WIND TUNNELS
Introduction

Wind tunnels are devices which provide air streams flowing under controlled conditions so that models of interest can be tested using them. From operational point of view, wind tunnels are generally classified as low-speed, high-speed, and special purpose tunnels.
Low-Speed Wind Tunnels

Low-speed tunnels are those with test-section speed less than 650 kmph. Depending upon the test-section size they are referred to as small size or full scale tunnels.

They are further classified into the following two categories: open-circuit tunnels, having no guided return of air, as shown in Figure 1, or closed-circuit or return-flow tunnel, having a continuous path for the air, as shown in Figure 2.
Figure: 1 Open circuit wind tunnel
Fan and motor

Test-section

Model

Guide vanes

Figure: 2 Closed circuit wind tunnel
Both open-circuit and closed-circuit tunnels can operate with either open-jet or closed-jet test-sections.

Open-jet is that test-section without side walls and closed-jet test-section is that with side walls.

The cross-section of the test-section can have different shapes such as, rectangular, circular, elliptical, octagonal, etc.

In low-speed tunnels, the predominant factors influencing the tunnel performance are inertia and viscosity. The effect of compressibility is negligible for these tunnels. Thus, if the Reynolds number of the experimental model and full scale prototype are equal, any difference in viscosity becomes unimportant.
High-Speed Wind Tunnels

Tunnels with test-section speed more than 650 kmph are called high-speed tunnels.

The predominant aspect in high-speed tunnel operation is that the influence of compressibility is significant. This means that, in high-speed flows it is essential to consider Mach number as a more appropriate parameter than velocity. A lower limit of high-speed might be considered to be the flow with Mach number approximately 0.5 (about 650 kmph) at standard sea level conditions.
Based on the test-section Mach number, $M$, range, the high-speed tunnels are classified as follows.

- $0.8 < M < 1.2$  Transonic tunnel
- $1.2 < M < 5$  Supersonic tunnel
- $M > 5$  Hypersonic tunnel
Like low-speed tunnels, high-speed tunnels are also classified as intermittent or open circuit tunnels and continuous return circuit tunnels, based on the type of operation. The power to drive a low-speed wind tunnel varies as the cube of the test-section velocity. Although this rule does not hold in the high-speed regime, the implication of rapidly increasing power requirements with increasing test-section speed holds for high-speed tunnels also.

Because of the power requirements, high-speed wind tunnels are often of the intermittent type, in which energy is stored in the form of pressure or vacuum or both and is allowed to drive the tunnel only a few seconds out of each pumping hour.
General Features

All modern wind tunnels have four important components;

- the **Effuser**
- the **Working or Test-section**
- the **Diffuser**
- the **Driving unit**
The Effuser

This is a converging passage located upstream of the test-section. In this passage fluid gets accelerated from rest (or from very low speed) at the upstream end of it to the required conditions at the test-section.

In general, effuser contains honey-comb and wire gauze screens to reduce the turbulence and produce an uniform air stream at the exit. Effuser is usually referred to as contraction cone.
Test-section

Model to be tested is placed here in the air-stream, leaving the down-stream end of the effuser, and the required measurements and observations are made.

If the test-section is bounded by rigid walls, the tunnel is called a closed throat tunnel. If it is bounded by air at different velocity (usually at rest), the tunnel is called open jet tunnel. The test-section is also referred to as working-section.
Diffuser

The diffuser is used to re-convert the kinetic energy of the air-stream leaving the working-section into pressure energy, as efficiently as possible. Essentially it is a passage in which the flow decelerates.
Driving Unit

If there were no losses, steady flow through the test-section could continue for ever, once it is established, without the supply of energy from an external agency. But in practice, losses do occur, and kinetic energy is being dissipated as heat in vorticity, eddying motion and turbulence. Moreover, as the expansion of the diffuser cannot continue to infinity, there is rejection of some amount of kinetic energy at the diffuser exit. This energy is also converted to heat in mixing with the surrounding air.
To compensate for these losses, energy from an external agency becomes essential for wind tunnel operation. Since power must be supplied continuously to maintain the flow, the fourth essential component namely, some form of driving unit is essential for wind tunnel operation. In low-speed tunnels this usually takes the form of a fan or propeller. Thus, for a low-speed tunnel, the simplest layout is similar to that shown in Figure 1.
The overall length of the wind tunnel may be shortened, and the rejection of kinetic energy at the diffuser exit eliminated, by the construction of some form of return circuit. Even then the driving unit is necessary to overcome the losses occurring due to vorticity, eddying motion and turbulence. The skin friction at the walls and other surfaces will be large since the velocity at all points in the circuit will be large (of the same order as the test-section velocity). Also, a construction ahead of the test-section, as shown in Figure 2, is necessary if the turbulence at the test-section has to be low, and particularly if the velocity distribution has to be uniform. To achieve this, usually guide vanes are placed in the corners.
Special Purpose Tunnels

These are tunnels with layout totally different from that of low-speed and high-speed tunnels. Some of the popular special purpose tunnels are: spinning tunnels, free-flight tunnels, stability tunnels and low-density tunnels.
Low-speed Wind Tunnels

A general utility low-speed tunnel has four important components, namely,

- the effuser
- the test-section
- the diffuser, and
- the driving unit.
The Effuser

This is basically a contraction cone, as shown in the Figure 3. Its application is to bring down the level of turbulence and increase the velocity of flow. The contraction ratio $n$ of an effuser is defined as

$$n = \frac{\text{Area at entry to convergent cone}}{\text{Area at exit of convergent cone}}$$

The contraction ratio usually varies from 4 to 20 for conventional low-speed tunnels.
**Test-Section**

The portion of the tunnel with **constant flow characteristics across its entire section** is termed the **test-section or working-section**. Since boundary layer is formed along the test-section walls, the walls are given a suitable divergence so that the net cross-sectional area of the uniform flow is constant along the length of the test-section.
**Diffuser**

The purpose of the diffuser is to **convert the kinetic energy of the flow coming out of the test-section to pressure energy**, before it leaves the diffuser, as efficiently as possible. Generally, the smaller the diffuser divergence angle, the more efficient is the diffuser. Near the exit, its cross-section should be circular to accommodate the fan.
Driving Unit

Generally the driving unit consists of a motor and a propeller or fan combination. The fan is used to increase the static pressure of the stream leaving the diffuser.

The wind tunnel fan, looking similar to the propeller of an airplane, operates under peculiar conditions that put it in a class by itself. Since the thrust of the fan and the drag of the various tunnel components vary with the square of the fan rpm, it would appear that to maintain an uniform velocity distribution in the test-section, speed adjustments should be made by varying the fan rpm rather than fan pitch.
Although this conclusion is justified in short tunnels of low contraction ratio, for large tunnels it is not true. In deed, many large tunnels which are equipped with both \textit{rpm} and \textit{pitch} change mechanisms use only the latter, being quick and simpler.
Power Losses in a Wind Tunnel

The total power loss in a wind tunnel may be split into the following components.

- Losses in cylindrical parts.
- Losses in guide vanes at the corner (in closed circuit tunnels).
- Losses in diffuser.
- Losses in contraction cone.
- Losses in honey comb, screens etc.
- Losses in test-section (jet losses in case of open jet).
- Losses in exit in case of open circuit tunnel.
Calculation of Percentage Energy Loss in the Various Parts of Wind Tunnel
Energy Loss Calculation

The loss of energy is expressed in terms of static pressure drop $\Delta p$, in the dimensionless form, called pressure drop coefficient $K$, as follows

$$K = \frac{\Delta p}{q} \quad (1)$$

where $q$ is the dynamic pressure of the flow. In terms of local velocity $V$, Eq. (1) becomes

$$K = \frac{\Delta p}{\frac{1}{2} \rho V^2}$$

where $\rho$ is the flow density.
It will be convenient to refer the local losses at different parts of the wind tunnel to the jet or test–section dynamic pressure, defining the coefficient of loss as

\[ K_0 = \frac{\Delta p}{q_0} = \frac{\Delta p}{q} \frac{q}{q_0} = K \frac{q}{q_0} \]

where \( q_0 \) is the test-section dynamic pressure. But \( q \propto V^2 \) and \( V \propto \frac{1}{A} \propto \frac{1}{D^2} \), therefore, the above equation can be rewritten as

\[ K_0 = K \left( \frac{D_0}{D} \right)^4 \]  \( \text{(2)} \)

where \( D_0 \) is the test–section diameter and \( D \) is the local tunnel diameter.
Using the above definitions, the section energy loss in the wind tunnel, $\Delta E$, may be expressed as,

$$
\Delta E = \Delta \rho AV \\
= KqAV \\
= K \frac{1}{2} \rho AV^3
$$

(3)

But $Kq = K_0 q_0$, therefore, Eq. (3) becomes

$$
\Delta E = K_0 q_0 A_0 V_0 = K_0 \frac{1}{2} \rho A_0 V_0^3
$$

(4)

where $A_0$ is the test–section area and $A$ is the local cross-sectional area.
Note that, in the above discussions on energy loss, the density of the flow is treated as invariant, even though it is valid only for incompressible flows. Such an assumption is usually made in the study of low-speed wind tunnels, since the Mach number involved is always less than 0.5, the compressibility effect associated with the flow will be only marginal and hence assuming the density, $\rho$, as an invariant will not introduce any significant error to the measured values.
Energy Ratio

The ratio of the energy of air stream at the test-section to the input energy to the driving unit is a measure of the efficiency of a wind tunnel. It is nearly always greater than unity, indicating that the amount of stored energy in the wind stream is capable of doing work at a higher rate than what it is doing in a wind tunnel, before being brought to rest. The energy ratio $ER$ is in the range 3 to 7, for most closed–throat wind tunnels.
The energy ratio is defined as

\[ ER = \frac{\text{Kinetic energy of jet}}{\text{Energy loss}} \]

\[ = \frac{\frac{1}{2} \rho A_0 V_0^3}{\sum K_0 \frac{1}{2} \rho A_0 V_0^3} \]

\[ = \frac{1}{\sum K_0} \]

Equation (5)

The definition of the energy ratio given by Eq. (5) excludes the fan and motor efficiency.

The magnitudes of the losses in the various components of a wind tunnel of circular cross-section may be calculated as follows.
Losses in Cylindrical Section

We know that the pressure drop in a cylindrical section of length $L$ can be expressed as

$$\Delta p = f \frac{L}{D} \frac{\rho}{2} V^2$$  \hspace{1cm} (6)

where $f$ is friction coefficient and $D$ is diameter of the cylindrical section.

Combining Eqs. (3.1) and (3.6), we get

$$K = \frac{\Delta p}{q} = \frac{fL}{D}$$

Substitution of the above equation into Eq. (2), results in

$$K_0 = \frac{fL}{D} \left( \frac{D_0}{D} \right)^4$$  \hspace{1cm} (7)
The value of the friction coefficient, $f$, may be computed from Von Karman formula (White, 1986), which gives

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left( Re_D \sqrt{f} \right) - 0.8$$

for smooth pipes at high Reynolds numbers.
Some numerical values of $f$, at specified Reynolds numbers are listed below.

<table>
<thead>
<tr>
<th>$Re_D$</th>
<th>400</th>
<th>$10^4$</th>
<th>$10^5$</th>
<th>$10^6$</th>
<th>$10^7$</th>
<th>$10^8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0.0899</td>
<td>0.0309</td>
<td>0.0180</td>
<td>0.0116</td>
<td>0.0081</td>
<td>0.0059</td>
</tr>
</tbody>
</table>
It is seen that $f$ drops by **only a factor of 5** over 10000-fold increase in Reynolds number. The above equation for $f$ in terms of $Re_D$ is usually used for calculating $f$. There are many alternate approximations in the literature from which $f$ can be computed explicitly from $Re_D$. However, for wind tunnel applications the above relation is good enough.

The friction coefficient variation with Reynolds number is shown as a plot in Figure 3.
Figure 3 Friction coefficient variation with Reynolds number
Equation (3.7) can be used to compute $K_0$ for (i) test-section and (ii) return passage (where there is no divergence in the section). For cross-sections other than circular, an equivalent diameter has to be used for the calculation of $Re$, $f$, etc.
Losses in Convergent Cone

Consider the contraction cone shown in Figure 4, with diameters $D_1$, and $D_0$ at its entrance and exit, respectively. The loss in convergent section is mainly due to friction. This loss may be expressed in terms of pressure loss as
Figure: 4 Contraction cone
\[ \Delta p = f \frac{L}{D_1} \frac{\rho}{2} \left( V_0^2 - V_1^2 \right) \]  

(8)

where \( V_1 \) and \( V_0 \) are the velocities at the inlet and exit of the convergent section, respectively.

