_w\left(\dfrac{p}{\rho_a N^2 D^2}\right)$



memory function $g_r(t) = -\dfrac{2}{\pi}\displaystyle\int_0^{\infty} B(\omega)\cos\omega t\, d\omega$

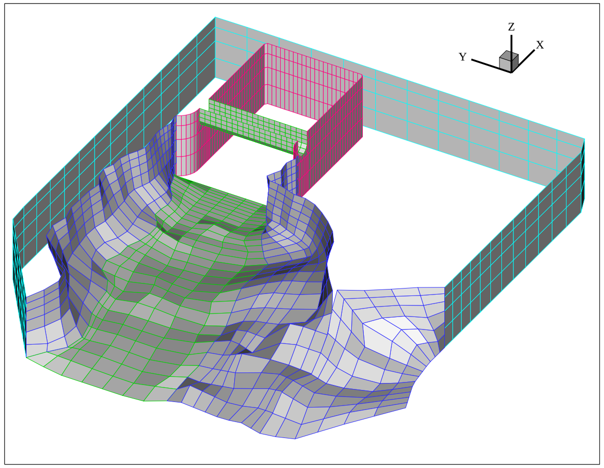
To be integrated numerically for $p(t)$

power output : $\Pi(t) = \dfrac{P_t(t)}{\rho_a N^3 D^5} = f_P\left(\dfrac{p(t)}{\rho_a N^2 D^2}\right)$

