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Wave Energy Converter through Piezoelectric Polymers

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Abstract: This note addresses the concept of wave energy conversion by means of piezoelectric material. The ocean surface waves represent an important source of energy power. A multiphysics simulation is used to focus on different aspects, namely the free surface wave, the fluid-structure-interactio, the mechanical energy input to the piezoelectric material and finally the electric power output using an equivalent open circuit model. The authors designed several feasible devices which are all similar, as long as they are forced by the wave action at a characteristic wave frequency. For the sake of simplicity, a basic system is analyzed here. The generic tool which is setup is useful for the design of piezoelectric polymer wave energy converter. The amount of energy generated by the piezoelectric materials appears very small, nevertheless some application of interest will be presented.

Keywords: Navier-Stokes, ALE, Fluid-Structure Interaction, Energy Harvesting, Piezoelectric Sensor.

1. Introduction

The capability of harvesting electrical energy from mechanical vibrations in a dynamic environment through piezoelectric transducers has been the topic of discussions for many years. Unused power exists in various forms such as vibrations, flowing water, wind, human motion and shock waves. Recent developments over global warming have renewed interest in the ocean energy conversion. The exploitation is heavily supported by the EU.

Effective systems for high energy production are based, for example, on overtopping and oscillating water column systems. This note is part of a study that investigates alternative methods, and in particular a device that is based on the piezoelectric effect. Such effect has been mainly used in relation with low consumption systems like i.e. portable electronic devices such as mp3 players, mobile phones, GPS receivers or sensors of remote sensing systems or transmitters which are conventionally powered by batteries. The advantage of a piezoelectric power supply is that it is ecological, embedded, and it does not need any maintenance.

In this study we used piezoelectric polymers (PVDF) to convert ocean wave energy into electrical power. Due to viscous and fluid pressure exerted by the wave movement, the system is bending. The resulting undulating motion of the system resembles like the movement of a sea plant in the ocean ground. See
Figure 1.

Figure 1: Seaweed

Piezoelectric polymers are commercially available and are relatively inexpensive. The capacity to sustain a high strain (3%) and to generate milliwatts to many watts depending on the mechanical system makes piezoelectric polymer an interesting material in ocean energy harvesting devices.

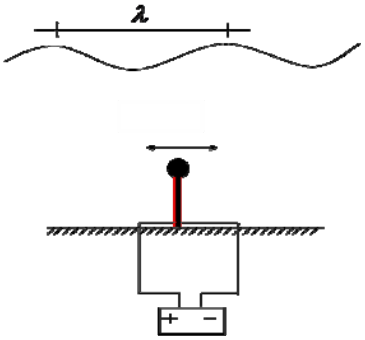

Figure 2 Principle of generation and storage

The simulation of the dynamic free surface flow is an interesting phenomenon and can only be solved if the full wave nonlinearity and the turbulent dissipation are taken into account. The simplest possible numerical model is based on the hypothesis of irrotational flow, which nevertheless does not account for viscous dissipation. Primary studies of this work have been dealing with assumption of the linear wave theory to determine the pressure field in the ocean ground. The velocity/pressure field was described with the total potential which is required to satisfy the Laplace partial differential equation. The implementation of the boundary conditions of this PDE on Comsol Multiphysics can be done only in a few minutes.

The second model, which is the object of this note, is based on the solution of the incompressible Navier-Stokes equation and the help of a moving mesh. Such model is fully non-linear and considers the actual wave shape. Coupling effects such as fluid-structure interaction or electromechanical structure modelling can be taken into account with this method.

In the following paragraphs a numerical model is presented which represents the initial approach for the design of a wave energy converter based on the piezoelectric phenomenon. To simulate the storage mechanism of a piezoelectric generator, an electrical open-circuit model is applied. The electromechanical coupling problem is not solved yet. Hopefully this next step will be shortly setup.