Usually, for contraction cones \( L \approx D_1 \), therefore, Eq. (8) becomes

\[ \Delta p = f \left[ 1 - \left( \frac{D_0}{D_1} \right)^4 \right] \frac{\rho}{2} V_0^2 \]  

(9)

i.e.,

\[ \frac{\Delta p}{\frac{1}{2} \rho \, V_0^2} = K_0 = f \left[ 1 - \left( \frac{D_0}{D_1} \right)^4 \right] \]
For contraction cones of good shapes with smooth walls, experimental results give $f = 0.005$, therefore,

$$K_0 = 0.005 \left[ 1 - \left( \frac{D_0}{D_1} \right)^4 \right]$$
Losses in Diffuser

The loss of energy in diffuser is due to (i) skin friction and (ii) expansion. Consider the divergent section shown in Figure 5, with $V_1$, $D_1$ and $V_2$, $D_2$ as the velocity and diameter at its entrance and exit, respectively.
Figure: 5 Diffuser section
Taking $f_n$ as the average value of the friction coefficient, the pressure loss in a divergent section may be expressed as

$$\Delta p = f_n \frac{\rho}{2} \int_0^{L_n} \frac{V_n^2}{D_n} dL_n$$

For an incompressible flow through the diffuser, by continuity, we have

$$D_n^2 V_n = D_1^2 V_1$$

Therefore,

$$V_n^2 = \frac{V_1^2 D_1^4}{D_n^4}$$

In the above equations the subscript $n$ stands for conditions at the exit of the $n^{th}$ section of the diffuser. This implies that, the diffuser may have $n$ sections with different divergence angles $\alpha$ between the opposite walls. In the present case, the diffuser shown in Figure 5 has only one portion with divergence angle $\alpha$, therefore, $D_n = D_2$, $V_n = V_2$. 
Substituting of the above expression for $V_n^2$ into the $\Delta \rho$ equation above, we get

$$\Delta \rho = f_n \frac{\rho}{2} \int_0^{L_n} V_1^2 D_1^4 \frac{dL_n}{D_n^5}$$  \hspace{1cm} (10)$$

From the geometry of the diffuser shown Figure 5, we have

$$D_n = D_1 + 2L_n \tan \left( \frac{\alpha}{2} \right)$$

In the differential form, this becomes

$$dD_n = 2 \tan \left( \frac{\alpha}{2} \right) dL_n$$
Therefore,

\[ dL_n = \frac{dD_n}{2 \tan \left( \frac{\alpha}{2} \right)} \]

Substitution of this into Eq. (10) results in

\[ \Delta p = f_n \frac{\rho}{2} V_1^2 D_1^4 \frac{1}{2 \tan \left( \frac{\alpha}{2} \right)} \int_{D_1}^{D_2} \frac{dD_n}{D_n^5} \]

On integration, this yields

\[ \Delta p = \frac{f_n}{8 \tan \left( \frac{\alpha}{2} \right)} \left[ 1 - \left( \frac{D_1}{D_2} \right)^4 \right] \frac{\rho}{2} V_1^2 \]

(11)
Therefore,

\[ K = \frac{\Delta p}{q} = \frac{f_n}{8 \tan \left( \frac{\alpha}{2} \right)} \left[ 1 - \left( \frac{D_1}{D_2} \right)^4 \right] \]
In terms of test-section dynamic pressure, the above equation becomes

\[ K_{01} = \frac{f_n}{8 \tan \left( \frac{\alpha}{2} \right)} \left[ 1 - \left( \frac{D_1}{D_2} \right)^4 \right] \left( \frac{D_0}{D_1} \right)^4 \]  

(12)

The expansion losses may be calculated from *Fleigners formula*, which expresses the pressure loss (except for tapered water pipe) as

\[ \Delta \rho = \frac{\rho}{2} (\sin \alpha) (V_1 - V_2)^2 \]  

(13)
Therefore,

\[ K = \sin \alpha \left[ 1 - \left( \frac{D_1}{D_2} \right)^2 \right]^2 \]

In terms of test-section dynamic pressure, the above \( K \) becomes

\[ K_{02} = \sin \alpha \left[ 1 - \left( \frac{D_1}{D_2} \right)^2 \right]^2 \left( \frac{D_0}{D_1} \right)^4 \]  \( \text{(14)} \)

The total loss coefficient for the diffuser is the sum of frictional and expansion loss coefficients. Thus,

\[ K_0 = K_{01} + K_{02} \]  \( \text{(15)} \)

The variation of pressure loss coefficients \( K_{01} \), \( K_{02} \), and \( K_0 \) with diffuser divergence angle \( \alpha \) is shown in Figure 6
It is seen from Figure 6 that, divergence angle $\alpha$ between 6 and 8 degrees proves to be the optimum for diffusers, resulting in minimum pressure loss.
Honey Combs

Wind tunnels have honey combs in the settling chamber, in order to improve the flow quality in the test-section. Usually, the honey combs are made of octagonal or hexagonal or square or circular cells with their length 5 to 10 times their width (diameter). Some typical honey combs used in wind tunnels are shown in Figure 7. The value of loss coefficient $K$, shown in Figure 7, are for honey combs with $a (\text{length/diameter}) = 6.0$, and equal tube areas. The loss in the honey combs is usually less than 5 per cent of the total loss of a tunnel.
Figure: 7 Honey combs
Guide Vanes

In wind tunnel design, it is not practical to make the corners of the return passage so gradual so that the air can follow the corner walls, with very small pressure loss. Such corners would require more space and more construction cost. Abrupt corners are therefore used, and their losses are kept to a minimum by means of corner or guide vanes. The losses in guide vanes are due to

- the skin friction of the vanes (approximately 33% of the total corner loss).
- rotational component due to change of flow direction (rest of the corner loss).

Abrupt corners without guide vanes may show a loss of even 100 per cent of velocity head. Well designed corners with guide vanes can reduce the loss by 15 to 20 per cent. Here, basically the corner is divided into many vanes of high aspect ratio, defined as the ratio of vane gap $G$ to height $h$, as shown in Figure 8.
Figure: 8 Corner-vane geometry
Commonly employed corner vanes have an aspect ratio of 6. In general, this criterion defines the vane gap since the height is known. Some typical vane profiles are shown in Figure 9, with the loss experienced under test conditions at Reynolds number around 40,000.
\[ \eta = 0.11 \]
\[ \eta = 0.138 \]
\[ \eta = 0.20 \]

Figure: 9 Corner vanes
In the corners, friction in the guide vanes accounts for about one-third of the total corner loss, and rotation losses account for the other two-thirds. For guide vanes of the type shown in Figure 9, the pressure loss coefficient $K_0$ may be calculated by the following formula, provided $\Delta p/q \approx 0.15$ (based on $Re = 500,000$, Pope and Allen, 1965).
\[ K_0 = \left[ 0.10 + \frac{4.55}{(\log_{10} R_e)^{2.58}} \right] \left( \frac{D_0}{D} \right)^4 \]  \hspace{1cm} (16)
Losses due to Open Jet Test-Section

Consider the open jet test-section, as shown in Figure 10.

Figure: 10 Open jet test-section
If the jet is assumed to be a closed jet, as shown by the dashed-lines, the losses will become very small. For example, let the length of the jet to be 1.5 times the diameter $D_0$. For a smooth wall the friction coefficient is very low and is of the order 0.008.Treating the jet as a cylindrical portion, the loss becomes, by Eq. (7),

$$K_0 = 0.008 \times 1.5 \times 100 = 1.2\%$$

Instead, when the jet is open the coefficient of friction is approximately 0.08 and therefore,

$$K_{\text{open jet}} = 0.08 \times 1.5 \times 100 = 12\%$$
That is, the jet loss for a open test-section is approximately 10 times that for a closed test-section. Further, for open test-section operation, due consideration must be given to the possibility of pulsations similar to the vibrations in an organ pipe, arising at the jet boundaries. This phenomenon, believed to be a function of jet length, can be quite serious. The simplest solution generally applied to overcome this problem is to provide vents in the diffuser, as shown in Figure 10, which connects it to atmosphere. Such an arrangement is called a breather.
Screens (wire gauze)

The prime uses of wire gauze screens in wind tunnels are the following.

(1a) To provide uniformity of velocity distribution in a duct. Consider the wire gauze screen shown in Figure 11.

![Figure: 11 Wire gauze screen]
The wire gauze offers higher resistance to a jet of a higher velocity and vice-versa. Therefore, by having number of screens, it is possible to achieve uniform velocity distribution. The resistance offered by the wire gauze to the flow is proportional to its velocity. In Figure 11, let the velocities at two locations in the flow field upstream of screen to be \( V_1 \) and \( V_2 \). Downstream of the screen, the velocities become

\[
V_1 - \Delta V_1 \approx V_2 - \Delta V_2
\]

since,

\[
\Delta V_1 \propto V_1 \quad \text{and} \quad \Delta V_2 \propto V_2
\]
(1b) In a diffusing duct, the gauze penetrates upstream, reducing the causes of non-uniformity as well as the non-uniformity itself.

For case 1a, the porosity of the screen should be carefully chosen. Wherever high value of $K_{\text{screen}}$ is required, large number of screens with small individual $K$ values should be used instead of less number of screens with high individual $K$ values. Screens should be placed at locations where the velocity should be small.
(2) To reduce the turbulence of the stream. For general purpose wind tunnel, the turbulence level should be less than 5 per cent (commonly acceptable limit). Fine gauzes are used in low turbulence wind tunnels. The components of turbulence are modified in the contraction on entering the gauze and if the porosity is suitably chosen the overall turbulence downstream of the screen can be reduced.

(3) To introduce high turbulence artificially, in a low-turbulence wind tunnel, to study the flow transition etc., with varying turbulence.

(4) To introduce known pressure drops when required in experiments.
Example 1

An open circuit subsonic wind tunnel of test-section 1.2 m × 0.9 m is run by a 110 kW motor. If the test-section speed is 90 m/s, calculate the energy ratio of the tunnel. Also, find the total loss in the tunnel in terms of test-section kinetic energy. Take the air density as the standard sea level value.
Solution

The energy ratio $ER$ is defined as

$$ER = \frac{\text{Kinetic energy of air stream in the test-section}}{\text{Input energy}}$$

$$= \frac{\frac{1}{2} \rho A_0 V_0^3}{\sum K_0 \frac{1}{2} \rho A_0 V_0^3}$$

$$= \frac{1}{\sum K_0}$$

$$= \frac{1}{1.225 \times 0.9 \times 1.2 \times 90^3}{110 \times 10^3}$$

$$= 4.38$$

The total loss $= \sum K_0 = 1/ER = 1/4.38 = 0.2283$. That is, total loss is $22.83\%$ of kinetic energy at the test-section.
Example 2

An open jet test-section of a subsonic wind tunnel expands freely into a still environment. The test-section length is 1.5 times the diameter of the contraction cone exit. The friction coefficient for the free jet is 10 times that of the closed throat with smooth wall. If the friction coefficient of the smooth wall is 0.008, determine the increase of loss when the jet is open, treating the jet as a cylindrical duct.

Solution

Given that, the length-to-diameter ratio of the jet is $L/D_0 = 1.5$ and the friction coefficient of the closed jet wall, $f = 0.008$. 
For cylindrical ducts, the pressure loss can be expressed as

\[ \Delta p = \frac{\rho V^2 L}{D_f} \]

Thus, the loss coefficient becomes

\[ K_0 = \frac{\Delta p}{\rho V^2} = \frac{L}{D_0 f} \]

where \( V_0 \) is the test-section velocity. Thus,

\[ K_0 = 1.5 \times 0.008 = 0.012 \]

For open jet, \( f = 0.08 \), therefore,

\[ K_0 = 1.5 \times 0.08 = 0.12 \]

Thus, increase of loss for the open jet is \((0.12 - 0.012) = 0.108\), i.e. \(10.8\%\).
Example 3

A subsonic open-circuit wind tunnel runs with a test-section speed of 40 m/s. The temperature of the lab environment is 16°C. If a turbulent sphere measures the turbulence factor, $TF$ (defined as the ratio of the theoretical critical Reynolds number for the sphere to the actual critical Reynolds number) of the tunnel as 1.2, determine the sphere diameter. Assume the test-section pressure as the standard sea level pressure.
Solution

Density of air in the test-section can be expressed as

$$\rho = \frac{p}{RT}$$

where $p$ is sea level pressure, and $R$ is the gas constant for air. The test-section temperature, $T$, can be determined as follows.

By energy equation, we have

$$h_0 = h + \frac{V^2}{2}$$
For perfect gas, \( h = c_p T \), thus,

\[
c_p T_0 = c_p T + \frac{V^2}{2}
\]

\[
T = T_0 - \frac{V^2}{2c_p}
\]

where \( T_0 \) is the test-section stagnation temperature, which is same as the lab temperature and \( c_p \) is the specific heat at constant pressure. Thus,

\[
T = 289.15 - \frac{40^2}{2 \times 1004.5}
\]

\[
= 288.35 \text{ K}
\]
The flow density at the test-section becomes

$$\rho = \frac{101325}{287 \times 288.35} = 1.224 \text{ kg/m}^3$$

The test-section Reynolds number is

$$Re = \frac{\rho V d}{\mu}$$

where $d$ is the sphere diameter and $\mu$ is dynamic viscosity coefficient at 288.35 K. The viscosity coefficient at 288.35 K is

$$\mu = 1.46 \times 10^{-6} \times \frac{288.35^{3/2}}{288.35 + 111}$$

$$= 1.79 \times 10^{-5} \text{ kg/(m s)}$$
The critical Reynolds number becomes

\[ Re_c = \frac{1.224 \times 40 \times d}{1.79 \times 10^{-5}} \]

The turbulence factor, \( TF \), is given by

\[ TF = \frac{385000}{Re_c} \]

\[ Re_c = \frac{385000}{1.2} = 320833.3 \]

Thus,

\[ d = \frac{Re_c \times 1.79 \times 10^{-5}}{1.224 \times 40} \]

\[ = \frac{320833.3 \times 1.79 \times 10^{-5}}{1.224 \times 40} = 0.1173 \text{ m} \]

\[ = 11.73 \text{ cm} \]
Example 4

A closed-return type wind tunnel of large contraction ratio has air at standard sea-level conditions in the settling chamber upstream of the contraction to the test-section. Assuming isentropic compressible flow in the tunnel, estimate the speed and the kinetic energy per unit area in the working section when the Mach number is 0.75.
Solution

The given sea-level pressure and temperature upstream of the test-section can be taken as the stagnation values, i.e. $p_0 = 101325$ Pa, and $T_0 = 288.15$ K. Also, the $p_0$ and $T_0$ are invariants, since the flow is isentropic.

By isentropic relation, we have

\[
T = T_0 \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}
\]

\[
= 288.15(1 + 0.2 \times 0.75^2)^{-1}
\]

\[
= 259 \text{ K}
\]
Also, the pressure in the test-section is given by

\[ p = p_0 \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} \]

\[ = \frac{101325}{(1 + 0.2 \times 0.75^2)^{3.5}} \]

\[ = 69769.65 \text{ Pa} \]

Therefore, the flow density in the test-section is

\[ \rho = \frac{p}{RT} = \frac{69769.65}{287 \times 259} \]

\[ = 0.9386 \text{ kg/m}^3 \]
Thus, the speed and the kinetic energy in the test-section becomes

$$V = Ma = 0.75 \times \sqrt{\gamma RT}$$
$$= 0.75 \times \sqrt{1.4 \times 287 \times 259}$$
$$= 242 \text{ m/s}$$

$$ke = \frac{1}{2} \rho V^3 = 0.5 \times 0.9386 \times 242^3$$
$$= 6.65 \text{ MW/m}^2$$
High-Speed Wind Tunnels

High-speed tunnels are those with test-section speed more than 650 kmph. The power to drive a low-speed wind tunnel varies as the cube of the test-section velocity. Although this rule is not valid for the high-speed regime, the implication of rapidly increasing power requirement with increasing test-section speed is true for high-speed tunnels also. Because of the power requirements, high-speed wind tunnels are often of the intermittent type in which energy is stored in the form of pressure or vacuum or both and is allowed to drive the tunnel only a few seconds out of each pumping hour.
High-speed tunnels are generally grouped into intermittent and continuous operation tunnels, based on the type of operation. The intermittent tunnels are further divided into blowdown tunnels and induction tunnels, based on type of the operational procedure.
Even though the flow in the Mach number range from 0.5 to 5.0 is usually termed high-speed flow, the tunnels with test–section Mach number less than 0.9 are generally grouped and treated under subsonic wind tunnels. Wind tunnels with Mach numbers from 1.5 to 5.0 are classified as supersonic tunnels and those with Mach number more than 5 are termed hypersonic tunnels. The wind tunnels in the Mach number range from 0.9 to 1.5 are called transonic tunnels.
The intermittent blowdown and induction tunnels are normally used for Mach numbers from 0.5 to about 5.0, and the intermittent pressure-vacuum tunnels are normally used for higher Mach numbers. The continuous tunnel is used throughout the speed range. Both intermittent and continuous tunnels have their own advantages and disadvantages.
Blowdown Type Wind Tunnels

Essential features of the intermittent blowdown wind tunnels are schematically shown in Figure 12.

