Complete electrical and optical simulation of electronic paper

Tom Bert a,*, Herbert De Smet a, Filip Beunis b, Kristiaan Neyts b

a TFCG Microsystems-Elis, Ghent University and IMEC vzw, Technologiepark (building 914), 9052 Zwijnaarde, Belgium
b Elis-Liquid Crystals and Photonics Group, St Pietersnieuwstraat 41, 9000 Ghent, Belgium

Received 20 April 2005; received in revised form 12 October 2005; accepted 13 October 2005
Available online 8 November 2005

Abstract

A complete model for the internal particle distribution of electrophoretic image displays (EPIDs) is presented. From this model, simulation formulas for electrical and optical properties are calculated. These formulas incorporate both time dependence and voltage dependence. They make it possible to optimize display properties since the model is built up with physical parameters. Results are shown that prove that this model is indeed capable of complete optical and electrical transient simulation.

The importance of centrifugation during display manufacturing is proven as a way to improve the switching speed of the display.

Keywords: Centrifugation; Electrophoresis; Diffusion

1. Introduction

Electrophoretic image displays (EPIDs) are a thin, flexible reflective display type based on the movement of colored pigments in a colored solvent. Its properties make it the most likely candidate in the search for a display that combines the benefits of printed paper and an image display [1]. Just like printed paper it is lightweight, flexible and easy to read in all lighting conditions. Just as an image display its content can be altered fast and an infinite number of times.

In an EPID colored pigments move in a contrasting liquid through the laws of electrophoresis in an externally applied field. When the observer side of the pixel is coated with pigment, the pixel has the pigment color. When the voltage switches sign, the pigments move to the opposite side and the pixel has the liquid color. A matrix of these pixels combined makes a two color reflective display.

2. Problem definition

Electronic paper has a lot of clear advantages. It is bistable: once attached to an electrode the pigments remain at that position even when no holding voltage is applied. This is a great improvement in the field of power consumption: constant refreshing of the image is no longer needed.

It is also very thin and lightweight: the liquid is placed between two ITO-covered sheets, only 100 μm apart. These sheets can be made of plastic, so the whole display can be flexible. Since, its reflective properties are based on the same technology as paper printing, reflection of the surrounding light on colored pigments, it has a very good reflectivity. Even in bright outdoor environments.

There are however also some major drawbacks:

- There is no threshold in the switching properties of the pigments. The pigments respond to every applied voltage. Therefore normal passive matrix addressing is not possible.
- The optical switching speed is very low. Even with applied voltages above 10 V, a pixel needs more than 500 ms to fully reach its contrasting color. This impedes the use of EPIDs at video rate.
- The system inside an electronic paper pixel has a very high electrochemical complexity. A full model of the internal particle distribution has not yet been developed. Neither the optical or electrical response properties are fully understood. Optimization of the switching speed is a pure chemical and physical process: a good understanding of the internal physics is necessary to do this.

In this article a solution for the last two problems will be given while we elaborate on the development of a complete simulation tool for EPIDs. A method to increase the internal
field will be given, this will increase the pigment velocity and hence the optical response.

3. Display model

Two kinds of charged particles are present inside the pixel of an EPID [2]

- Charged pigments: neutral pigments covered with surfactant ions. The pigments are macromolecules with a radius of approximately 0.5 μm. After addition of surfactant, a shell of surfactant ions attaches itself to the pigment. It hereby acquires a charge of about 3000e \( (e = 1.60E−19C) \) which makes it possible to move the pigment in an electrical field. This also avoids flocculation of the pigment particles.
- Charged inverse micelles: these are formed through flocculation of surfactant molecules. They form ions with a charge of 1e and a radius that is much smaller than that of the pigments.

The charged pigments and inverse micelles have opposite charge: on application of an external field they move in opposite directions.