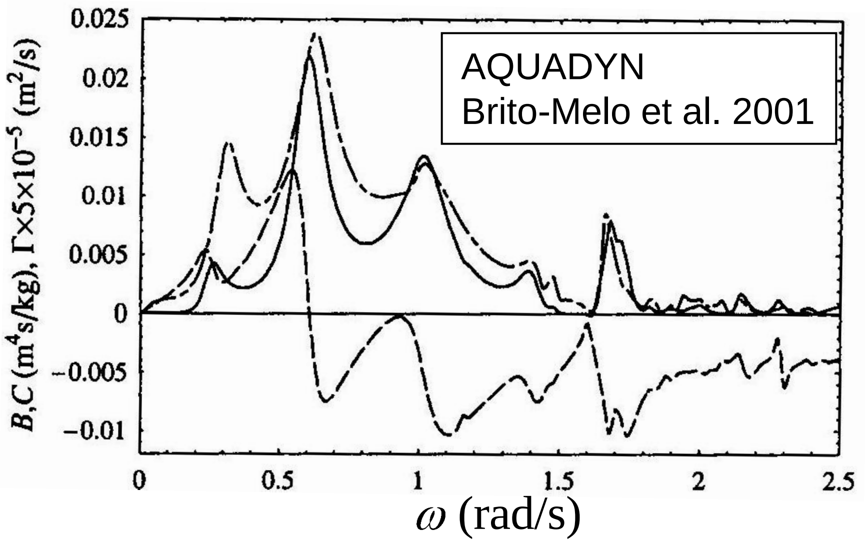


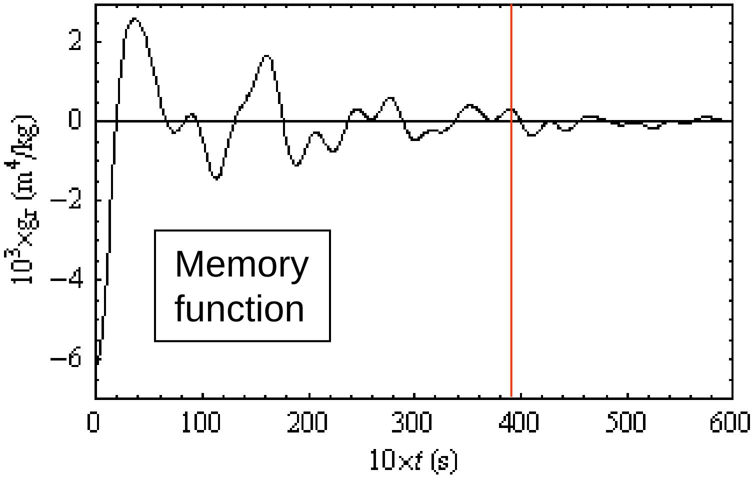
OWC Dynamics

Numerical application



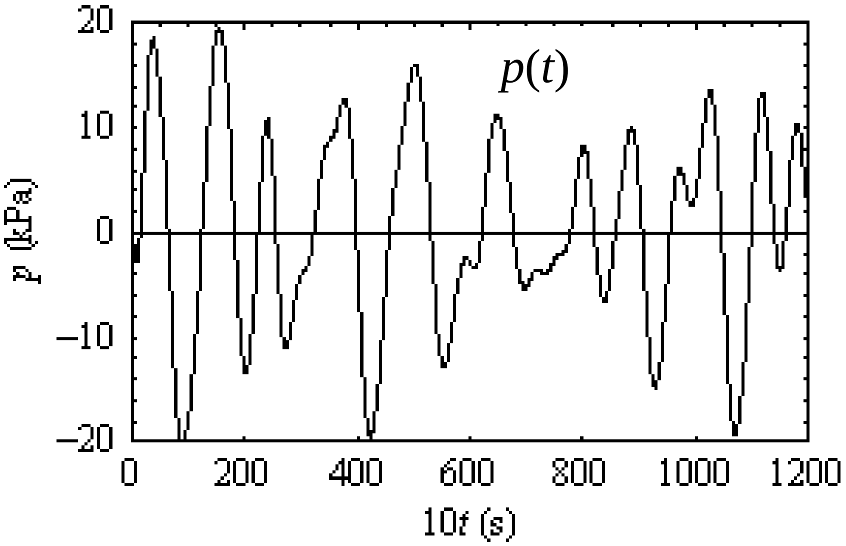
Pico OWC



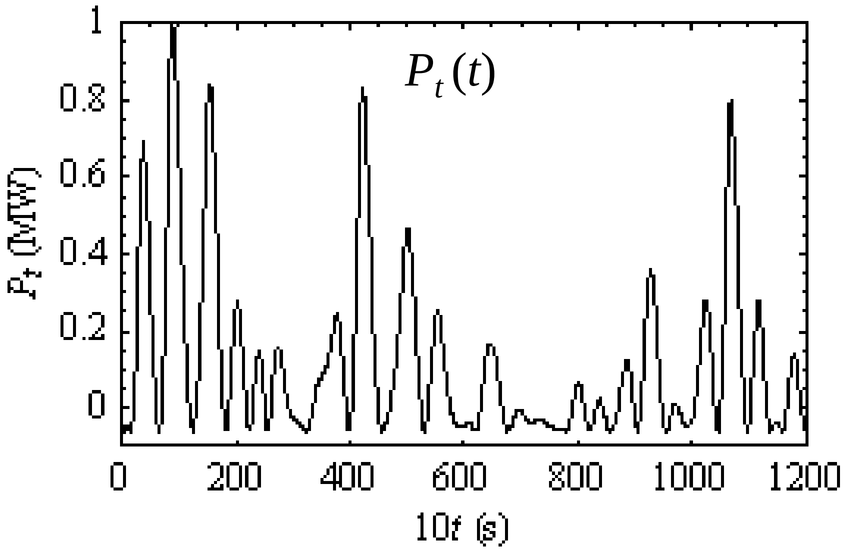


OWC Dynamics

Numerical application



Air pressure in chamber



Power

Results from time-domain modelling of <u>impulse turbine</u> over $\Delta t = 120$ s

- Turbine $D = 1.5$ m, $N = 115$ rad/s (1100 rpm)
- Sea state $H_s = 3$ m, $T_e = 11$ s
- Average power output from turbine 97.2 kW

OWC Dynamics

Stochastic modelling

- **Irregular waves**
- **Linear air-turbine**

- Much less time-consuming than time-domain analysis
- Appropriate for optimization studies