Figure: 12 Schematic layout of intermittent blowdown tunnel
Advantages

The main advantages of blowdown type wind tunnels are the following.

- They are the simplest among the high-speed tunnel types and most economical to build.
- Large size test-sections and high Mach numbers (up to $M = 4$) can be obtained.
- Constant blowing pressure can be maintained and running time of considerable duration can be achieved.

These are the primary advantages of intermittent blowdown tunnels. In addition to these, there are many additional advantages for this type of tunnels, like, a single drive may easily run several tunnels of different capabilities, failure of a model usually will not result in tunnel damage. Extra power is available to start the tunnel and so on.
Disadvantages

The major disadvantages of blowdown tunnels are the following.

- Charging time to running time ratio will be very high for large size tunnels.

- Stagnation temperature in the reservoir drops during tunnel run, thus changing the Reynolds number of the flow in the test-section.

- Adjustable (automatic) throttling valve between the reservoir and settling chamber is necessary for constant stagnation pressure (temperature varying) operation.

- Starting load is high (no control possible).

- Reynolds number of flow is low due to low static pressure in the test-section.
The commonly employed reservoir pressure range is from 100 to 300 psi for blowdown tunnel operations. As large as 2000 psi is also used where space limitations required.
Induction Type Tunnels

In this type of tunnels, a vacuum created at the downstream end of the tunnel is used to establish the flow in the test-section. A typical induction tunnel circuit is shown schematically in Figure 12.

![Diagram of Induction Tunnel](image)

**Figure:** 13 Schematic diagram of induction tunnel
Advantages

The advantages of induction tunnels are the following.

- Stagnation pressure and stagnation temperature are constants.
- No oil contamination in air, since the pump is at the downstream end.
- Starting and shutdown operations are simple.
Disadvantages

The disadvantages of induction type supersonic tunnels are the following.

- Size of the air drier required is very large, since it has to handle a large mass flow in a short duration.
- Vacuum tank size required is also very large.
- High Mach numbers ($M > 2$) are not possible because of large suction requirements for such Mach numbers.
- Reynolds number is very low, since the stagnation pressure is atmospheric.
The above mentioned blowdown and induction principles can also be employed together for supersonic tunnel operation to derive the benefits of both the types.
Continuous Supersonic Wind Tunnels

The essential features of a continuous flow supersonic wind tunnel is shown in Figure 14.

Figure: 14 Schematic of closed-circuit supersonic wind tunnel
Like intermittent tunnels, the continuous tunnels also have some advantages and disadvantages.

The main advantages of continuous supersonic wind tunnels are the following.

- **Better control** over the Reynolds number possible, since the shell is pressurized.
- Only a **small capacity** drier is required.
- Testing conditions can be held the same over a long period of time.
- The test-section can be designed for **high Mach numbers** ($M > 4$) and large size models.
- Starting load can be **reduced** by starting at low pressure in the tunnel shell.
Major **disadvantages** of continuous supersonic tunnels are the following.

- Power required is **very high**.
- Temperature stabilization requires **large size** cooler.
- Compressor drive to be designed to **match** the tunnel characteristics.
- Tunnel design and operation are more **complicated**.
It is seen from the foregoing discussions that, both intermittent and continuous tunnels have certain specific advantages and disadvantages. Before going into the specific details about supersonic tunnel operation, it will be useful to note the following details about supersonic tunnels.
Axial flow compressor is better suited for large pressure ratio and mass flow.

Diffuser design is critical since increasing diffuser efficiency will lower the power requirement considerably. Supersonic diffuser portion (geometry) must be carefully designed to decrease the Mach number of the flow to be as low as possible, before shock formation. Subsonic portion of the diffuser must have an optimum angle, to minimize the frictional and separation losses.
Proper nozzle **geometry** is very important to obtain good distribution of Mach number and freedom from flow angularity in the test-section. Theoretical calculation to high accuracy and boundary layer compensation, etc., have to be carefully worked out for large test-sections. Fixed nozzle blocks for different Mach numbers is simple but very expensive and quite laborious for change over in the case of large size test-sections. Flexible wall type nozzle is **complicated and expensive** from design point of view and Mach number range is limited (usually $1.5 < M < 3.0$).

Model size is determined from the **test-rhombus**, shown in Figure 15.
The model must be accommodated \textit{inside} the rhombus formed by the incident and reflected shocks, for proper measurements.
Losses in Supersonic Tunnels

The total power loss in a continuous supersonic wind tunnel may be split into the following components.

1. Frictional losses (in the return circuit).
2. Expansion losses (in diffuser).
3. Losses in contraction cone and test-section.
4. Losses in guide vanes.
5. Losses in cooling system.
7. Losses due to model and support system drag.
The first five components of losses represent the usual low-speed tunnel losses. All the five components together constitute only about 10 per cent of the total loss. Components 6 and 7 are additional losses in a supersonic wind tunnel and usually amount to approximately 90 per cent of the total loss, with shock wave losses alone accounting to nearly 80 per cent and model and support system drag constituting nearly 10 per cent of the total loss. Therefore, it is customary in estimating the power requirements to determine pressure ratio required for supersonic tunnel operation that, the pressure ratio across the diffuser alone is considered and a correction factor is applied to take care of the rest of the losses.
The pressure ratio across the diffuser multiplied by the correction factor must therefore be equal to the pressure ratio required across the compressor to run the tunnel continuously. The relationship between these two vital pressure ratios, namely the diffuser pressure ratio, \( p_{01}/p_{02} \), and compressor pressure ratio, \( p_{0c}/p_{03} \), may be related as follows.

\[
\text{Compressor pressure ratio} \quad \frac{p_{0c}}{p_{03}} = \frac{p_{01}}{p_{02}} \frac{1}{\eta} \quad (17)
\]

where

- \( p_{0c} = \) stagnation pressure at compressor exit,
- \( p_{03} = \) stagnation pressure at compressor inlet,
- \( p_{01} = \) stagnation pressure at diffuser inlet, and
- \( p_{02} = \) stagnation pressure at diffuser exit, and

\[
\eta = \frac{\text{Diffuser losses}}{\text{Total loss}}
\]

is the correction factor.
The value of $\eta$ varies from 0.6 to 0.85, depending on the kind of shock pattern through which the pressure recovery is achieved in the diffuser. The variation of compressor pressure ratio, $p_{0c}/p_{03}$, with the test-section Mach number, $M$, is shown in Figure 16.
Supersonic Wind Tunnel Diffusers

Basically diffuser is a device to convert the kinetic energy of a flow to pressure energy. The diffuser efficiency may be defined, in two ways, as

1. Polytropic efficiency $\eta_d$.
2. Isentropic efficiency $\eta_\sigma$. 
Polytropic Efficiency

It is known that, at any point in a diffuser a small change of kinetic energy of unit mass of fluid results in increase of pressure energy as per the equation

$$\Delta p = \eta_d d \left( \frac{V^2}{2} \right)$$  \hspace{1cm} (18)

and the pressure ratio is given by

$$\frac{p_{02}}{p_1} = \left( \frac{T_{02}}{T_1} \right)^{\frac{\gamma}{\gamma-1} \eta d} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma-1} \eta d}$$  \hspace{1cm} (19)

where $p_1$ and $p_{02}$ are the static and stagnation pressures upstream and downstream of the point under consideration, respectively, and $\eta_d$ is the polytropic efficiency. $M_1$, $T_1$, and $T_{02}$, respectively, are the Mach number, static and stagnation temperatures at the appropriate locations.
Isentropic Efficiency

The isentropic efficiency of a diffuser may be defined as

\[ \eta_\sigma = \frac{\text{Ideal KE required for observed power}}{\text{Actual KE transferred}} \]

and

\[
\text{Ideal KE from } p_1 \text{ to } p_{02} (\text{without loss}) = \int_{p_1}^{p_{02}} \frac{dp}{\rho} = \frac{\gamma}{\gamma - 1} \frac{p_1}{\rho_1} \left[ \left( \frac{p_{02}}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]
\]

(20)
Note that, in Eqs. (3.19) and (3.20) the velocity at the diffuser outlet is assumed to be negligible, that is why the pressure at location 2 is taken as $p_{02}$, the stagnation pressure. With Eq. (20) the isentropic efficiency, $\eta_\sigma$, becomes

$$\eta_\sigma = \frac{\gamma}{\gamma - 1} \rho_1 \frac{p_1}{\rho_1} \left[ \frac{p_{02}}{p_1} \frac{\gamma - 1}{\gamma} - 1 \right] = \frac{2}{\gamma - 1} \frac{1}{M_1^2} \left[ \left( \frac{p_{02}}{p_1} \right) \frac{\gamma - 1}{\gamma} - 1 \right]$$
From the above equation, the pressure ratio $p_{02}/p_1$ becomes

$$\frac{p_{02}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \eta_\sigma\right) \frac{\gamma}{\gamma - 1}$$  \hspace{1cm} (21)

From Eqs. (3.19) and (3.21), we get

$$\left(1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\eta_d} = \left(1 + \frac{\gamma - 1}{2} M_1^2 \eta_\sigma\right)$$  \hspace{1cm} (22)
Let $H$ to be total pressure (total head) upstream of the test-section, and $p_1$ to be the static pressure there, then we have by isentropic relation,

$$\frac{H}{p_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right) \frac{\gamma}{\gamma - 1}$$

(23)

Therefore, the overall pressure ratio, $H/p_{02}$, for the tunnel becomes

$$\frac{H}{p_{02}} = \frac{H}{p_1} \frac{p_1}{p_{02}}$$
But this is also the compressor pressure ratio required to run the tunnel. Hence, using Eqs. (3.21) and (3.23), the compressor pressure ratio, \( p_\sigma \), can be expressed as

\[
p_\sigma = \frac{H}{p_{02}} = \left[ \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2 \eta_\sigma} \right] \frac{\gamma}{\gamma - 1}
\]  

(24)

For continuous and intermittent supersonic wind tunnels, the energy ratio, \( ER \), may be defined as follows.
1. For continuous tunnel

\[ ER = \frac{KE \text{ at the test-section}}{\text{Work done in isentropic compression per unit time}} \]

Using Eq. (24), this may be expressed

\[ ER = \frac{1}{\left( \frac{\gamma^{-1}}{p_0^{\gamma}} - 1 \right) \left( \frac{2}{(\gamma - 1)M_1^2} + 1 \right)} \]  \hspace{1cm} (25)

2. For intermittent tunnel

\[ ER = \frac{(KE \text{ in test-section})(\text{Time of tunnel run})}{\text{Energy required for charging the reservoir}} \]  \hspace{1cm} (26)
From the above discussions we can infer that,

- For $M < 1.7$, induced flow tunnels are more efficient than blowdown tunnels.

- In spite of this advantage, most of the supersonic tunnels even over this Mach number range are operated as blowdown tunnels and not as induced flow tunnels. This is because vacuum tanks are more expensive than compressed air storage tanks.
Effects of Second Throat

A typical supersonic tunnel with second throat is shown schematically in Figure 17.

Figure: 17 Schematic of supersonic wind tunnel with second throat
The second throat, shown in Figure 17, is used to provide isentropic deceleration and highly efficient pressure recovery after the test-section. Neglecting frictional and boundary layer effects, a wind tunnel can be run at design conditions indefinitely, with no pressure difference requirement to maintain the flow, once started. But this is an ideal situation which cannot be encountered in practice. Even under the assumptions of this ideal situation, during start-up a pressure difference must be maintained across the entire system, shown in Figure 17, to establish the flow.
For the supersonic tunnel sketched in Figure 17, the following observation may be made.

- As the pressure ratio $p_{0e}/p_{0i}$ is decreased below 1.0, the flow situation is the same as that in a convergent-divergent nozzle, where $p_{0i}$ and $p_{0e}$ are the stagnation pressures at the nozzle inlet and the diffuser exit, respectively.

- Now, any further decrease in $p_{0e}/p_{0i}$ would cause a shock to appear downstream of nozzle throat.

- Further decrease in $p_{0e}/p_{0i}$ moves the shock downstream, towards the nozzle exit.

- With a shock in the diverging portion of the nozzle, there is a severe stagnation pressure loss in the system.

- To pass the flow after the shock, the second throat must be at least of an area $A_2^*$. 
The worst case causing maximum loss of stagnation pressure is that with a normal shock in the test-section. For this case, the second throat area must be at least $A_2^*$. If the second throat area is less than this, it cannot pass the required flow and the shock can never reach the test-section, and will remain in the divergent part of the nozzle.

Under these conditions, supersonic flow can never be established in the test-section.

As $p_{0e}/p_{0i}$ is further lowered, the shock jumps to an area in the divergent portion of the diffuser which is greater than the test-section area, i.e. the shock is swallowed by the diffuser.