As a first approximation, we assume that the charged micelles and pigments follow a normal (Gaussian) distribution inside the pixel. This is in very good accordance with reality [5] and greatly improves the ease of computation. A 1D simulation will be done: particles only move along the x-axis. Since, the pixel dimension parallel to the field is much smaller than perpendicular to the field, it is safe to make this 1D assumption. By adjusting the standard deviation it is possible to model a uniform (large standard deviation) or packed (small standard deviation) distribution [3,4].

To model diffusion a time-dependent standard deviation is defined: the charge distribution spreads out with a diffusion velocity \( v_{\text{diffusion}} \). The standard deviation \( \sigma(t) \) then equals \( (v_{\text{diffusion}}t^2) \).

To model drift due to the external field a time dependent average value for the normal distribution is defined by means of an EPID [2]

\[
R(t_d) = \frac{1}{\sqrt{2\pi\sigma_{\text{delay}}}} e^{-\frac{1}{2}((t_d-T_{\text{delay}})^2)/(\sigma_{\text{delay}}^2)}
\]

\( R(t_d) \) is the number of particles with a certain delay time \( t_d \), \( T_{\text{delay}} \) is the average delay time, \( \sigma_{\text{delay}} \) the standard deviation.

3.1. Particle distribution

The distribution of the charged particles inside the pixel is modeled as follows:

\[
n(x, t) = \frac{N}{\sqrt{2\pi v_{\text{diffusion}} t}} e^{-\frac{1}{2}(x-v_{\text{drift}}t)^2/(v_{\text{diffusion}} t^2)}
\]

\( N \) is the total amount of particles that is present. Micelles and pigments have the same distribution, however with different parameters: they are different in number, have different properties and move in the opposite direction.

The mean deviation of a group of particles with mobility \( \mu \) spreading out through diffusion (starting from a delta Dirac distribution) varies as [5].

\[
\sigma = \sqrt{2D\tau}
\]

where the relationship between diffusion coefficient \( D \) and mobility \( \mu \) is given by

\[
D = \frac{kT}{q\mu}
\]

This means that the diffusion velocity is given by

\[
v_{\text{diffusion}}(t) = \frac{d\sigma}{dt} = \frac{D}{\sqrt{2Dt}}
\]

When starting from a distribution with a distribution larger than 0 (no Dirac delta), the following formulas have to be used

\[
\sigma(t) = \sqrt{2D(t + t_0)}
\]

and

\[
v_{\text{diffusion}}(t) = \frac{d\sigma}{dt} = \frac{D}{\sqrt{2D(t + t_0)}} = \frac{D}{\sqrt{2D(t + \sigma_0^2/2D)}}
\]

where \( \sigma_0 \) is the starting standard deviation (at \( t=0 \)).

The relationship between \( \sigma_0 \) and \( V_1 \) (the voltage applied in the previous half period) can be approached through Coulomb’s law:

\[
E = \frac{Q}{4\pi\varepsilon R^2} \Rightarrow \sigma_0 \approx R = \sqrt[3]{\frac{Q}{4\pi\varepsilon}} \sqrt[3]{\frac{1}{V_1}}
\]

The charges are pushed apart through Coulomb forces and pushed together by \( V_1 \). The standard deviation of the distribution gives an idea of how far apart the charges are. Therefore:

\[
\sigma_0 \approx \sqrt[3]{\frac{Q}{4\pi\varepsilon}} \sqrt[3]{\frac{\text{cst}}{V_1}}
\]

where ‘cst’ is an unknown constant which has meter (m) as a unit.

Using Poisson’s law it is possible to obtain how this varying charge distribution interferes with the internal field.

\[
\frac{dE}{dx} = \frac{\rho}{\varepsilon} \Rightarrow \frac{dE}{dx} = \frac{dE}{dr} \frac{dr}{dx} = \frac{NQ}{\varepsilon} \left( \frac{1}{\sigma^2} \right) \Rightarrow \frac{dE}{dr} \left( \sqrt{2Dt} \right) = \frac{NQ}{\varepsilon} \left( \frac{1}{2\sqrt{2Dt}} \right)^{3/2}
\]