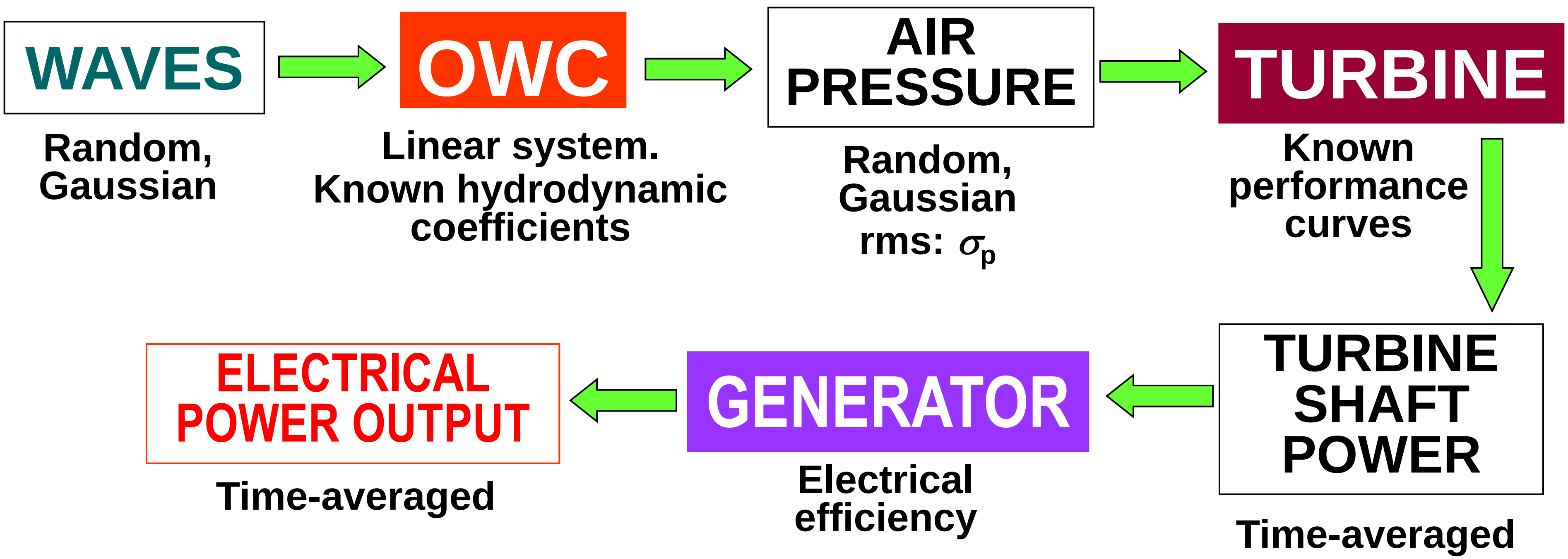
- **A.F. de O. Falcão, R.J.A. Rodrigues, "Stochastic modelling of OWC wave power performance", *Applied Ocean Research*, Vol. 24, pp. 59-71, 2002.**
- **A.F. de O. Falcão, "Control of an oscillating water column wave power plant for maximum energy production", *Applied Ocean Research*, Vol. 24, pp. 73-82, 2002.**
- **A.F. de O. Falcão, "Stochastic modelling in wave power-equipment optimization: maximum energy production versus maximum profit". *Ocean Engineering*, Vol. 31, pp. 1407-1421, 2004.**

OWC Dynamics

Stochastic modelling

Wave climate represented by a set of sea states

- For each sea state: H_s, T_e, freq. of occurrence ϕ.
- Incident wave is random, Gaussian, with known frequency spectrum.



Gaussian process (e.g. surface elevation ζ)

Probability density function (pdf) :

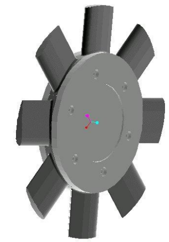
$$f(\zeta) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left(-\frac{\zeta^2}{2\sigma_\zeta^2} \right)$$

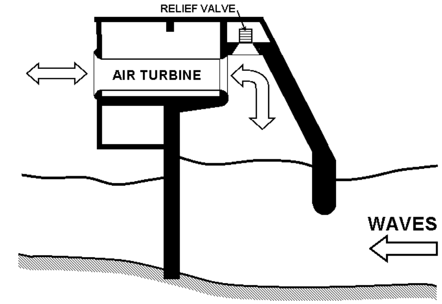
$$\sigma_\zeta^2 = \int_{-\infty}^{\infty} S_\zeta(\omega) d\omega$$ = variance

spectral density

σ_ζ = standard deviation

OWC Dynamics





Stochastic model:

- Linear turbine (Wells turbine)
- Random Gaussian waves

Pierson-Moskowitz spectrum $S_\zeta(\omega) = 263 H_s^2 T_e^{-4} \omega^{-5} \exp(-1054 T_e^{-4} \omega^{-4}).$