To maximize the pressure recovery in the diffuser, $p_{0e}/p_{0i}$ can now be increased, which makes the shock to move upstream to the diffuser throat, and the shock can be positioned at the location where the shock strength is the minimum.
From the above observations, it is evident that the second throat area must be large enough to accommodate the mass flow, when a normal shock is present in the test-section. Assuming the flow to be one-dimensional in the tunnel sketched in Figure 17, it can be shown from continuity equation that,

\[ \rho_1^* a_1^* A_1^* = \rho_2^* a_2^* A_2^* \]
The flow process across a normal shock is adiabatic and therefore,

\[ T_1^* = T_2^* \]

and

\[ \frac{\rho_1^*}{\rho_2^*} = \frac{p_1^*}{p_2^*} = \frac{p_{01}}{p_{02}} \]

Also,

\[ a_2^* = a_1^* \]

since \( T_1^* = T_2^* \).
Therefore, the minimum area of the second throat required for starting the tunnel becomes

\[
\frac{A_2^*}{A_1^*} = \frac{p_{01}}{p_{02}}
\]  

(27)

where \( p_{01} \) and \( p_{02} \) are the stagnation pressures upstream and downstream, respectively, of the normal shock just ahead of the second throat. The pressures \( p_{01} \) and \( p_{02} \) are identically equal to \( p_{0i} \) and \( p_{0e} \), respectively. Instead of the ratio of the throats area, it is convenient to deal with the ratio of test-section area \( A_1 \) to diffuser throat area \( A_2^* \). This is called the diffuser contraction ratio, \( \psi \).
Thus, the maximum permissible contraction ratio for starting the tunnel is given by

\[ \psi_{\text{max}} = \frac{A_1}{A_2^*} = \frac{A_1}{A_1^*} \times \frac{A_1^*}{A_2^*} = \frac{A_1}{A_1^*} \times \frac{p_{02}}{p_{01}} = f(M_1) \]  

(28)

when the second throat area is larger than the minimum required for any given condition, the shock wave is able to 'jump' from the test-section to the downstream side of diffuser throat. This is termed shock swallowing.
The complete test-section has supersonic flow, which is the required state for a supersonic wind tunnel test-section. However, the second throat and part of the diffuser also have supersonic flow. Apparently we have only shifted the shock from test-section to the diffuser. This again will result in considerable loss. In principle, it is possible to bring down the loss to a very low level by reducing the area of the second throat, after starting the tunnel. As $A_2^*$ is reduced, the shock becomes weaker (as seen from Eq. (3.27)) and moves upstream towards the second throat. When $A_2^* = A_1^*$, the shock just reaches the second throat, and its strength becomes vanishingly small. This is the ideal situation, resulting in supersonic flow in the test-section and isentropic flow in the diffuser.
At this stage, we should realize that the above model is based on the assumption that the flow is one-dimensional and inviscid, with a normal shock in the test-section. A more realistic model might have to take into account the non-stationary effects of the shock, the possibility of oblique shocks, and the role of boundary layer development. Further, reduction of $A_2^*$ to $A_1^*$, which is the ideal value, is not possible in practice. However, some contraction after starting is possible, up to a limiting value at which the boundary layer effects prevent the maintenance of sufficient mass flow for maintaining a supersonic test-section, and beyond that the flow breaks down.
Experimental studies confirm, in a general way, the theoretical considerations outlined above, although there are modifications owing to viscous effects. The skin friction at the wall of course causes some additional loss of stagnation pressure. Some of the diffuser problems outlined here may be avoided to a large extent by

- Using variable-geometry diffuser.
- Using variable-geometry diffuser in conjunction with variable-geometry nozzle.
- Driving the shock through the diffuser throat by means of a large-amplitude pressure pulse.
- Taking advantage of effects which are not one-dimensional.
Compressor Tunnel Matching

Usually the design of a continuous supersonic wind tunnel has either of the following two objectives.

1. Choose a compressor for specified test-section size, Mach number, and pressure level.
2. Determine the best utilization of an already available compressor.

In the first case, wind tunnel characteristics govern the selection of compressor and in the second case it is the other way about. In either case the characteristics to be matched are the *overall pressure ratio* and *mass flow*. 
The compressor characteristics are usually given in terms of the \textit{volumetric flow} $V$ rather than mass flow. Therefore, it is convenient to give the wind tunnel characteristics also in terms of $V$. We know that, the volume can be expressed as

$$V = \frac{m}{\rho}$$

since the density $\rho$ varies in the tunnel circuit, the volumetric flow also varies for a given constant mass flow $m$. 
For the compressor, we specify the intake flow as

\[ V_i = \frac{m}{\rho_i} \]  \hspace{1cm} (29)

which is essentially the same as the volume flow at the diffuser exit. On the other hand, the volume flow at the supply section (wind tunnel settling chamber) is

\[ V_0 = \frac{m}{\rho_0} \]  \hspace{1cm} (30)
Using throat as the reference section, the mass flow can be expressed as

\[
m = \rho^* a^* A^* = \left( \frac{2}{\gamma + 1} \right) \frac{\gamma + 1}{2(\gamma - 1)} \rho_0 a_0 A^* \tag{31}
\]

where \( a^* \) and \( a_0 \) are the sonic speeds at the throat and stagnation state, respectively.

With Eq. (31), Eq. (30) can be rewritten as

\[
V_0 = \left( \frac{2}{\gamma + 1} \right) \frac{\gamma + 1}{2(\gamma - 1)} a_0 A^*
\]

\[
= \left( \frac{2}{\gamma + 1} \right) \frac{\gamma + 1}{2(\gamma - 1)} \sqrt{\gamma RT_0} \frac{A^*}{A} A = \text{constant} \sqrt{T_0} A \left( \frac{A^*}{A} \right)
\]
From this equation it is seen that, the volume flow rate $V_0$ depends on the stagnation temperature, test-section area, and test-section Mach number (since $A/A^*$ is a function of $M$).

The compressor intake flow and the supply section (settling chamber) flow may easily be related, using Eqs. (3.29) and (3.30), to result in

$$\frac{V_i}{V_0} = \frac{\rho_0}{\rho_i} = \frac{p_0}{p_i} \times \frac{T_i}{T_0} = \Lambda$$  \hspace{1cm} (32)

since $T_i = T_0$, $\Lambda$ is simply the pressure ratio at which the tunnel is actually operating. This pressure ratio $\Lambda$ must always be more than the minimum pressure ratio required for supersonic operation at any desired Mach number.
Equation (3.32) gives the relation between the operating pressure ratio, $\Lambda$, and the compressor intake volume, $V_0$, as

$$\Lambda = \left(\frac{1}{V_0}\right) V_i$$

The plot of $\Lambda$ verses $V_i$ is a straight line, through the origin, with slope $1/V_0$, as shown in Figure 18 (Liepmann and Roshko, 1956).
Figure: 18 Wind tunnel and compressor characteristics (a) matching of wind tunnel compressor characteristics (one test-section condition): n, matching point; b, matching point with by-pass; 0, match point at minimum operating pressure ratio. (b) Operation over a range of $M$, using multistage compressor.
The power requirement for a multistage compressor is given by

\[
HP = \left( \frac{1}{746} \right) \left( \frac{N \gamma}{\gamma - 1} \right) \dot{m}RT_s \left[ \left( \frac{p_{0c}}{p_{03}} \right) ^{\frac{\gamma - 1}{\gamma N}} - 1 \right]
\]

(33)

where \(\dot{m}\) is the mass flow rate of air in kg/s, \(p_{03}\) and \(p_{0c}\) are the total pressures at the inlet and outlet of the compressor, respectively, \(N\) is the number of stages, and \(T_s\) is the stagnation temperature.
Example 4

Determine the minimum possible diffuser contraction ratio and the power required for a 2 stage compressor to run a close circuit supersonic tunnel at $M = 2.2$. The efficiency of the compressor is 85 per cent, $p_{01} = 4$ atm, $T_0 = 330$ K and $A_{TS} = 0.04 \, m^2$. 
Solution

Compressor pressure ratio is

\[
\frac{p_{0c}}{p_{03}} = \frac{p_{01}}{p_{02}} \frac{1}{\eta}
\]

Given that, \(M = 2.2\), \(\eta = 0.85\), \(N = 2\), \(T_0 = 330\) K, \(p_{01} = 4\) atm, \(A_{TS} = 0.04\) m\(^2\).

For \(M_1 = 2.2\), \(\frac{p_{02}}{p_{01}} = 0.6281\), from normal shock table, and \(\frac{A_1}{A_1^*} = 2.005\), from isentropic table.
Therefore, the maximum possible contraction ratio becomes

\[ \psi_{\text{max}} = \frac{A_1}{A_1^*} \times \frac{p_{02}}{p_{01}} \]

\[ = 2.005 \times 0.6281 \]

\[ = 1.26 \]
The mass flow rate is given by

\[ \dot{m} = \frac{0.6847}{\sqrt{R T_0}} p_0 A^* \]

\[ = \frac{0.6847 \times 4 \times 101325}{\sqrt{287 \times 330}} \times 0.04 \times 2.005 \]

\[ = 17.99 \text{ kg/s} \]
The power required to run the tunnel is

\[
\text{Power} = \frac{1}{746} \times \frac{2 \times 1.4}{0.4} \times 17.99 \times 287 \\
\times 330 \left[ \left( \frac{1}{0.6281/0.85} \right)^2 - 1 \right] \\
= 1499.57 \text{ hp}
\]
Basic Formulas for Supersonic Wind Tunnel Calculations

From our discussions so far, it is easy to identify that the following are the important relations required for supersonic tunnel calculations.

\[
\frac{p_1}{p_2} = \left( \frac{\rho_1}{\rho_2} \right)^{\gamma} = \left( \frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma - 1}}
\]

\[
a = \sqrt{\gamma RT} = 20.04 \sqrt{T} \text{ m/s}
\]

\[
\frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{V^2}{2} = \text{constant} = \frac{\gamma}{\gamma - 1} \frac{p_t}{\rho_t}
\]

where \( p_t \) and \( \rho_t \) are the stagnation pressure and density, respectively.
\[ \frac{p_2}{p_1} = \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma - 1}} \]

\[ \frac{p_t}{p} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \]

\[ \frac{\rho_t}{\rho} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{1}{\gamma - 1}} \]

\[ \frac{T_t}{T} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \]

where \( p, \rho \) and \( T \) are the local pressure, density, and temperature, respectively, and \( p_1 \) and \( p_2 \) are the pressures upstream and downstream of a normal shock.
The Mass Flow

Mass flow rate is one of the primary considerations in sizing a wind tunnel test-section and the associated equipments, such as compressor and diffuser. The mass flow rate is given by

\[ \dot{m} = \rho AV \]

From isentropic relations, for air with \( \gamma = 1.4 \), we have

\[ \rho = \rho_t (1 + 0.2M^2)^{-\frac{5}{2}} \]

where \( \rho_t \) is the total or stagnation density. By perfect gas state equation, we have

\[ \rho_t = \frac{p_t}{R T_t} \]
Therefore,

\[
\rho = \left( \frac{p_t}{R T_t} \right) (1 + 0.2M^2)^{-\frac{5}{2}}
\]

where, \( R = 287 \text{ m}^2/(\text{s}^2 \text{ K}) \) is the gas constant for air, \( p_t \) is the total pressure in pascal, and \( T_t \) is the total temperature in kelvin.

Also, the local temperature and velocity are given by

\[
T = T_t(1 + 0.2M^2)^{-1}
\]

\[
V = M(1.4 R T)^{\frac{1}{2}}
\]
Substituting the above expression for $T$ into $V$ expression, we get

$$V = M \left[ \frac{1.4 R T_t}{(1 + 0.2 M^2)} \right]^{\frac{1}{2}}$$

Using the above expressions for $V$ and $\rho$ in the $\dot{m}$ equation, we get the mass flow rate as

$$\dot{m} = \left( \frac{1.4}{R T_t} \right)^{\frac{1}{2}} \frac{M \rho_t A}{(1 + 0.2 M^2)^3}$$  \hspace{1cm} (34)
This equation is valid for both subsonic and supersonic flows. When the mass flow rate being calculated is for subsonic Mach number, Eq. (3.34) is evaluated using test-section Mach number in conjunction with the total temperature and pressure. For supersonic flows, it is usually convenient to make the calculations at the nozzle throat, where the Mach number is 1.0. Further, it should be noted that blowdown tunnels are usually operated at a constant pressure during run. The main objective of constant pressure run is to obtain a steady flow while data being recorded. Thus, the total pressures to be used in the evaluation of Eq. (34) are the minimum allowable (or required) operating pressures.
Example 5

A continuous wind tunnel operates at Mach 2.5 at test-section, with static conditions corresponding to 10,000 m altitude. The test-section is 150 mm $\times$ 150 mm in cross-section, with a supersonic diffuser downstream of the test-section. Determine the power requirements of the compressor during start-up and during steady-state operation. Assume the compressor inlet temperature to be the same as the test-section stagnation temperature.
Solution

At the test-section, $M = 2.5$. At 10000 m altitude, from atmospheric table, we have

$$p = 26.452 \text{ kPa}, \quad T = 223.15 \text{ K}$$

These are the pressure and temperature at the test-section.

From isentropic table, for $M = 2.5$, we have

$$\frac{p}{p_0} = 0.058528 \quad \frac{T}{T_0} = 0.4444$$
Therefore, the stagnation pressure and temperature at the test-section are,

\[
p_0 = \frac{26.452}{0.058528} = 451.95 \text{ kPa}
\]

\[
T_0 = \frac{223.15}{0.44444} = 502.1 \text{ K}
\]
During steady-state operation, the mass flow rate through the test-section is

\[ \dot{m} = \rho A V \]

\[ = \frac{\rho}{RT} A M \sqrt{\gamma RT} \]

\[ = \frac{26452}{287 \times 223.15} (0.15 \times 0.15)(2.5)\sqrt{1.4 \times 287 \times 223.15} \]

\[ = 6.96 \text{ kg/s} \]

From isentropic table, for \( M = 2.5 \), we have

\[ \frac{A}{A^*} = 2.63671 \]
Therefore,

\[ A^* = \frac{0.15 \times 0.15}{2.63671} \]

\[ = 0.00853 \text{ m}^2 \]

This is the area of the first throat.
During start-up, a shock wave is formed when the flow becomes supersonic. The pressure loss due to this shock is maximum when it is at the test-section.