1.1 Methods

A 2D-model has been set up for the simulation of a free surface flow with fluid-structure interaction. With the help of the arbitrary Lagrangian Eulerian (ALE) technique, the coupled deformation of the structure and the surrounding fluid flow can be solved.

The model consists of a fluid part, solved by the incompressible Navier-Stokes equations in the wave channel and a structural mechanics part which is solved in the obstacle. The model uses both the static 2D piezoelectric and the solid plain strain application mode in the Structural Mechanics Module.

Due to the oscillating strain of the cantilever bending sensor the polarization is proportional to the deformation and causes an electrical potential difference over the piezoelectric polymer material.

1.2 Geometry

The numerical simulation considers a 2D model in Froude scale 1:20. The choice of this geometrical scale is related to frequently used dimensions of water tanks in hydraulic laboratories. This scale reduction allows furthermore faster computation time due to lower Reynolds numbers.

The obstacle consists of a sandwiched cantilever beam 30mm long, placed in a 0.6 m deep wave flume; a flexible foam core 3.75mm thick is sandwiched by two 1.25mm thick and 30mm long piezoelectric polymer layers, see Figure 3.

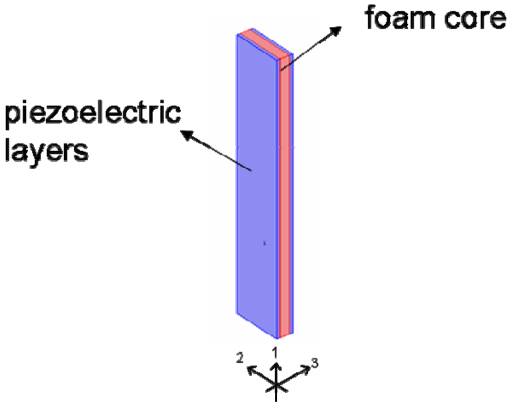

Figure 3 Piezoelectric generator: blue PVDF, red foam core.

The cantilever is orientated along the 1-axis. The polarization direction of the piezoelectric polymer material is aligned with the 3-axis. Mass and material properties of the foam core are chosen in the manner that the natural frequency of the mechanical system f_m=0.92Hz is near the characteristic wave frequency f_w=0.89Hz in the model scale. The analysis of the energy spectrum of wave frequency has not been topic of this study.

1.3 Mesh

The domain is discretized by 4946 triangular elements for total degrees of freedom of 44469.

Element refinement had to be done at the free fluid surface boundary to avoid inverted mesh warnings. The element quality of the worst element is 0.6912.

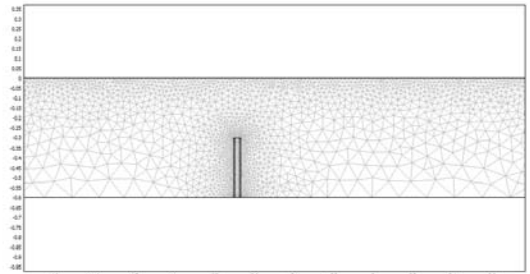

Figure 4 FEM-Mesh

3. Physics

3.1 Boundary conditions for the fluid

There are five types of boundaries in the model domain. The fluid is free to move on the top boundary. The viscous stress in the surrounding environment is neglected.

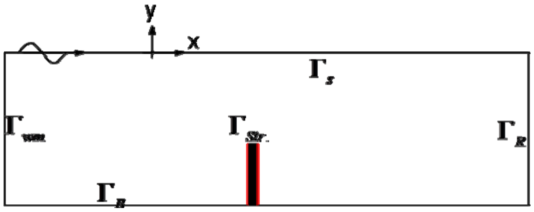

Figure 5 Definition sketch

Γ_s : Natural boundary (no tension)
Γ_B : No slip in the horizontal boundaries
Γ_R : Slip in the vertical boundaries
$\Gamma_{Str.}$: No Slip/Slip Structural displacement
Γ_{wm} : Incoming linear waves