For linear system, $p(t)$ is random Gaussian, with variance

$$\sigma_p^2 = \int_0^\infty S_\zeta(\omega) |\Gamma(\omega)\Lambda(\omega)|^2 d\omega \quad \text{where } \Lambda = \left[\left(\frac{KD}{\rho_a N} + B \right) + i \left(\frac{\omega V_0}{\rho_a c_a^2} + C \right) \right]^{-1}$$

$$|Q_{exc}(\omega)| = \Gamma(\omega) A_w$$

excitation coeff.; wave ampl.

and pdf

$$f(p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{p^2}{2\sigma_p^2} \right)$$

$$\overline{P_t} = \int_{-\infty}^{\infty} f(p) P_t(p)\, dp = \frac{2\rho_a N^3 D^5}{\sqrt{2\pi}\sigma_p} \int_0^\infty \exp\left(-\frac{p^2}{2\sigma_p^2} \right) f_P\left(\frac{p}{\rho_a N^2 D^2} \right) dp$$

Time-averaged turbine power output :

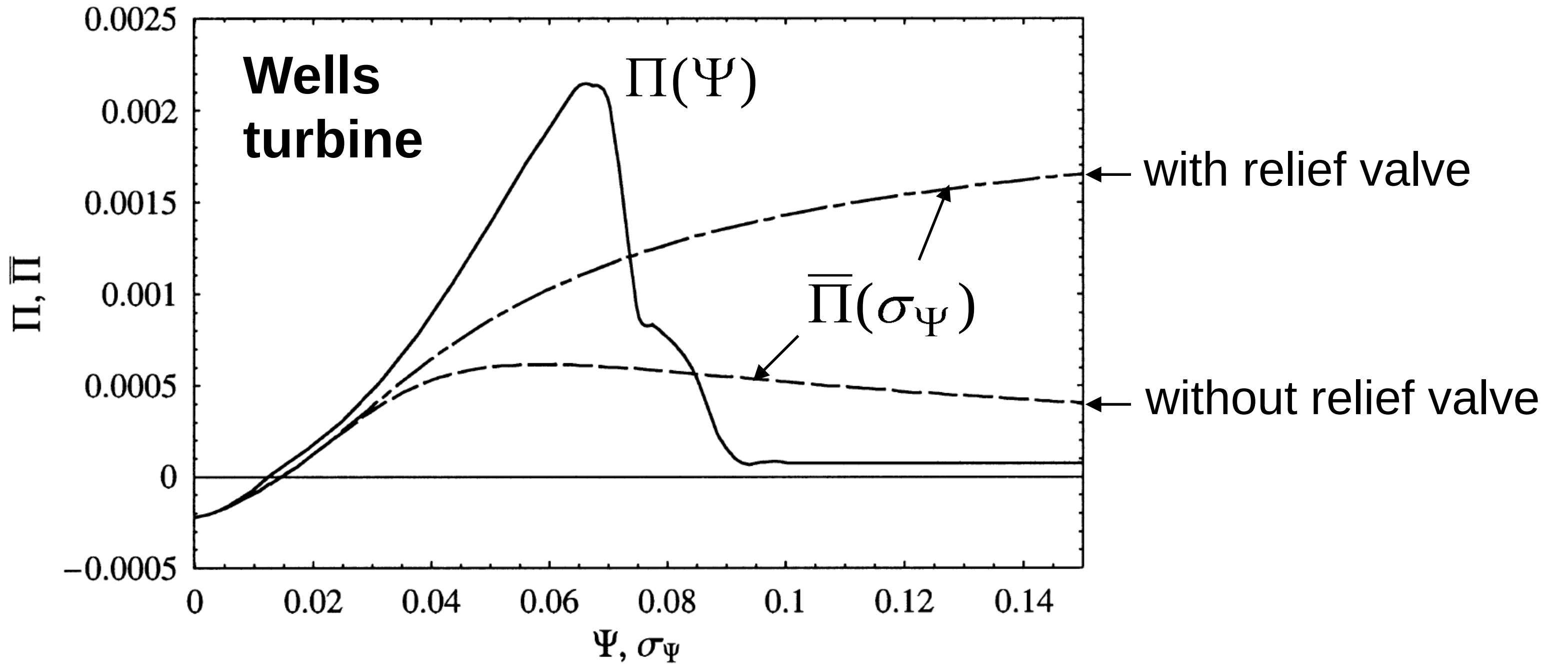
$$\bar{P}_t = \frac{\rho_a N^3 D^5}{\sqrt{2\pi}\sigma_p} \int_{-\infty}^{\infty} \exp\left(-\frac{p^2}{2\sigma_p^2}\right) f_P\left(\frac{p}{\rho_a N^2 D^2}\right) dp$$