For \( M = 2.5 \), from normal-shock table, we have

\[
\frac{p_{02}}{p_{01}} = 0.499
\]

Also, we know that,

\[
\frac{p_{02}}{p_{01}} = \frac{A_1^*}{A_2^*}
\]
Therefore,

\[
\frac{A_1^*}{A_2^*} = 0.499
\]

\[
A_2^* = \frac{A_1^*}{0.499} = \frac{0.00853}{0.499}
\]

Thus,

\[
\frac{A}{A_2^*} = \frac{0.15 \times 0.15}{0.00853} \times 0.499
\]

\[
= 1.316
\]

For this area ratio, from isentropic table, we get \( M = 1.68 \). This is the Mach number ahead of the shock when the shock is at the second throat.
For $M = 1.68$, from normal shock table, we have

$$\frac{p_{02}}{p_{01}} = 0.86394$$

This pressure loss must be compensated by the compressor. The power input required for the compressor to compensate for this loss is

$$\text{Power} = h_o - h_i = C_p (T_o - T_i)$$

where the subscripts $o$ and $i$, respectively, refer to compressor outlet and inlet conditions.
For an isentropic compressor,

\[
\frac{T_o}{T_i} = \left( \frac{p_o}{p_i} \right)^{\frac{\gamma - 1}{\gamma}}
\]

\[
T_o - T_i = T_i \left( \left( \frac{p_o}{p_i} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right)
\]

\[
= 502.1 \left( \left( \frac{1}{0.86394} \right)^{0.286} - 1 \right)
\]

\[
= 21.44 \text{ K}
\]
Thus, the power input required becomes

\[ \text{Power} = 1004.5 \times 21.44 = 21543.8 \text{ J/kg} \]

The horse power required for the compressor is

\[
\text{Power} = \frac{\dot{m} \ W}{746} = \frac{6.96 \times 21543.8}{746} = 201 \text{ hp}
\]

This is the running horse power required for the compressor.
During start-up, $M_1 = 2.5$, the corresponding $p_{02}/p_{01} = 0.499$, from normal shock table.

The isentropic work required for the compressor during start-up is

$$W = \left[ \left( \frac{1}{0.499} \right)^{0.286} - 1 \right] 502.1 \, C_p$$

$$= 110.4 \, C_p$$

$$= 1004.5 \times 110.4$$

$$= 110896.8 \, J/kg$$
Thus, the power required is

\[
\text{Power} = \frac{6.96 \times 110896.8}{746}
\]

\[= \boxed{1034.7 \text{ hp}}\]
Example 6

Estimate the settling chamber pressure and temperature and the area ratio required to operate a Mach 2 tunnel under standard sea-level conditions. Assume the flow to be one-dimensional and the tunnel is operating with correct expansion.
Solution

The tunnel is operating with correct expansion. Therefore, the sea-level pressure and temperature become the pressure and temperature in the test-section (i.e. at the nozzle exit). Thus, $p_e = 101.325$ kPa and $T_e = 15^\circ C$.

This problem can be solved by using the appropriate equations or by using gas tables. Let us solve the problem by both the methods.
Solution using equations

Let subscripts \( e \) and \( 0 \) refer to nozzle exit and stagnation states, respectively.
From isentropic relations, we have the temperature and pressure ratio as

\[
\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} \frac{M_e^2}{e}
\]

\[
= 1 + \frac{1.4 - 1}{2} \times 2^2 = 1.8
\]

\[
T_0 = 1.8 \times T_e = 1.8 \times 288.15 = 518.67 \text{ K}
\]
\[
\frac{p_0}{p_e} = \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{\gamma/\gamma-1}
\]

\[
= 1.8^{3.5} = 7.824
\]

\[
p_0 = 7.824 p_e
\]

\[
= 792.77 \text{ Pa}
\]
Form isentropic relations we have the area ratio as

\[
\left( \frac{A_e}{A_{th}} \right)^2 = \frac{1}{M_e^2} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right) \right]^{\frac{\gamma + 1}{\gamma - 1}}
\]

\[
= \frac{1}{2^2} \left[ \frac{2}{2.4} \times 1.8 \right]^6 = 2.8476
\]

\[
\frac{A_e}{A_{th}} = 1.687
\]
Solution using gas tables

From isentropic table, for $M_e = 2$, we have

\[
\frac{p_e}{p_0} = 0.1278, \quad \frac{T_e}{T_0} = 0.55556, \quad \frac{A_e}{A_{th}} = 1.6875
\]
Thus,

\[ p_0 = \frac{p_e}{0.1278} \]
\[ = \frac{101325}{0.1278} = 792.84 \text{ Pa} \]

\[ T_0 = \frac{T_e}{0.55556} \]
\[ = \frac{288.15}{0.55556} = 518.67 \text{ K} \]
Blowdown Tunnel Operation

In a blowdown tunnel circuit, the pressure and temperature of air in the compressed air reservoir (also called storage tank) change during operation. This change of reservoir pressure causes the following effects.

- The tunnel stagnation and settling chamber pressures fall correspondingly.
- The tunnel is subjected to dynamic condition.
- Dynamic pressure in the test-section falls and hence, the forces acting on the model change during test.
- Reynolds number of the flow changes during tunnel run.
Usually three methods of operation are adopted for blowdown tunnel operation. They are

- Constant **Reynolds number** operation.
- Constant **pressure** operation.
- Constant **throttle** operation.

The ratio between the settling chamber initial pressure $p_{bi}$ and reservoir initial pressure $p_{0i}$ is an important parameter influencing the test-section Reynolds number. Let

$$\frac{p_{bi}}{p_{0i}} = \frac{\text{Settling chamber initial pressure}}{\text{Reservoir initial pressure}} = \alpha$$
The variation of Reynolds number with tunnel running time \( t \), as a function of \( \alpha \) will be as shown in Figure 19.

![Figure 19 Reynolds number variation with tunnel running time](image-url)
As seen from Figure 19, the Reynolds number increases with running time for constant pressure operation, and decreases with running time for constant throttle operation. The change in Reynolds number results in the change of boundary layer thickness, and which in turn causes area and Mach number change in the test-section. Usually Mach number variation due to the above causes is small.
Reynolds Number Control

By definition, Reynolds number is the ratio between the inertia and viscous forces.

\[ Re = \frac{\text{Inertia force}}{\text{Viscous force}} \]

It can be shown that,

\[ Re = \frac{\rho VL}{\mu} \]

where \( \rho \), \( V \), and \( \mu \) are the density, velocity, and viscosity, respectively, and \( L \) is a characteristic dimension of the model being tested.
The above equation may be expressed as

\[
\frac{Re}{L} = \frac{\rho V}{\mu}
\]  

(35)

Also, the viscosity coefficient may be expressed as

\[
\mu = C T^m V = C T^m M \sqrt{\gamma R T} = C_1 f(M) \sqrt{T}
\]

In the above expression, \( C_1 \) and \( C \) are constants, \( m \) is viscosity index, \( \gamma \) is isentropic index, and \( R \) is gas constant.

Let \( \rho_b \) and \( \rho_{bi} \) be the *instantaneous and initial* pressures in the settling chamber, respectively, and \( T_b \) and \( T_{bi} \) be the corresponding temperatures.
With the above relations for $\mu$, Eq. (35) can be expressed as

$$\frac{R_e}{L} = g_1 f(M, m) \left[ \frac{\rho_b/\rho_{bi}}{(T_b/T_{bi})(m+\frac{1}{2})} \right]$$

(36)

where $g_1$ is a function of initial (starting) conditions ($\rho_{bi}, \rho_{ti}$). From Eq. (36) it is seen that the Reynolds number during tunnel run is influenced only by the quantities within the square brackets. These quantities can easily be held constant by suitable manipulation of a throttling valve located between reservoir and settling chamber, as shown in Figure 20.
Reservoir
\[ p_0 \]
\[ T_0 \]

Settling chamber
Throttle valve
Quick opening valve

**Figure:** 20 Blowdown tunnel layout
The throttling process may be expressed by the following equation.

\[ p_{bi} = \alpha p_0^\beta \]  

(37)

where \( p_{bi} \) and \( p_0 \) are the total pressures after (stagnation pressure in the settling chamber) and before (stagnation pressure in the reservoir) throttling, respectively, and \( \alpha \) and \( \beta \) are constants.
The function $g$ in Eq. (36), at settling chamber conditions, is

$$g_i = \left( \frac{\gamma p_{bi}}{a_{bo} \mu_{bo}} \right)$$

where $a_{bo}$ is the proportionality constant and $\mu_{bo}$ is the viscosity coefficient of air in the settling chamber.

The function $f(M, m)$, from isentropic relations, is

$$f(M, m) = \frac{M}{\left[ 1 + \frac{\gamma - 1}{2} M^2 \right] \left( \frac{\gamma + 1}{2(\gamma - 1)} - m \right)}$$
Now, applying the polytropic law for the expansion of gas in the storage tank, we can write

\[ \frac{p_0}{p_{0i}} = \left( \frac{T_0}{T_{0i}} \right)^{n - 1} \]

where subscripts 0 and 0i refer for instantaneous and initial conditions in the reservoir and \( n \) is the polytropic index.

Also, from Eq. (37), we have

\[ \frac{p_b}{p_{bi}} = \left( \frac{p_0}{p_{0i}} \right)^{\beta} \]
Therefore, with the above relations, Eq. (36) can be expressed as,

\[
\frac{Re}{L} = g_1 f(M, m) \left[ \left( \frac{p_0}{p_{0i}} \right)^\beta \left( \frac{(2m + 1)(n - 1)}{2n} \right) \right]
\]

(38)

This is the general relation between test-section Reynolds number and reservoir pressure. From this equation, the following observations can be made.
For $R_e = \text{constant}$; 
$$\beta = \frac{(2m + 1)(n - 1)}{2n}$$

For constant $p_b$ operation, $p_b = \alpha p_0^\beta = \text{constant}$, and $\beta = 0$. Thus, Eq. (38) simplifies to

$$\frac{R_e}{L} = K_3 \left( \frac{p_0}{p_0} \right)^2 \frac{(2m + 1)(n - 1)}{2n}$$

where $K_3$ is a constant. This implies that, $R_e$ increases with time $t$, since $p_0$ decreases with $t$.

For constant throttle operation, $\beta = 1$ and

$$p_b = \alpha p_0^\beta = \alpha p_0$$

Therefore,

$$\frac{R_e}{L} = K_3 \left( \frac{p_0}{p_{0i}} \right)^2 \left[ 1 \frac{- (2m + 1)(n - 1)}{2n} \right]$$

$$0 < \frac{(2m + 1)(n - 1)}{2n} < 1$$

This implies that, $R_e$ decreases with $t$ for constant throttle operation.
From the above observations it can be inferred that, for a given settling chamber pressure and temperature, the running time is

- The **shortest** for constant throttle operation.
- The **longest** for constant Reynolds number operation.
- In **between the above two** for constant pressure operation.
Optimum Conditions

For optimum performance of a tunnel in terms of running time \( t' \), the drop in reservoir pressure should be as slow as possible. To achieve this slow rate of fall in reservoir pressure, the pressure regulating valve should be adjusted after the tunnel has been started, in such a manner that the pressure in the settling chamber is the minimum pressure, \( p_{bmin} \), required for the run.

The performance of the tunnel; the test-section Mach number \( M \) versus the tunnel run time \( t \), for different methods of control mentioned above should be evaluated for the entire range of operation. These performance can be recorded in the form of graphs for convenient reference. From such graphs, the best suited method of operation for any particular test and the required settings of the throttle valve (\( \alpha, \beta \), etc.) can be chosen. A typical performance chart will look like the one shown in Figure 21.
Figure: 21 Wind tunnel performance chart
For a given test-section Mach number $M$ there is a $p_b$ minimum in the settling chamber, given by the pressure ratio relation. The Reynolds number in the test-section depends on this $p_b$ value and constant Reynolds number operation is possible only if,

- $p_{bi}$ value is so chosen that as $t'$ proceeds (increases) both $p_0$ and $p_b$ reach $p_{b_{min}}$ value simultaneously (to result in an optimum constant Reynolds number).
- $p_{bi} > p_{b_{opt}}$ the reservoir pressure will become equal to $p_b$ at some instant and then onwards constant Reynolds number operation is not possible.
- $p_{bi} < p_{b_{opt}}$ the $p_b = p_{b_{min}}$ state will be reached at time $t'$ when $p_0 > p_b$ and supersonic operation will not be possible further.
Running Time of Blowdown Wind Tunnels

Blowdown supersonic wind tunnels are usually operated with either constant dynamic pressure \((q)\) or constant mass flow rate \((\dot{m})\).

For constant \(q\) operation, the only control necessary is a pressure regulating valve (PRV) that holds the stagnation pressure in the settling chamber at a constant value. The stagnation pressure in the storage tank falls according to the polytropic process; with the polytropic index \(n = 1.4\) for short duration runs, with high mass flow, approaching \(n = 1.0\) for long duration runs with thermal mass\(^1\) in the tank.

\(^1\)Thermal mass is a material which has high value of thermal capacity.
For constant mass flow run, the **stagnation temperature and pressure in the settling chamber must be held constant**. For this, either a heater or a thermal mass external to the storage tank is essential. The addition of heat energy to the pressure energy in the storage tank results in longer running time of the tunnel. Another important consequence of this heat addition is that the constant settling chamber temperature of the constant-mass run keeps the test-section Reynolds number at a constant value.
For calculating the running time of a tunnel, let us make the following assumptions.

- Expansion of the gas in the storage tank is polytropic.
- Gas temperature in the storage tank is held constant with a heater.
- Gas pressure in the settling chamber is kept constant with a pressure regulating valve.
- No heat is lost in the pipe lines from the storage tank to the test-section.
- Expansion of the gas from the settling chamber to test-section is isentropic.
- Test-section speed is supersonic.
The mass flow rate \( \dot{m} \) through the tunnel, as given by Eq. (3.34), is

\[
\dot{m} = \left( \frac{1.4}{R T_t} \right)^{1/2} \frac{M p_t A}{(1 + 0.2M^2)^3}
\]

where \( M \) is the test-section Mach number, \( p_t \) and \( T_t \), respectively, are the pressure and temperature in the settling chamber.

We know that, for supersonic flows it is convenient to calculate the mass flow rate with nozzle throat conditions. At the throat, \( M = 1.0 \) and the Eq. (34) becomes

\[
\dot{m} = 0.0404 \frac{p_t A^*}{\sqrt{T_t}} \tag{39}
\]
The value of gas constant used in the above equation is $R = 287 \frac{m^2}{(s^2 \cdot K)}$, which is the gas constant for air.

The product of mass flow rate and run time gives the change of mass in the storage tank. Therefore,

$$\dot{mt} = (\rho_i - \rho_f) V_t$$

(40)

where $V_t$ is the tank volume and $\rho_i$ and $\rho_f$ are the initial and final densities in the tank, respectively.
From Eq. (40), the running time $t$ is obtained as

$$t = \frac{(\rho_i - \rho_f)}{\dot{m}} V_t$$

Substituting for $\dot{m}$ from Eq. (39) and arranging the above equation, we get

$$t = 24.728 \frac{\sqrt{T_t}}{\rho_t} \frac{V_t}{A^*} \rho_i \left(1 - \frac{\rho_f}{\rho_i}\right)$$  \hspace{1cm} (41)
For polytropic expansion of air in the storage tank, we can write

$$\frac{\rho_f}{\rho_i} = \left( \frac{p_f}{p_i} \right)^{\frac{1}{n}}; \quad \rho_i = \frac{p_i}{RT_i}$$

where subscripts $i$ and $f$ denote the initial and final conditions in the tank, respectively.