Here, the following approximation was used:

\[
\rho = \frac{dQ}{dV \text{Volume}} \approx \frac{NQ}{(dx)^2} \approx \frac{NQ}{\sigma^2}.
\]
This gives the following time dependence of the internal field screening due to diffusion:

$$\Delta E(t) \approx \frac{NQ}{\varepsilon} \frac{1}{\sqrt{2D}} \sqrt{\frac{1}{t + \frac{Q^*\text{cst}}{8\pi\varepsilon D^2}}} \quad (11)$$

And when starting from a non-zero distribution:

$$\Delta E(t) \approx \frac{NQ}{\varepsilon} \frac{1}{\sqrt{2D}} \sqrt{t + \frac{Q^*\text{cst}}{8\pi\varepsilon D^2}} \quad (12)$$

It is important to notice that the value of this screening field can never exceed the value of the externally applied field. At its maximum strength the Poisson field completely blocks the external field, resulting into a zero internal field.

This formula shows how the redistribution of charges leads to field screening. Using this formula easy to measure properties can be calculated: average velocity, delay time and diffusion velocity.

### 3.1.1. Average velocity

For the average particle velocity we can write (using Eq. (12)):

$$v = \mu E = \mu (E_{\text{external}} - \Delta E(t))$$

$$= \mu \left( E_{\text{external}} - NQ \frac{1}{\sqrt{2D\varepsilon}} \sqrt{t + \frac{Q^*\text{cst}}{8\pi\varepsilon D^2}} \right). \quad (13)$$

The external electrical field is only dependent on the applied voltage $V_2$.

Using the equations derived above and a first order approximation we find that

$$v_{\text{average}} = \frac{\mu}{d} V_2 - \frac{\mu N}{\varepsilon} \sqrt{\frac{4\pi\varepsilon Q}{\text{cst}}} \sqrt{V_1} \quad (14)$$

where $d$ is the thickness of the pixel.

The average particle velocity:

- Is both dependent on $V_2$ and $V_1$: it becomes bigger with increasing $V_2$ and smaller as a bigger $V_1$ is applied. $V_2$ increases the drift velocity, $V_1$ increases the field screening.
- Becomes lower when more charged particles are present (bigger $N$): this is because the field screening becomes stronger when more charged particles are present inside the pixel. The average velocity also drops as the charge of the particles ($Q$) increases. This is for the same reasons.

### 3.1.2. Diffusion velocity

From Eq. (6):

$$\sigma(t) = \sqrt{2D \left( t + \frac{Q^*\text{cst}}{2D} \right)} = \sqrt{2D \left( t + \frac{Q^*\text{cst}}{8D\pi\varepsilon V_1} \right)} \approx \sqrt{\frac{D}{2}}$$

$$\times \sqrt{8D\pi\varepsilon V_1} = \sqrt{\frac{4D^2\pi\varepsilon}{Q^*\text{cst}}} \quad (15)$$

Through a first order approach (last step) we find

$$v_{\text{diffusion}} = \sqrt{V_1} \sqrt{\frac{4D^2\pi\varepsilon}{Q^*\text{cst}}} \quad (16)$$

Hence, the diffusion velocity:

- Is only dependent on $V_1$ and increases with $V_1$. As a bigger $V_1$ is applied, the charges are packed more closely together and they diffuse at a higher velocity.
- Increases with increasing mobility: when charges are more mobile they can get to the uniform distribution faster.

### 3.1.3. Delay time

The delay time is the time difference between a particle that moves at its maximum velocity in the externally applied field and one that moves in the internal field that has a lower value due to field screening [6].

$$\Delta t = \frac{\Delta s}{v_{\text{slow}}} - \frac{\Delta s}{v_{\text{fast}}}$$

$$= \frac{\Delta s}{v_{\text{fast}} - \Delta v} - \frac{1}{v_{\text{fast}}} \approx \frac{\Delta s}{v_{\text{fast}} - \Delta v} \quad (17)$$

Using Eq. (13) we find $v_{\text{fast}} = \mu E_{\text{external}} = \mu (V_2/d)$.