In dimensionless form :

$$\bar{\Pi} = \frac{1}{\sqrt{2\pi}\sigma_\Psi} \int_{-\infty}^{\infty} \exp\left(-\frac{\Psi^2}{2\sigma_\Psi^2}\right) f_P(\Psi) d\Psi$$

$$\sigma_\Psi = \frac{\sigma_p}{\rho_a N^2 D^2}$$ dimensionless pressure variance

$$\bar{\Pi} = \frac{\bar{P}_t}{\rho_a N^3 D^5}$$ dimensionless time-averaged power



Time-averaged turbine power output :

$$\bar{P}_{\mathrm{t}} = \frac{\rho_{\mathrm{a}} N^3 D^5}{\sqrt{2\pi}\sigma_p} \int_{-\infty}^{\infty} \exp\left(-\frac{p^2}{2\sigma_p^2}\right) f_P\left(\frac{p}{\rho_{\mathrm{a}} N^2 D^2}\right) \mathrm{d}p$$

In dimensionless form :

$$\bar{\Pi} = \frac{1}{\sqrt{2\pi}\sigma_\Psi} \int_{-\infty}^{\infty} \exp\left(-\frac{\Psi^2}{2\sigma_\Psi^2}\right) f_P(\Psi) \mathrm{d}\Psi$$

$$\sigma_\Psi = \frac{\sigma_p}{\rho_{\mathrm{a}} N^2 D^2}$$ dimensionless pressure variance

$$\bar{\Pi} = \frac{\bar{P}_{\mathrm{t}}}{\rho_{\mathrm{a}} N^3 D^5}$$ dimensionless averaged power

How to control the rotational speed N for maximum $\bar{P}_{\mathrm{t}}$?

$$\left.\begin{aligned} \sigma_\Psi &= \frac{\sigma_p}{\rho_{\mathrm{a}} N^2 D^2} \\ \bar{\Pi} &= \frac{\bar{P}_{\mathrm{t}}}{\rho_{\mathrm{a}} N^3 D^5} \end{aligned}\right\} \Rightarrow \frac{1}{D^3 \sigma_p} \frac{\mathrm{d}\bar{P}_{\mathrm{t}}}{\mathrm{d}N} = \frac{3\bar{\Pi}}{\sigma_\Psi} + \left(\frac{N}{\sigma_p}\frac{\mathrm{d}\sigma_p}{\mathrm{d}N} - 2\right)\frac{\mathrm{d}\bar{\Pi}}{\mathrm{d}\sigma_\Psi}$$

$\frac{\mathrm{d}\bar{P}_{\mathrm{t}}}{\mathrm{d}N}$ = 0 for maximum energy production

$$\sigma_\Psi = \frac{\sigma_p}{\rho_a N^2 D^2}$$

$$\bar{\Pi} = \frac{\bar{P}_t}{\rho_a N^3 D^5}$$

$$\frac{1}{D^3 \sigma_p} \frac{d\bar{P}_t}{dN} = \frac{3\bar{\Pi}}{\sigma_\Psi} + \left(\frac{N}{\sigma_p} \frac{d\sigma_p}{dN} - 2 \right) \frac{d\bar{\Pi}}{d\sigma_\Psi}$$

= 0 for maximum energy production

$$\frac{\sigma_\Psi}{\bar{\Pi}} \frac{d\bar{\Pi}}{d\sigma_\Psi} = \frac{3}{2 - \frac{N}{\sigma_p} \frac{d\sigma_p}{dN}}$$