The boundary conditions on the structure are imposed in terms of velocity. On the left hand side the wave kinematics is prescribed directly:

$$X(t) = k \cdot \frac{\cosh(k \cdot (h+y))}{\cosh(k \cdot h)} \sin(k \cdot x - \omega \cdot t) \qquad (1)$$

$$Y(t) = k \cdot \frac{\sinh(k \cdot (h+y))}{\cosh(k \cdot h)} \cdot \cos(k \cdot y - \omega \cdot t) \qquad (2)$$

3.2 Boundary conditions for the mesh

In order to follow the motion of the fluid with the moving mesh, it is necessary to constrain the mesh motion to the fluid motion and to the structure motion. The grid movements are described by the ALE application mode (1st PDE), with the Laplace smoothing method.

3.3 Electrical boundary conditions for the piezoelectric polymer

The electrostatic boundary conditions of this model are that the inner surfaces of the triple layer are grounded and the outer as well as the top and bottom surfaces are isolated. The piezoelectric polymer material in this model has a transversely isotropic material behaviour. The orientation of the polarization is aligned with the 3- direction, see Figure 3.

4. Piezoelectric Generator Characteristics

Considering the mechanical stress and strain in the 1-direction and the electrical field displacement in the 3-direction, the constitutive equations can be written as:

$$\begin{aligned} S_1 &= s_{11}{}^E \cdot T_1 + d_{31} \cdot E_3 \\ D_3 &= \varepsilon_0 \cdot \varepsilon_{33}{}^T \cdot E_3 + d_{31} \cdot T_1 \end{aligned} \qquad (3)$$

S is the mechanical strain vector (6×1), s^E is the elasticity tensor (6×6), T is the mechanical stress vector (6×1) (N/m^2), E is the electrical field vector (3×1) (V/M), ε^T is the dielectric constants (3×3) (Farad/m) for constant T, d is the piezoelectric constant (3×6) (C/N or m/V), D is the electrical displacement vector (3×1) (C/m^2). The subscripts 1 and 3 refer to the directions described above. In a piezoelectric generator operating in a bending d_{31} mode, each volume element obeys the above equations. The piezoelectric strain coefficient d_{31} describes the polarization P or electric displacement D in direction 3 perpendicular to the sheet. Piezoceramics have considerably higher piezoelectric coefficients, but their stiffness makes them impractical for hydraulic or wave generation applications, [7].

4.1 Transducer design and power generation

d_{31}	25×10^{-12} C/N
s_{11}	1.5×10^{8} N/m^2
ε	12
ε_0	8.85×10^{-12}
k^2_{31}	0.00088277
g	0.235

Table 1 Sensor Parameters of PVF$_2$ [7]

The dimensionless electromechanical coupling constant $k^2_{31} = s_{11}\cdot d^2_{31}/\varepsilon_0\varepsilon$ is the transduction rate of a piezoelectric material; ε and ε_0 are the dielectric permittivity of the material and of the air and g is the piezoelectric voltage coefficient, given as $g = d/\varepsilon_0\varepsilon$. The transducer design should be such that the product of $d_{31}\cdot g$ is enhanced. The second factor of the transducer design is that the piezoelectric structure should be able to withhold cyclic stresses without any fracture.

The power for a piezoelectric material per unit volume with a matched resistive electrical load is given by eq(4);

$$P = \frac{\pi\cdot d^2_{31}\cdot S_1^2\cdot s_{11}^2\cdot\omega}{2\cdot\varepsilon_0\cdot\varepsilon} \qquad (4)$$

with ω as the operating frequency. When a piezoelectric material is stressed electric charge is generated, the circuit model in Figure 6 can be used for representing the electrical behaviour of the piezoelectric material coupled with an external load.