From Eq. (12) for $t=0$ (since the delay time occurs then) we find:

$$\Delta v = \mu \Delta E(t) \approx \mu \frac{NQ}{\varepsilon} \frac{1}{\sqrt{2D}} \sqrt{\frac{1}{t + \frac{Q^*\text{cst}}{8\pi\varepsilon D^2}}}$$

$$= \mu N \sqrt{\frac{4\pi Q}{\varepsilon^2 \text{cst}}} \sqrt{V_1}$$

This gives for the total delay time, from Eq. (17):

$$\Delta t = \frac{\Delta s}{\mu} \frac{1}{V_2 - N \sqrt{\frac{4\pi Q}{\varepsilon^2 \text{cst}}} / \sqrt{V_1}} \quad (18)$$

By using a first order approximation this becomes:

$$\Delta t \approx \frac{\Delta s}{\mu} d^2 N \sqrt{\frac{4\pi Q}{\varepsilon^2 \text{cst}}} \sqrt{\frac{V_1}{V_2}} \quad (19)$$

$\Delta s$ is a non-physical length that is used to match a difference in velocity to a difference in time. It cannot be measured nor has it a value in the pixel. Its value will have to be matched to fit measurements when optimizing the simulation.

The average delay time:

- Increases with $V_1$: as charges are packed more together it takes longer for them to loosen. For $V_1 = 0$ micelles are
uniformly distributed at the beginning, there is no field screening, all charges immediately move at the drift velocity, there is no delay time.

- It decreases with $V^2$: as a bigger drift force is applied, charges loosen more quickly. When an infinitely big drift force is applied, there is no delay time since, the external field then overtakes all internal fields.

When calculated, the same voltage dependence for the standard deviation of the delay time as for the average delay time is found.

3.2. Optimization and summary

In all the previous formulas more physical properties of the particles can be incorporated by using the following relationship between mobility ($\mu$), viscosity ($\eta$), charge ($Q$) and particle size ($R$):

$$\mu = \frac{Q}{6\pi\eta R}$$  \hspace{1cm} (20)

Together with Eq. (4), this gives for the formulas calculated above:

- Average velocity (from Eq. (14))

$$v_{\text{average}} = \frac{Q}{6\pi\eta R} \left( \frac{1}{d} V_2 - \frac{N}{e} \sqrt{\frac{4\pi \varepsilon Q}{\text{cst}} \sqrt{V_1}} \right)$$  \hspace{1cm} (21)

- Diffusion velocity (from Eq. (16))

$$v_{\text{diffusion}} = \sqrt{V_1} \sqrt{\frac{4\pi \varepsilon \frac{kT}{Q^* \text{cst}}}{6\pi \eta R}}$$  \hspace{1cm} (22)

- Average delay time and standard deviation of the delay time (from Eq. (19)):

$$\Delta t = \frac{6\pi\eta R}{Q} \Delta x d^2 N \sqrt{\frac{4\pi Q \sqrt{V_1}}{e^* \text{cst}} \frac{V_1}{V_2}}$$ \hspace{1cm} (23)

Both the screening field and delay time increase with the amount of charge that is present (defined by $N$ and $Q$). It is therefore important to remove all excess charge that is present inside the pixel. This can be done by carefully centrifuging the liquid. The charged pigments are much heavier than the micelles. They can be separated from the rest of the liquid after centrifugation. When the supernatant is replaced with fresh, uncharged solvent, the minimum amount of charge is present in the liquid: only charged pigments. Then the internal field approaches the external field in the best way and the optical response is at its fastest. Hence, centrifugation is a simple way to improve the display response speed. In a previous article we already proved that centrifugation can also be used to improve the image quality by reducing pattern formation in the display liquid [9].

3.3. Current simulation

The distribution of the inverse micelles inside the pixel and how this distribution varies in time and with the applied voltages $V_1$ and $V_2$ was calculated above. Since, the movement of charged micelles forms the major contribution to the current, the voltage and time dependence of the current can be calculated from the charge movement.