For given turbine is function of σ_Ψ

For given OWC, turbine and sea state, is function of N

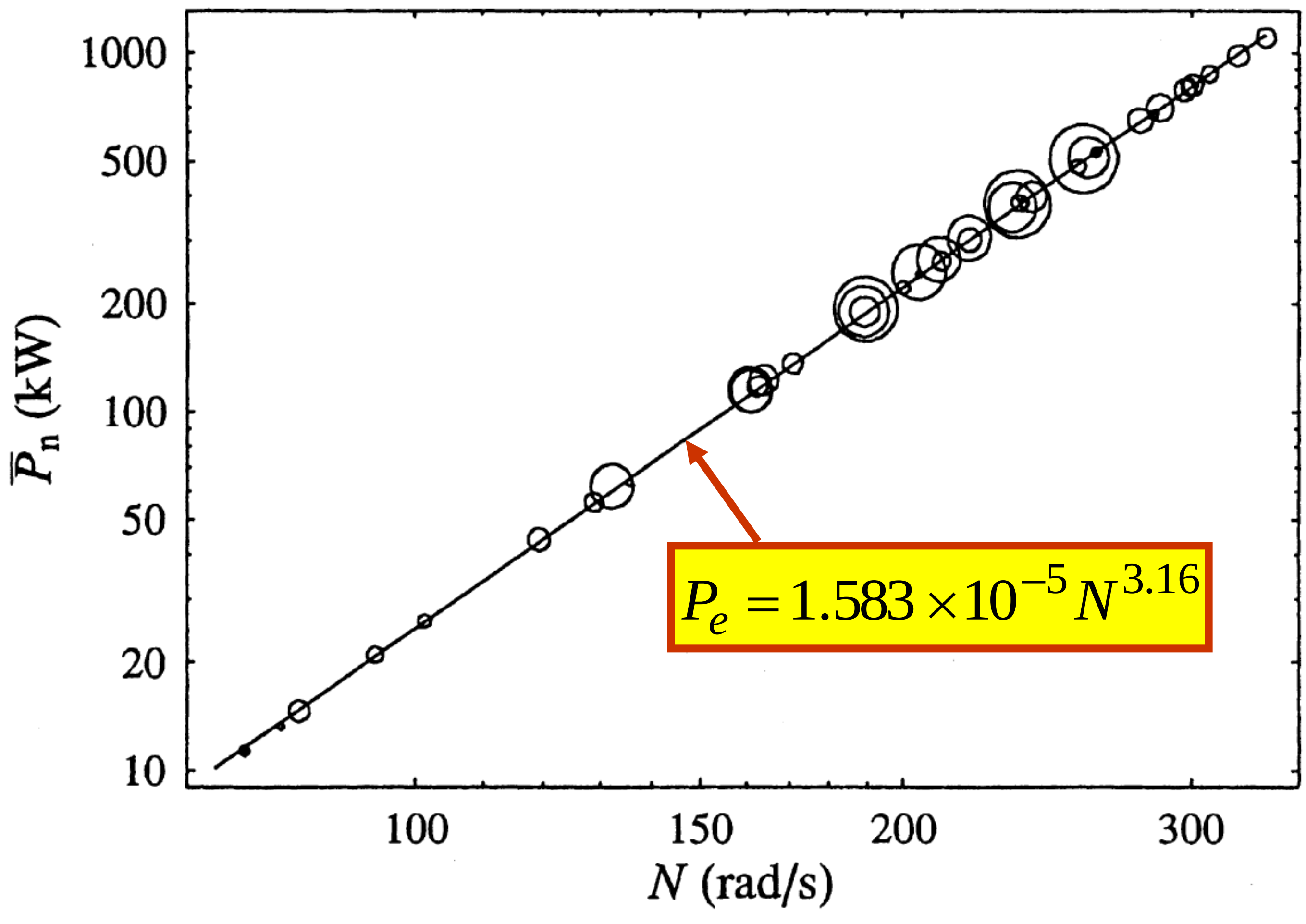
We obtain optimal N and maximum $\bar{P}_t$.

Control algorithm:

- **Set electrical power** $P_e = \bar{P}_t = \text{function}\,(N)$

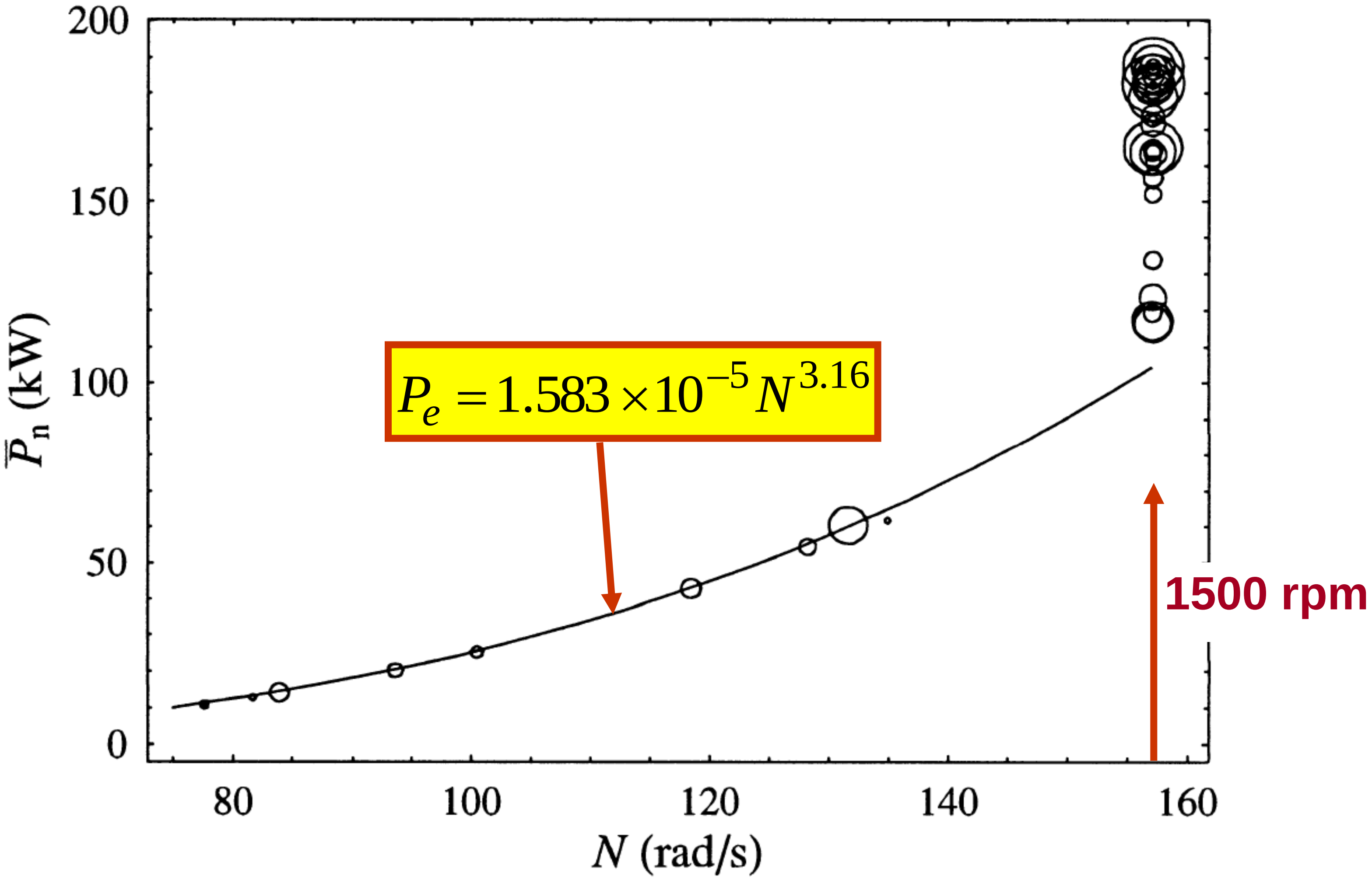
Example: Pico OWC plant with 2.3m Wells turbine

Local wave climate represented by 44 sea states (44 circles)



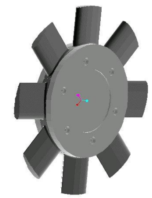
Maximum rotational speed may be constrained by:

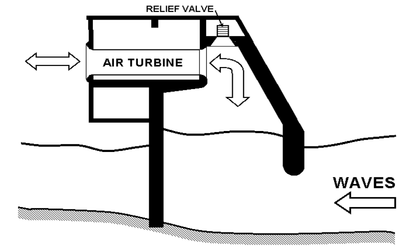
- Centrifugal stresses in turbine and electrical generator
- Mach number effects (shock waves)