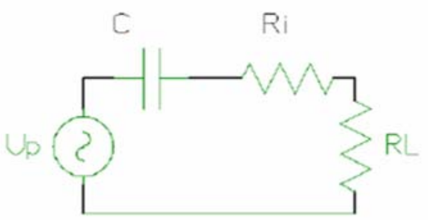

Figure 6 Piezoelectric circuit model connected with a load resistor.

If we eliminate stress T$_1$ from the eq(1) in favour of the strain, and neglect the coupling constant which is small for piezopolymer, we obtain

$$U_p = E\cdot x_1 = -\left(\frac{s_{11}\cdot x_1\cdot d_{31}}{\varepsilon_0\cdot\varepsilon}\right)\cdot S_1 \qquad (5)$$

by setting $D_3 = 0$, x_1 is the thickness of the sheet. Assuming that the piezoelectric material generates a sinusoidal signal, the voltage source is equal to the open circuit voltage.

$$U_P(t) = U_0\cdot\sin(\omega\cdot t) \qquad (6)$$

The voltage across the R$_L$ in Figure 3 is given by

$$U_{RL}(t) = \frac{Z_R(s)}{Z_T(s)}\cdot U(t) \qquad (7)$$

Where $Z_R(s) = 1/R_L$ is the impedance of the resistor and $Z_T(s) = 1/\sqrt{((R_L+R_i)^2 + (1/\omega C)^2}$ is the total impedance of the circuit. The average power output of the resistor can be found using

$$\langle P\rangle = U^2_{RL}(t)/R_L \qquad (8)$$

The average power with the optimal load resistance can be found by $\partial\langle P\rangle/\partial R_L = 0$.

$$R_L = \frac{\sqrt{1+(\omega\cdot C\cdot R_i)^2}}{\omega\cdot C} \qquad (9)$$

$$\langle P\rangle = \frac{U^2{}_p}{2\cdot R_L\left[1+(1/\omega\cdot C\cdot R_L)^2\right]} \qquad (10)$$

The output varies with load and is maximum at the matching impedance, neglecting the inner losses of the circuit R_i; the optimal load resistance is given as:

$$R_{L,opt} = \frac{1}{\omega\cdot C} \qquad (11)$$

The output power is measured across the resistive load directly without any amplification circuit. This method is usually used to characterize the performance of different PVDF materials, [5].

5. Wave simulation and results

Waves are generated in a 10m long and 0.6m deep channel with an amplitude of H = 0.03m and a period of T = 1.1sec. The wave steepness is lower than 2% to reduce non-linearities. The particle velocity near the structure can be seen in Figure 7.

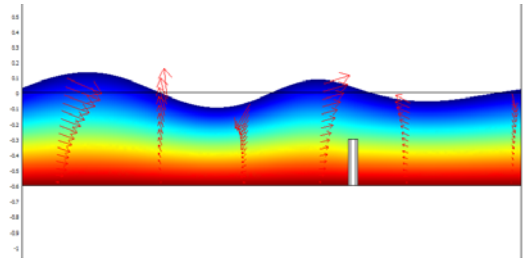

Figure 7 Partical velocity of the wave motion

The displacement field of the structure due to the wave motion is shown in Figure 8. The resulting voltage can be seen in Figure 9

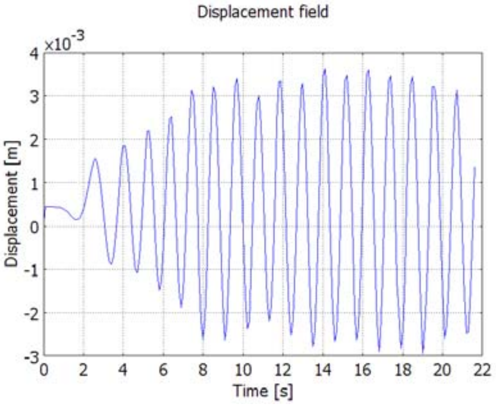

Figure 8 Displacement field at the upper right corner of the structure.