Substitution of the above relations into Eq. (41) results in

$$t = 0.086 \frac{V_t}{A^*} \frac{\sqrt{T_t}}{T_i} \frac{\rho_i}{\rho_t} \left[ 1 - \left( \frac{p_f}{p_i} \right)^{\frac{1}{n}} \right]$$

with $V_t$ in m$^3$, this gives the run time in seconds for the general case of blowdown tunnel operation with constant mass flow rate condition.
From Eq. (42) it is obvious that, for $t_{\text{max}}$ the condition required is $p_t$ minimum. At this stage we should realize that, the above equation for running time has to be approached from practical point of view and not from purely mathematical point of view. Realizing this, it can be seen that the tunnel run does not continue until the tank pressure drops to the settling chamber stagnation pressure $p_t$, but stops when the storage pressure reaches a value which is appreciably higher than $p_t$, i.e., when $p_f = p_t + \Delta p$. This $\Delta p$ is required to overcome the frictional and other losses in the piping system between the storage tank and the settling chamber. Value of $\Delta p$ varies from about $0.1p_t$ for very-small-mass flow runs to somewhere around $1.0p_t$ for high-mass flow runs.
The proper value of the polytropic index $n$ in Eq. (42) depends on the rate at which the stored high-pressure air is used, total amount of air used, and the shape of the storage tank. The value of $n$ tends towards 1.4 as the storage tank shape approaches spherical shape. With heat storage material in the tank (i.e., for isothermal condition), the index $n$ approaches unity. Equation (3.42) may also be used with reasonable accuracy for constant-pressure runs in which the change in total temperature is small, since these runs approach the constant-mass flow rate situation.
Example 7

Determine the running time for a Mach 2 blowdown wind tunnel with test-section cross-section of 300 mm × 300 mm. The storage tank volume is 20 m³ and the pressure and temperature of air in the tank are 20 atm and 25°C, respectively. The tank is provided with a heat sink material inside. Take the starting pressure ratio required for Mach 2.0 to be 3.0, the loss in pressure regulating valve (PRV) to be 50 per cent and the polytropic index \( n = 1.0 \).
Solution

Given that, the settling chamber pressure required to start the tunnel is $p_t = 3.0 \times 101.3$ kPa. The pressure loss in the PRV is 50%, therefore, $p_f = 1.5 \times 303.9 = 455.85$ kPa. 

From isentropic tables, for $M = 2.0$, we have $A^*/A = 0.593$. Therefore,

$$A^* = 0.593 \times 0.09 = 0.0534 \text{ m}^2$$

By Eq. (42), the running time, $t$, is given by

$$t = 0.086 \left( \frac{20}{0.0534} \right) \left( \frac{\sqrt{298}}{298} \right) \left( \frac{2026}{303.9} \right) \left[ 1 - \left( \frac{455.85}{2026} \right) \right]$$

$$= 9.64 \text{ s}$$
Hypersonic Tunnels

Hypersonic tunnels operate with test-section Mach numbers above 5. Generally they operate with stagnation pressures in the range from 10 to 100 atmosphere and stagnation temperatures in the range from 50°C to 2000°C. Contoured nozzles which are more often axially symmetric are used in hypersonic tunnels.

Models that can be tested in hypersonic tunnels are usually larger than those meant for test in supersonic tunnels. The model frontal area can go up to 10 per cent of the test-section cross-sectional area. Model size will probably be restricted by the wake behind it, which takes too much flow area in the diffuser and blocks it during tunnel starting.
Use of dry and heated air is necessary for hypersonic operation to avoid condensation effects and liquefaction during expansion to the high Mach number and corresponding low-temperatures. The requirement of heated air is the major factor making hypersonic tunnel operation more complicated than supersonic tunnel operation. To get a feel about the drastic changes in the flow properties at hypersonic speeds, let us examine the parameters listed in Table 1, for isentropic index $\gamma = 1.4$. 
<table>
<thead>
<tr>
<th>$M$</th>
<th>$A/A^*$</th>
<th>$p_0/p$</th>
<th>$T_0/T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25.00 E 00</td>
<td>529.10 E 00</td>
<td>6.000</td>
</tr>
<tr>
<td>10</td>
<td>53.59 E 01</td>
<td>424.39 E 03</td>
<td>21.000</td>
</tr>
<tr>
<td>15</td>
<td>37.55 E 02</td>
<td>660.15 E 03</td>
<td>46.000</td>
</tr>
<tr>
<td>20</td>
<td>15.38 E 03</td>
<td>478.29 E 04</td>
<td>80.998</td>
</tr>
<tr>
<td>25</td>
<td>46.31 E 03</td>
<td>224.54 E 05</td>
<td>126.000</td>
</tr>
</tbody>
</table>
Now, we should note that the above table is based on isentropic relations. As we know, in the isentropic relations the index $\gamma$ is treated as a constant. However, we are familiar with the fact that $\gamma$ is constant only for gases which are thermally as well as calorically perfect; simply termed perfect gases (Gas Dynamics, Rathakrishnan, 2nd edi., 2008). Therefore, in hypersonic flows if the test-section flow temperatures are to be at room temperature levels, the storage temperature has to be increased to very high values, which will pose metallurgical problems. Because of these considerations the temperatures in the test-section of hypersonic tunnels are usually quite low, in spite of the fact that the storage temperature is kept appreciably above the ambient temperature.
Example 8

Find the test-section temperature for a hypersonic stream of air at Mach 7 with stagnation temperature at 700 K.
Solution

From isentropic table ($\gamma = 1.4$), for $M = 7.0$, we have

$$\frac{T}{T_0} = 0.09592$$

Therefore, the temperature of air in the test-section is

$$T = (0.09592)(700) = 67.144\,\text{K}$$

$$= -205.856\,\text{°C}$$
It is seen from Table 1 that, the pressure ratios involved in hypersonic tunnel flow process are very high. In order to achieve these pressure ratios, it is customary to **employ combination** of high pressure and vacuum together, in hypersonic tunnel operations. A typical hypersonic tunnel circuit is schematically shown in Figure 22.
**Figure: 22 Hypersonic tunnel circuit**
From the above discussions on hypersonic tunnel operation, the following observations can be made.

- The pressure ratio, area ratio, and temperature ratio for $M > 5$ increase very steeply with increase of $M$. Usually both high pressure tank at the nozzle inlet and vacuum tank at the diffuser end are necessary for hypersonic operations.

- The very low-temperatures encountered in the test-section results in liquefaction of air and hence preheating of air to 700 K to 1000 K is common in hypersonic tunnel operation. For air, up to Mach 8 preheating to about 1300 K is satisfactory. For $M > 10$, gas like helium is better suited.
Because of very low density in the test-section, optical flow visualization of viscous shock waves etc., becomes more difficult.

Shock wave angle (e.g. on wedge, cone, etc.,) changes appreciably with moisture content of air and hence, measurements have to be done with extra care.

The heating of air introduces additional problems, like material requirement for settling chamber, nozzle, test-section glass window, and distribution of parts (the tunnel structure, test-section walls, etc.,) to stand high temperatures.
Because of low pressure and temperature, the flow at the test-section has low Reynolds number and hence, the boundary layer thickness increases to a large extent.

Determining the exact value of Mach numbers at high Mach number is quite difficult, since heating expands tunnel walls and therefore, area ratio is changed. In addition, the boundary layer (which is quite thick) makes it more difficult to calculate $M$. Also, the specific heats ratio $\gamma$ is changing due to the drastic changes of temperature encountered in the tunnel and hence, accurate computation of the total pressure, $p_0$, and static pressure, $p$, are difficult.
Hypersonic Nozzle

For hypersonic operations axisymmetric nozzles are better suited than two-dimensional nozzles. For high Mach numbers of the order 10 the throat size becomes extremely narrow and forming the shape itself becomes very difficult. Because of high temperatures, material to be used also poses problem. Material liners scale and pit easily at these high speeds. Though porcelain coated nozzles are good for these high temperatures and speeds, they do not take the smoothness which is required for hypersonic speeds. A minimum diameter of around 12 mm for the nozzle throat is arrived at from practical considerations. A typical shape of hypersonic nozzles is schematically shown in Figure 23.
Figure: 23 Hypersonic nozzle

\[ \phi 100 \text{ mm} \quad \phi 12 \text{ mm} \quad \phi 120 \text{ mm} \]
Instrumentation and Calibration of Wind Tunnels

Calibration of wind tunnel test-section to ensure uniform flow characteristics everywhere in the test-section is an essential requirement in wind tunnel operation. In this section, let us see the calibration required and the associated instrumentation to study the flow characteristics in the test-sections of subsonic and supersonic wind tunnels. The instruments used for calibration will be only briefly touched upon in this section. The detailed study of these measuring devices will be done in the chapters to follow.
Low-Speed Wind Tunnels

We know that, a low-speed air stream is characterized by the distribution of its dynamic pressure, static pressure, total pressure, temperature, flow direction, and turbulence level. From these details, the flow velocity and Reynolds number for any specific model can be computed. In other words, the instrumentation and calibration of low-speed tunnels involve the determination of the following.

1. Speed setting; calibration of true air speed in the test-section.
2. Flow direction; determining the flow angularity (pitch and yaw) in the test-section.
3. Turbulence level.
4. Velocity distribution; determination of flow quality.
5. Wake survey; determination of flow field in the wake of any model.
Speed Setting

Consider the flow in a subsonic wind tunnel, schematically shown in Figure 24.

Figure: 24 Open circuit tunnel
Measure the static pressure at the entry and exit of the contraction cone, at stations AA and BB. Applying the incompressible Bernoulli equation, we can write

\[ p_A + q_A = p_B + q_B - k_1 q_B \]

where, \( p_A \) and \( p_B \) are the static pressures at sections AA and BB and \( q_a \) and \( q_B \) are the corresponding dynamics pressures, respectively, and \( k_1 \) is the pressure drop coefficient due to frictional loss between stations AA and BB.
Therefore,
\[
\Delta p = p_A - p_B = q_B - q_A - k_1 q_B
\]  \hspace{1cm} (43)

From incompressible flow continuity equation, we have

\[
A_A V_A = A_B V_B
\]

Squaring and multiplying both sides by \(\frac{\rho}{2}\), we get

\[
\frac{\rho}{2} A_A^2 V_A^2 = \frac{\rho}{2} A_B^2 V_B^2
\]

But \(\frac{\rho}{2} V^2 = q\) is the dynamics pressure, thus,

\[
q_A A_A^2 = q_B A_B^2
\]
Let

\[ \frac{A_B^2}{A_A^2} = k_2 \]

Therefore,

\[ q_A = k_2 \, q_B \]

Using this relation in Eq. (43), we obtain

\[ \Delta p = q_B - k_1 \, q_B - k_2 \, q_B = (1 - k_1 - k_2) \, q_B \quad (44) \]

Applying Bernoulli equation between stations BB and CC, we get

\[ q_B A_B^2 = q_t A_t^2 \]

\[ q_B = \left( \frac{A_t}{A_B} \right)^2 \quad q_t = k_3 \, q_t \]

where subscript $t$ refers to test-section.
Using this, Eq. (44) can be rewritten as

$$\Delta p = p_A - p_B = (1 - k_1 - k_2) k_3 q_t$$  \hspace{1cm} (45)

The pressure drop, $\Delta p$, and the test-section velocity, $V_t$, in Eq. (45) can be measured independently, by wall static pressure taps and a standard pitot-static tube placed in the empty test-section, respectively. The typical variations of test-section velocity, $V_t$ and dynamic pressure, $q_t$ with static pressure drop is given in Figure 25. From this figure, the advantage of using $q_t$ instead of $V_t$ is obvious from its linear variation with $\Delta p$. 
Figure: 25 Test-section dynamic pressure and velocity variation with static pressure drop in the contraction cone
Flow Direction

The flow in the test-section has to be uniform throughout its cross-section, with all streamlines parallel to the tunnel axis, when the tunnel is run without any model in the test-section. Therefore, ensuring the flow direction along the tunnel axis is essential before putting the tunnel in to use. A yaw head probe can be used to measure the angularity of the flow. The yaw head probes used, for measuring the flow angularity in a wind tunnel rest-section, are usually sphere type or claw type.
Yaw Sphere

Schematic diagram of a typical sphere type yaw meter is shown in Figure 26.
For measuring the angularity of planar or two-dimensional flows we need a two-hole yaw sphere. Whereas, yaw meter with four-holes is necessary for three-dimensional flows, since the yaw and pitch of the stream need to be measured simultaneously. The procedure followed for angularity measurement is the following.
A sphere with two orifices located 90° apart on the forward port is to be placed in the wind tunnel test-section by a support.

The instrument axis has to be adjusted in the vertical plane till both the pressure taps measure equal pressure (i.e. $p_A = p_B$, refer Figure 3.26). The measurement of equal pressure by these two pressure taps implies that the axis of the yaw meter is aligned to the flow direction. Now the angle between the tunnel axis and yaw sphere axis gives the angle between the flow direction and tunnel axis.

Align the instrument axis parallel to the tunnel axis and note the pressure difference ($p_A - p_B$). The instrument may be calibrated by standard experiments for yaw head, defined as the ratio between $\Delta K$ and $\Delta \psi$, where $\Delta K = \Delta p/q$ and $\psi$ is the yaw angle.
Theoretical and actual values of $\Delta p/q$ for a spherical yaw head are compared in Figure 27.

Figure: 27 Variation of $\Delta p/q$ with $\psi$
The yaw head ($\Delta K / \Delta \psi$) varies from 0.04 to 0.07/degree. Therefore, for each instrument its yaw head constant, $\Delta K / \Delta \psi$, should be determined experimentally.

In addition to the yaw holes, a total-head orifice may also be provided at the front of the yaw head sphere. From Figure 3.27, it can be noted that the total pressure head orifice reading will be correct only for small flow deflection angles. In deed, at 5 degree yaw the total head reading is down by 1.2 per cent.
Claw Yaw Meter

The principle and the functioning of claw type yaw meter is similar to that of the spherical yaw meter. A typical claw type yaw meter is shown schematically in Figure 28.