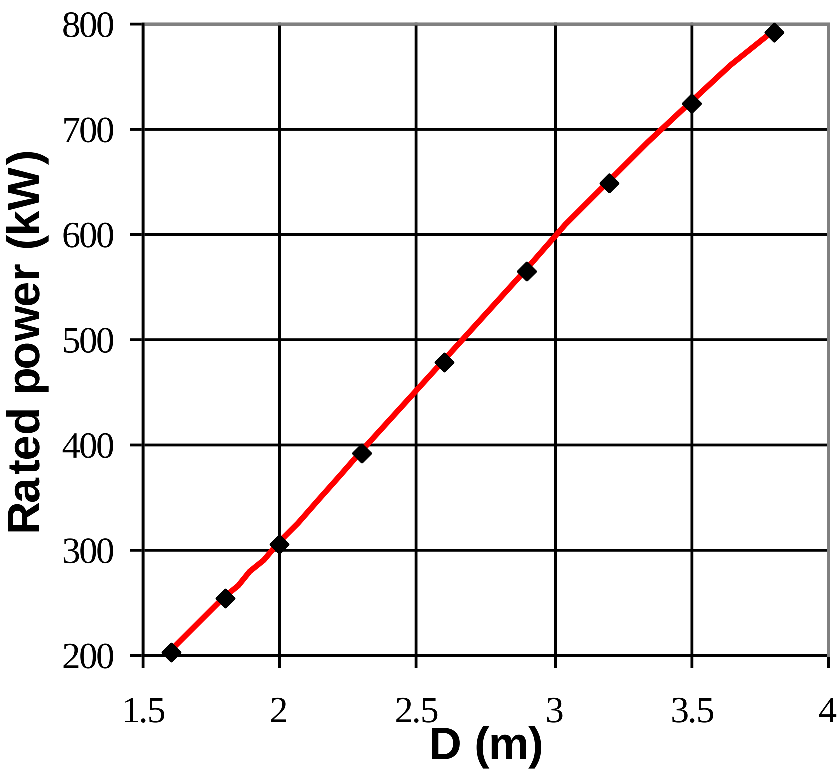
OWC Dynamics

Application of stochastic model

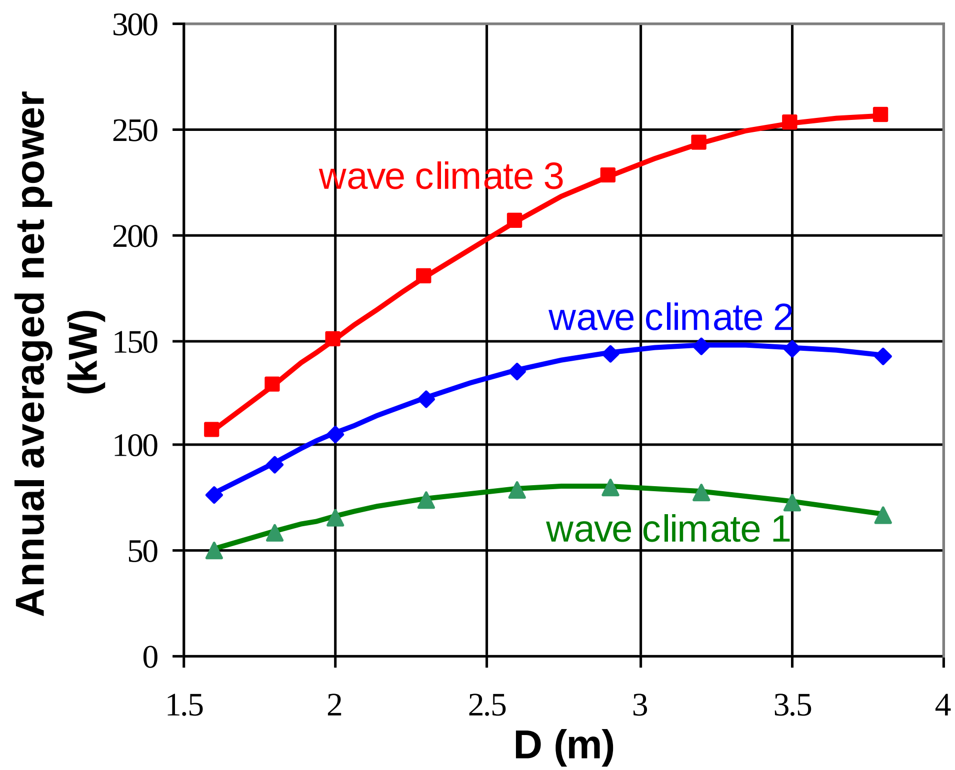




Wells turbine size range   1.6m < *D* < 3.8m



Plant rated power
(for H_s = 5m, T_e=14s)



Annual averaged
net power (electrical)

This presentation can be downloaded from:

http://hidrox.ist.utl.pt/doc_fct/Lancaster_pres.ppt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