In Figure 10 we can observe the theoretical power average output for the piezoelectric power generator in function with the wave frequency and the load resistance. The power increases monotonically with frequency and resistance. This is justified if the generator is connected directly to a resistive load.

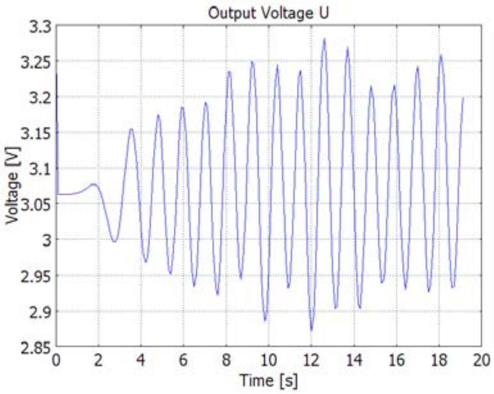

Figure 9 Voltage Field at the bottom right edge of the structure.

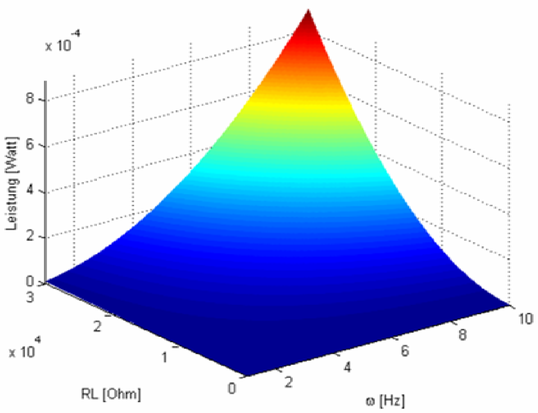

Figure 10 Theoretical power average output of the piezoelectric power generator

The results of the piezoelectric energy harvesting device are summarized in the following Table (2).

$x_1=1.25mm$	PVF_2
C (nF)	0.147
U_{peak} (V)	3.28
U_{RL} (V)	2.44

Table 2 Output voltage of a PVF_2 transducer in open circuit

6. Outlook

When piezoelectric devices are used as a power generator the circuit must have both capacitive and inductive elements. To overcome the low coupling factor k^2_{31} of PVDF and to maximize amplitude of the oscillation, the system must achieve mechanical and electrical resonance. The electrical resonance of a LCR-circuit has approximate angular frequency of $\omega = 1/\sqrt{LC}$. This frequency is far from the mechanical eigenfrequency of the system. Therefore direct electrical resonance is not possible for the piezoelectric wave energy converter. Low frequency of the wave motion necessitates impractically large inductor values. The numerical implementation of an adequate electrical circuit to model the energy storage and the electromechanical coupling has not been presented in this study. Experiments should be made to determine whether the above power and voltage densities can be approached.

7. Conclusions

This note presents a numerical model which is useful for the design of the piezoelectric polymer wave energy generator. The nonlinear wave motion is described by the NSE which is solved on a moving grid. The fluid-structure interaction is coupled with a piezoelectric material. Due to the wave motion and the pressure difference, the numerical model has showed up an oscillating movement of the structure with cantilever boundary conditions at the ocean ground. The wave amplitude is relatively small (H=0.03m), numerical runs with higher wave heights has been made, although the model turned out to be unstable and inverted mesh warnings could not be eliminated by a finer mesh.

The electrical behavior of the piezoelectric model is represented by a voltage generator connected with a capacitor and a resistor in series. This kind of voltage generator does not represent well the characteristics of piezoelectric material, as it is suggested in [10].

However the electrical circuit used in this work is the most suitable model for piezoelectric material applied for structural application. The electromechanical coupling problem is not fully solved yet, although interesting results could be presented. The obtained amount of energy generated by the studied piezoelectric device is very small; indeed the proposed system is oversimplified with the aim of describing the governing model equations.

8. References

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9. Acknowledgements

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