Because of less interfering geometrical construction, it is generally used to measure the direction, rotation and so on of the flow field at any point near model surfaces.
Turbulence

From our discussions on turbulence in chapter 2, it is evident that basically turbulence is a measure of chaos or disorderliness in the flow. Further, generation of turbulent flow is easier compared to laminar flow. In fact, generation of laminar flow in the laboratory is a cumbersome task. In nature, most of the flows; air flow over plains, sea breeze, water flow in rivers, streams and so on, are turbulent. Therefore, it becomes imperative to simulate the level of turbulence in the actual flow field, where a prototype is going to operate, in the wind tunnel test-section, where a scale model of the prototype is tested, for establishing dynamically similar flow conditions between the prototype and model flow fields.
If there is any disagreement between tests made in different wind tunnels at the same Reynolds number and between tests made in wind tunnels and in actual field, then correction has to be applied on the effect of the turbulence produced in the wind tunnel by the propeller, guide vanes, screens and wire meshes, and vibration of the tunnel walls. From experience it is known that, turbulence causes the flow pattern in the tunnel to be similar to the flow pattern in free air at a higher Reynolds number. Hence, the wind tunnel test Reynolds number could be said to have a higher effective Reynolds number compared to free flight in the actual flow field.
In other words, a tunnel with a certain level of turbulence exhibits a flow pattern at a test-section Reynolds number $Re$, which will be identical to a free field flow at a Reynolds number which is much higher than the test-section Reynolds number $Re$. That is, the actual Reynolds number of the test-section is equivalent to a much higher free field Reynolds number. This increase is called the \textit{turbulence factor}, $TF$, defined as 

$$Re_e = TF \times Re_c$$  \hspace{1cm} (46)$$

where the subscript $e$ stands for effective Reynolds number, and $Re_c$ is the measured critical Reynolds number in the tunnel test-section.
Now it is clear that, measurement of turbulence in the test-section is essential for determining the $Re_e$. The turbulence may be measured with

1. Turbulence sphere
2. Pressure sphere
3. Hot-wire anemometer
Turbulence Sphere

The drag coefficient, $C_D$, of a sphere is greatly influenced by changes in flow velocity. The $C_D$ for a sphere decreases with increasing air speed, since the earlier transition to turbulent flow results in reduced wake behind the sphere. This action decreases the form or pressure drag, resulting in lower total drag coefficient. The decrease of drag coefficient is rapid in a range of speed in which both the drag coefficient and drag go down. The Reynolds number at which the transition occurs at a given point on the sphere is a function of the turbulence already present in the air stream, and hence the drag coefficient of the sphere can be used to measure turbulence of the air stream.
The procedure usually adopted for this measurement is the following.

- Measure the drag, $D$, for a small sphere of about 150 mm diameter, at many speeds of the test-section.
- After subtracting the buoyancy, the drag coefficient may be calculated from the relation,

$$ C_D = \frac{D}{\frac{1}{2} \rho V^2 \left( \frac{\pi d^2}{4} \right) } $$

where $d$ is the sphere diameter.
- The sphere drag coefficient is plotted against Reynolds number $Re$, as shown in Figure 29.
- The $Re$ at which $C_D = 0.30$ is noted and termed the critical Reynolds number, $Re_c$. 
Figure: 29 $C_D$ versus $Re$ for drag sphere
For the drag sphere placed in an undisturbed flow, the theoretical value of $C_D = 0.3$ occurs at the theoretical critical Reynolds number of $Re = 385,000$. Therefore,

$$TF = \frac{385,000}{Re_c}$$

(47)

where $Re_c$ is the actual critical Reynolds number for the tunnel. Using this equation and Eq. (46), the effective Reynolds number can be calculated.
Example 3.9

A subsonic wind tunnel of square test-section runs at 30 m/s, with pressure 97.325 kPa and temperature 22°C, in the test-section. A turbulence sphere with theoretical surface finish offering 4 per cent blockage experiences critical Reynolds number at this state. Determine the test-section height.
Solution

The flow density in the test-section is

\[ \rho = \frac{p}{RT} = \frac{97.325 \times 10^3}{287 \times 295} = 1.15 \text{ kg/m}^3 \]

The viscosity of air at 22°C is

\[ \mu = 1.46 \times 10^{-6} \frac{T^{3/2}}{T + 111} \]

\[ = 1.46 \times 10^{-6} \frac{(295)^{3/2}}{295 + 111} \]

\[ = 1.822 \times 10^{-5} \text{ kg/(m s)} \]
For a turbulence sphere with theoretical surface finish, the theoretical critical Reynolds number is 385,000. Therefore,

\[ 385000 = \frac{\rho V d}{\mu} \]

\[ d = \frac{385000 \times (1.822 \times 10^{-5})}{1.15 \times 30} \]

\[ = 0.203 \text{ m} \]

The projected area of the sphere is

\[ A = \frac{\pi \times 0.203^2}{4} = 0.18 \text{ m}^2 \]

The blockage is 4 per cent. Thus, the test-section area is

\[ A_{TS} = \frac{0.18}{0.04} = 4.5 \text{ m}^2 \]

This gives the test-section height as \[ \boxed{2.25 \text{ m}} \]
Pressure Sphere

In this method, no force measurement is necessary. Therefore, the difficulty of finding the support drag is eliminated. The pressure sphere has an orifice at the front stagnation point and four equally spaced interconnected orifices at 22\(\frac{1}{2}\) degrees from the theoretical rear stagnation point. A lead from the front orifice is connected across a manometer to the lead from the four rear orifices. After the pressure difference due to the static longitudinal pressure gradient is subtracted, the resultant pressure difference \(\Delta p\) for each \(Re\) at which measurements were made is divided by the dynamic pressure for the appropriate \(Re\). The resulting dimensionless pressure difference, also known as the pressure coefficient, \(C_p\) (\(C_p = \Delta p/q\)), is plotted against \(Re\), as shown in Figure 30.
Figure: 30 $C_p$ variation with $Re$

$Re_{cri} = 299,000$
It has been proved that, \( \Delta p/q = 1.22 \) corresponds to \( C_D = 0.30 \) and hence this value of \( \Delta p/q \) determines the critical Reynolds number \( Re_c \). The turbulence factor may now be found from Eq. (46).

At this stage, it is proper to question the accuracy of the turbulence measurement with turbulence sphere or pressure probe, since we know that turbulence is first of all a random phenomenon and further the flow transition from laminar to turbulent nature takes place over a range of Reynolds number and not at a particular Reynolds number. The answer to this question is that, in all probability the turbulence factor will change slightly with tunnel speed. This variation of turbulence factor may be obtained by finding it with spheres of several different diameters.
The turbulence will also vary slightly across the jet of the test-section. Particularly **high turbulence** is usually noted at the center of the jet of double-return tunnels, because this air has scraped over the walls of the return passage (Pope and Allen, John Wiley, 1965).

Turbulence factor varies **from 1.0 to about 3.0**. Values **above 1.4 possibly indicate that** (Pope and Allen, John Wiley, 1965) the air has very high turbulence for good testing results. Turbulence should be very low for research on low-drag airfoils. The variation of the turbulence factor with the degree of turbulence is shown in Figure 31.
Figure: 31 Turbulence factor variation with turbulence level
The turbulence in a tunnel may be kept at low level by the following means.

- Using maximum number of fan blades.
- With anti-swirl vanes.
- With a very long, gradual nacelle.
- Providing the maximum possible distance between the propeller and test-section.
Limitations of Turbulence Sphere

From experimental studies it is established that, the accuracy of turbulence sphere measurements are not adequate when

- The turbulence level is less than the degree of turbulence which corresponds to a turbulence factor of about 1.05.
- The Mach number is greater than 0.35.

Under these circumstances, we have to resort to devices which are capable of handling these situations. One such popular device is the hot-wire anemometer.
Hot-Wire Anemometer

The hot-wire anemometer can be used to measure freestream turbulence directly. It can be used for the measurement of turbulence at very low-speeds also. It measures the instantaneous values of speed accurately and hence the turbulence, defined as

\[
\text{Turbulence} = \frac{\text{Deviation from mean speed}}{\text{Mean speed}}
\]

can be found directly. The time lag is practically negligible. The hot-wire anemometer works on the principle that, "the rate of heat loss from a wire heated electrically and placed in an air stream is proportional to the stream velocity". The wire used is platinum or tungsten wire of about 0.015 mm diameter and about 10 mm long.
The rate of heat dissipation $H$ is given by

$$H = I^2 R = \left( A + B \sqrt{V} \right) (T - T_a)$$  \hspace{1cm} (48)

where $I$ is the electric current through the wire, $R$ is the wire resistance, $A$ and $B$ are constants to be found by calibration experiments, $V$ is the flow velocity, and $T$ and $T_a$ are the wire and room temperatures, respectively. Equation (3.48) is known as the Kings’ formula, which is the fundamental relation for hot-wire anemometry. The details of hot-wire anemometer, its working principle, measurement procedure, application and limitations are discussed in chapter 5.
Rakes

For the simultaneous measurement of a large number of total-head readings, a bank of total-head tubes, generally termed rake is commonly employed. Usually, brass or stainless steel tubes of diameter about 1.5 mm, arranged with a lateral spacing of around 3 mm on a single unit support, as shown in Figure 32, is used as a rake for measurements in wake behind a body and so on. It is particularly important that the lateral spacing be exact, since even a slight misalignment of a tube can cause considerable error in wake survey measurement. The total head tubes should be cut off to have square edged entry, and their length is immaterial.

Even though the static pressure in the wakes rapidly reach the freestream static pressure, it is advisable to make frequent checks of this. Hence, it is customary that several static tubes are kept along with the total-head tubes, as shown in Figure 32.
Figure: 32 Total pressure rake
Surging

One of the most troublesome problems associated with wind tunnels is *tunnel surging*. It is a low-frequency vibration in velocity that may run as high as 5 per cent of the dynamic pressure $q$. In practice, large number of tunnels suffer from this defect one time or other; some live with it; some find a cure.

Surging is mainly due to the separation and reattachment of flow in the diffuser, and usually it can be cured by providing substantial number of tripper strips in the diffuser. After a cure has been found, we may successively cut back on tripper numbers and size to save power. Another means to overcome surge is by employing some sort of boundary-layer control which corrects the surge rather than hides its effects.
The following are the difficulties caused by surging.

- It makes the wind tunnel balances **widely** trying to keep up.
- It **disturbs** the reference pressure in pressure measurements.
- Surging makes the validity of assigning a Reynolds number to the test **doubtful**.
- It usually makes the dynamic testings **impossible**.
Wind Tunnel Balance

Basically, wind tunnel balance is a device to measure the actual forces and moments acting on a model placed in the test-section stream. Based on the constructional details, the wind tunnel balances are broadly classified into,

- Wire type balance.
- Strut type balance.
- Platform type balance.
- Yoke type balance.
- Strain gauge type balance.
Irrespective of its type of construction, a wind tunnel balance should have certain basic **features and characteristics** for proper measurements. They are the following.

1. **The balance should be capable of measuring the various loads (1, 2, 3, 4, 5 or 6 components) acting on the model with very high degree of accuracy.**
2. The interaction between the different load components should be kept small.
3. **The balance should have provision to vary** the angle of incidence, pitch, yaw, roll, etc., of the model which is mounted on the balance, within the normal range.
4. **The balance and the supporting structures should be designed for very high rigidity**, so that the deflection of the parts under the influence of maximum load is negligible.
5. **Damping devices** should be incorporated in the measuring system.
6. **Use of bearings (ball, roller type etc.)** should as far as possible be avoided, since they cause large hysteresis and zero error.
The **range of loads and accuracy** for the balance are determined from the tunnel size, test-section velocity, model size, configuration, etc.
Wire Balances

In wire type wind tunnel balances only wires are used to support the model. All the load components are transmitted to the measuring device by these wires. Wire type balances are probably the simplest and easiest to build. But they have several disadvantages due to the use of too many bearings and bell crank systems, friction of the wires, and high damping requirements. Further, it is extremely bulky since the support system should be very rigid.
The disadvantages of wire balance are the following.

- **Large tare drag** because of the exposed wires. It is difficult to streamline the wires and hence it is difficult to determine the magnitude of tare drag accurately.
- Bearings and linkages cause zero error.
- Wires have the tendency to crystallize and break.
- Space occupied is very large.
In spite of these disadvantages, the wire balances are still in use because of its simplicity and low cost. The usual arrangements for wire type balance are the following.

1. Models are tested in inverted position to avoid unloading of wires.
2. Since the wires can take only tension, it is essential to arrange the wires such that, for all types of loading (± lift, ± drag, etc.,) the necessary wires are in tension. The number of wires required, in view of the above requirements, is considerable for securing a model.
3. Conventional three-point attachment is provided on the model. Pulleys, ball bearings, lever arms, etc., are freely used in the transfer of loads to the measuring devices.
4. The support structure is usually very heavy and bulky to give the necessary rigidity.

A typical six-component wire balance, supporting an airfoil model, is shown in Figure 33.
Figure: Figure 33 A wire balance
The balance shown in Figure 33 mounts the models at the quarter chord. The forces and moments acting on the wing section mounted on the balance are

\[
\begin{align*}
\text{Lift} & = L_W + M_F \\
\text{Drag} & = D \\
\text{Side force} & = C + E + F \\
\text{Pitching moment} & = -M_F \times l \\
\text{Rolling moment} & = (C - F)(h/2) \\
\text{Yawing moment} & = -E \times l
\end{align*}
\]
Strut-Type Balances

The wire-type balances are seldom used in large tunnels because of the disadvantages associated with them. The strut-type balances are proved to be suitable for such tunnels. Based on the structural construction, strut-type balances are broadly classified into

1. Yoke type.
2. Platform type.
3. Pyramid type.

As indicated by the name, struts are used to support the model and transmit the loads to the measuring devices. Conventionally, models are mounted on the balance with three point mounting (two on the wings and one on the rear fuselage for aircraft models, for instance). A typical strut type balance with an aircraft model mounted on it is shown schematically in Figure 34.
Figure: 34 Simplified schematic of strut-type balance
Advantages of strut type balances are the following.