When incorporating a delay time, the formula for micelle distribution becomes:

$$n(x, t, V_1, V_2) = R(t_d, V_1, V_2)n_{\text{micelles}}(x, t - t_d, V_1, V_2)$$  \hspace{1cm} (24)

Due to the forces of drift and diffusion the micelles acquire a certain time and position dependent velocity. This is given by:

$$v(x, t, V_1, V_2) = \frac{dx}{dt} = \frac{dn(x, t, V_1, V_2)}{dn(x, t, V_1, V_2)/dx}$$ \hspace{1cm} (25)

The current is given by the total amount of charged particles...
that is present inside the pixel, weighted with their velocity:

\[
J(t, V_1, V_2) = \int_0^d v(x, t, V_1, V_2)n(x, t, V_1, V_2)\,dx
\]  

(26)

With this formula the current response in time on application of a voltage \(V_2\) after a voltage \(V_1\) has been applied can be calculated.

3.4. Optical simulation

Using the knowledge of the pigment distribution in time and how it is voltage dependent, a formula to calculate the optical response in time can be developed. Again, the pigments are assumed to be normally distributed (as in Eq. (2))

The intensity of the reflected light on a particle varies exponentially with the distance to the observer. When the pigment distribution and a delay time are incorporated, this yields for the reflected light intensity, hence for the optical response:

\[
I(t, V_1, V_2) = I_0 \int_0^t \int_0^d R(t_d) \left[ n(x, t - t_d, V_1, V_2)e^{-\alpha(d - x)}\,dx \right] \,dt_d
\]  

(27)

\(I_0\) is the input intensity coming from the light source, \(\alpha\) is the extinction coefficient of the medium and \(a\) is a geometrical factor that varies with the incidence angle of the light and dielectric properties of the medium.

![Fig. 2. Simulation (Fig. 2(a), left) and measurement (Fig. 2(b), right) of the transient current response on application of a voltage block wave with varying amplitude.](image)

![Fig. 3. Simulation (Fig. 3(a), left) and measurement (Fig. 3(b), right) of the transient current dependence on \(V_2\), the applied voltage, with constant \(V_1\), the previously applied voltage.](image)
4. Computer simulation

We have incorporated our calculations in a simulation tool. Some results are given below.

Fig. 1 shows the agreement between the model and measurements in optical response. The simulation curve is a very good match to the measurement. Delay time, response speed and shape are simulated correctly. The dependence of the optical response on the physical parameters (charge, viscosity, ...) can be visualized through temperature dependent measurements. When we compare the variation of the optical response when temperature varies to the theoretical prediction of the optical response calculating the temperature dependence of the parameters, the physical correctness of the model is confirmed. This temperature dependent modeling will be described in a later article.

Figs. 2–4 show the agreement between the model and measurements in electrical response. The dependence on \( V_1 \) and \( V_2 \) of the height, shape and position of the current peak is simulated correctly. The dependence on the amplitude of a block wave with varying amplitude is correctly simulated by the model. Again can the model correctly simulate the temperature dependence (hence parameter dependence) of the electrical transient responses. This confirms the correctness of the electrical model and will be elaborated on in a later article.

5. Conclusion

- We managed to model the internal physics of electronic paper. A complete study and explanation for both electrical and optical properties is given. The formulas contain physical, measurable parameters.
- The importance of the presence of the charged inverse micelles is shown: removal of these particles by careful centrifugation is very important in order to maximize the internal field strength and hence the pigment velocity.
- The pigments are much heavier than the micelles: through centrifugation it is possible to separate these heavy particles from the rest, hereby creating a liquid without contaminating ionic particles. Another benefit of careful removal of the contaminating charged particles is the lowering of the electrical current flow through the pixel. This will improve the power consumption.
- Using this model it will be possible to calculate in advance the use of other display materials (liquid viscosity, pigment radius, ...) and process parameters to optimize the display production.

Acknowledgements

This research was financed by a PhD grant of the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen), and partially funded by the Federal Office for Scientific, Technical and Cultural Affairs (OSTC, Belgium) through the IUAP (phase V) Contract No. P5/11.

References