1. The struts being rigid, their deflection can be kept at very small value.

2. The arrangement enables the struts to be faired so that the tare and interference drag are minimized.

3. By using cross spring pivot instead of knife edge, ball bearings etc., in the beam, the zero error can be minimized.

4. By choosing proper linkages, interaction loads can be reduced to a small value.

5. Weight of the support structure can be kept very low, since efficient (tubular, for instance) members can be easily adapted.
Schematic of the cross spring pivot used in strut type balances is shown in Figure 35.

The principle of load measurement for yoke, platform and pyramid type are the same, and only the method of model suspension is different in them.

Figure: 35 Cross spring pivot
Consider the platform type balance shown in Figure 36. There is only one mounting point on the fuselage, by which the model is mounted, thus, the interference is minimized.

Figure: 36 Platform type balance
The platform is supported at three points, as shown in the Figure 3.36. Model is supported on the platform by a single streamlined strut. The loads are calculated as follows.

\[
\text{Lift} = Z_a + Z_b + Z_c
\]

\[
\text{Drag} = X_d + X_c
\]

\[
\text{Side force} = Y_f
\]

\[
\text{Rolling moment} = M_x = (Z_a - Z_b) \frac{l}{2}
\]

\[
\text{Pitching moment} = M_y = Z_c \times m
\]

\[
\text{Yawing moment} = M_z = (X_e - X_d) \frac{l}{2}
\]
Yoke Type Balance

Consider the yoke type balance shown in Figure 37. There are nine measurements made; $Z_A$, $Z_B$, $X_C$, $X_D$, $X_E$ and $Y_F$, for finding the six components of the load acting on the model. The model is mounted on yoke, as shown in Figure 37.
Figure: 37 Yoke type balance
The loads are calculated as follows.

Lift $= Z_A + Z_B$

Drag $= X_C + X_D + X_E$

Side force $= Y_F$

Rolling moment $= (Z_A - Z_B) \frac{l}{2}$

Pitching moment $= X_E \times m$

Yawing moment $= (X_C - X_D) \frac{l}{2}$
Pyramid Type Balance

In pyramid type balances, the model mounting is about one point connecting the model to the platform (by four arms), as shown in Figure 38.
The pyramid type balance, compared to platform or yoke type, has a specific advantage. In this balance, the forces and moments acting on the model are measured with respect to a single point (attachment point), and hence by locating this point at an advantageous position like aerodynamic center of model, the readings obtained are free from error associated with the conversion of a measured quantity to the required quantity. An accuracy of the order of ±0.1%, at full load, can easily be achieved with pyramid type balance.
Strain Gauge Balance

Balances with strain gauges as load-sensing element are termed strain gauge balances. Based on strain gauge fixing on the model, the strain gauge balances are generally classified as

1. Internal balance.
2. Semi-internal balance.
In an internal balance, all the measuring elements are located inside the model, in a semi-internal balance, the measuring elements are located partially inside and partially outside, and in an external balance, all the measuring elements are located outside the model. The types of strain gauges commonly used in wind tunnel balances are shown in Figure 39.
Figure: 39 Strain gauge balance
The gauges consist of a grid of very fine wire (10 to 30 micron in diameter) or very thin foil (thickness less then 30 micron) embedded on a sheet of bakelite having a thickness comparable to that of a thick paper. The grid material is usually constantan, nichrome, or nichrome with small additions of iron and aluminium. The grid length varies from about 1 mm to several centimeters.
Strain Gauge Operation Theory

When the bakelite body of the gauge is intimately connected to the surface of a structure, it will **stretch or contract** as the outer fibers of the structure to which it is attached. The grid wires embedded in the bakelite will stretch or contract with the bakelite body and thus with the outer fibers of the structure. As the grid wires are stretched, their cross-sectional area decreases, **causing an increase** in electrical resistance.
Similarly, as the grid wires are compressed, their cross-sectional area increases, causing a decrease in electrical resistance. In both events the changes in resistance is actually more than the changes that the area would indicate, because of the change in the length.
From experience it has been found that, the change in resistance of the types of strain gauges normally used in wind tunnel balances is directly proportional to the stress in the outer fibers of the structure to which it is attached.

From the above discussion, it is evident that great care must be exercised in the installation of the strain gauge. Strain gauge in wind tunnel balances are normally located on a member in which the desired component of loading is a bending moment. A typical strain gauge installation is schematically shown in Figure 40.
Figure: 40 Strain gauge installation on a balance member
Two gauges are placed side by side on the compression as well as the tension surface of the member. The four gauges are wired together in to a bridge circuit, as shown in Figure 41.

\[ R = \text{Resistance, ohms} \]

**Figure: 41 Strain gauges in a bridge circuit**
The supply voltage is generally between 5 and 10 volts and the current may be either direct or alternating.

The signal voltage from a strain gauge bridge can be calculated using Ohms law, as follows. The current flow through gauges 1 and 3 and through gauges 2 and 4 of Figure 41 are

\[ I_{13} = \frac{E_0}{R_1 + R_3} \]

\[ I_{24} = \frac{E_0}{R_2 + R_4} \]
The drop in the voltage across gauges 1 and 2 are

\[ \Delta E_1 = I_{13}R_1 = E_0\frac{R_1}{R_1 + R_3} \]

\[ \Delta E_2 = I_{24}R_2 = E_0\frac{R_2}{R_2 + R_4} \]

The signal voltage is equal to

\[ E_s = (E_0 - \Delta E_1) - (E_0 - \Delta E_2) \]
This can be reduced to

$$\frac{E_s}{E_0} = \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_3)(R_2 + R_4)}$$

with the matched gauges on a symmetrical section of the balance having both axial and bending loads, the resistances of the individual gauges are

$$R_1 = R_4 = R_0 - \Delta R_b + \Delta R_a$$

$$R_2 = R_3 = R_0 + \Delta R_b + \Delta R_a$$

where $R_0$ is the initial resistance of the gauge, $\Delta R_a$ and $\Delta R_b$ are the increments in the gauge resistance due to the axial and bending stresses, respectively.
Using these, we get

\[
\frac{E_s}{E_0} = \frac{\Delta R_b}{R_0 + \Delta R_a}
\]  \hspace{1cm} (49)

Since $\Delta R_a$ is normally very small compared to $R_0$, it can be neglected in the above equation.
Basic Equations of Strain Gauge Transducer

The fundamental equations associated with strain gauges are

\[ GF = \frac{\Delta R}{R} \frac{R}{\Delta L/L} \]  

(50)

\[ e = \frac{\sigma}{E} = \frac{\Delta L}{L} \]  

(51)

where \( GF \) is the gauge factor (strain gauge), \( R \) is the resistance of strain gauge, \( L \) is length of strain gauge, \( \Delta R \) and \( \Delta L \) are the changes in resistance and length, respectively, due to load, \( E \) is Youngs modulus, \( \sigma \) is stress and \( e \) is strain.
From Equations (56) and (57), the stress can be expressed as

\[ \sigma = \frac{\Delta R}{R} \frac{E}{GF} \]  \hspace{1cm} (52)

From this equation it is seen that, for computing the load on a member an accurate measurement of \( \Delta R \) only is necessary. Wheatstone type bridge circuit (modified) is used for this measurement.
The Dynamometer Block

This unit is designed as a system of transducers to convert the six components of load; the normal force, tangential force, side force, rolling moment, pitching moment and yawing moment, into corresponding electrical quantities.
Strain Gauge Signal–Measuring Devices

The transducer, namely the strain gauge used here is a device to convert the quantity to be measured (force etc..) into an electrical quantity. For a detailed description of devices for measuring strain gauge signals the reader is encouraged to refer to books devoted on this topic. However, a brief description of the principle involved is given in the following subsection. The measurement principle is as shown below.

Dynamometer block $\rightarrow$ Balancing bridge $\rightarrow$ Recorder
A typical balancing bridge circuit is shown in Figure 42.

**Figure: 42 Balancing bridge circuit**
The principle involved here is the comparison of strain gauge signal voltage with a known reference voltage which is varied until the reference and the signal voltages are equal. A voltage $E_1$ is applied across the resistance of a potentiometer. This voltage is a small fraction of the voltage, $E_0$, applied to the strain gauge bridge, but larger than the strain gauge signal voltage, $E_s$. The voltage $E_1$ is divided to provide a potential $E_{\text{ref}} = E(R/R_1)$ between one end of the resistor and the movable contact of the potentiometer.
One of the strain gauge signal leads is attached to the end of the resistor and the other to the movable contact of the potentiometer, through a galvanometer. The movable contact of the potentiometer is moved until the galvanometer indicates no current flow. At this point of zero current flow, the reference potential $E_{\text{ref}}$ equals the strain gauge signal voltage $E_s$. Knowing the voltage $E_1$ and the variation of potentiometer resistance with movable contact position, the signal variation from contact position and the relation $E_s = E_{\text{ref}} = E_1 \left( \frac{R}{R_1} \right)$ can be determined.
In wind tunnel operations, with a system of the type described above, the current flow that operates the galvanometer is amplified and drives a motor. The motor drives a movable contact of the potentiometer to a null position. Additional circuitry is provided to eliminate the necessity of reversing signal leads when the signal voltage from the strain gauge changes polarity. Though the modern measuring systems are highly sophisticated compared to the one described, the basic principle underlying them is the same as that briefed above.
Balance Calibration

All balances (wire, strut or strain gauge type) are required to be calibrated after assembly, and checked periodically. It is customary to check the calibration before running the wind tunnel for a model test. The calibration of wind tunnel balances consists of the following procedure.

1. Applying known loads, in stepwise, in fixed directions for each component and noting the balance readings. The load is usually applied by means of wire-pulley arrangement or pull-rod with flexer pivots at the end depending on the size of the unit. Alignment of the load applying unit (wire or pull rod) should be adjusted very accurately. Dead weights (standardized units 10 kg ± 5 g, 5 kg ± 1 g, etc.) are used for varying the magnitude of the load.
1. For each component of the load applied, the readings of all the indicators are noted after balancing the system.

2. In a similar manner, the experiments for other load components are carried out.

3. Combined loads are applied in discrete steps (3 or 6 components) and the various readings are noted.

4. From the test data, the interaction and percentage error of the different load components are computed and plotted in the form of graphs.

5. The deflection of the system (balance, support structure) should be measured under different loading conditions and the correction factor determined.
Wind Tunnel Boundary Correction

We know that, the condition under which a model is tested in a wind tunnel are not the same as those in free air. Even though there is practically no difference between having the model stationary and the air moving instead of vice versa, there is longitudinal static pressure gradient usually present in the test-section, and the open or close jet boundaries in most cases produce extraneous forces that must be subtracted from the measured values.

The variation of static pressure along the test-section produces a drag force known as horizontal buoyancy. It is usually small in closed test-section, and is negligible in open jets.
The presence of the lateral boundaries around the test-section (the walls of the test-section) produces the following.

- A lateral constraint to the flow pattern around a is body known as solid blocking. For closed throats solid blocking is the same as an increase in dynamic pressure, increasing all forces and moments at a given angle of attack. For open test-sections it is usually negligible, since the air stream is free to expand.

- A lateral constraint to flow pattern about the wake is known as wake blocking. This effect increases with increase of wake size. For closed test-section wake blocking increases the drag of the model. Wake blocking is usually negligible for open test-sections.

- An alteration to the local angle of attack along the span.
Calibration of Supersonic Wind Tunnels

Supersonic tunnels operate in the Mach number range of about 1.4 to 5.0. They usually have operating total pressures from about atmospheric to 2 MPa (≈ 300 psi) and operating total temperatures of about ambient to 100°C. Maximum model cross-section area (projected area of the model, normal to the test-section axis) of the order of 4 per cent of the test-section area is quite common for supersonic tunnels. Model size is limited by tunnel choking and wave reflection considerations.
When proper consideration is given to choking and wave reflection while deciding on the size of a model, there will be no effects of the wall on the flow over the models (unlike low-speed tunnels), since the reflected disturbances will propagate only downstream of the model. However, there will be a buoyancy effect if there is a pressure gradient in the tunnel.
Luckily, typical pressure gradients associated with properly designed tunnels are small, and the buoyancy effects in such tunnels are usually negligible. The Mach number in a supersonic tunnel with solid walls cannot be adjusted, because it is set by the geometry of the nozzle. Small increase in Mach number usually accompany large increase in operating pressure (the stagnation pressure in the settling chamber in the case of constant back pressure or the nozzle pressure ratio in the case of blowdown indraft combination), in that the boundary layer thickness is reduced and consequently the effective area ratio is increased.
During calibration as well as testing, the condensation of moisture in the test gas must be avoided. To ensure that condensation will not be present in significant amounts, the air dew-point in the tunnel should be continuously monitored during tunnel operation. The amount of moisture that can be held by a cubic meter of air increases with increasing temperature, but is independent of the pressure.
The moist atmospheric air cools as it expands isentropically through a wind tunnel. The air may become supercooled (cooled to a temperature below the dew-point temperature) and the moisture will then condense out. If the moisture content is sufficiently high, it will appear as a dense fog in the tunnel.
Condensation causes changes in local Mach number and other flow characteristics such that, the data taken in a wind tunnel test may be erroneous. The flow changes depend on the amount of heat released through condensation. The functional dependence of Mach number and pressure on the heat released may be expressed as

\[
\frac{dM^2}{M^2} = \frac{1 + \gamma M^2}{(1 - M^2)} \left( \frac{dQ}{H} - \frac{dA}{A} \right)
\]

\[
\frac{dp}{p} = -\frac{\gamma M^2}{(1 - M^2)} \left( \frac{dQ}{H} - \frac{dA}{A} \right)
\]

where \( M \) is the Mach number, \( \gamma \) is the specific heats ratio, \( dQ \) is the heat added through condensation, \( H \) is the enthalpy, \( A \) is the duct cross-sectional area, and \( p \) is the static pressure.
From the above relations it is seen that, at subsonic speeds the Mach number decreases and the pressure increases with condensation, whereas at supersonic speeds the reverse is taking place. It is important to note that, the presence of water vapour without condensation is of no significance as far as the temperature ratio, pressure ratio, and Mach number determination with the isentropic relations are concerned. The condensation depends on the amount of moisture in the air stream, the static temperature of the stream, the static pressure of the stream, and the time duration during which the stream is at a low temperature.
The amount of moisture that may be contained in normal atmospheric air usually varies in the range of 0.004 to 0.023 kg/(kg of dry air). The air temperature at the test-section of a supersonic tunnel is usually quite low. For example, let us assume that the total temperature of the stored air is $40^\circ$C. From isentropic table (for $\gamma = 1.4$), we can see that for a test-section Mach number of 1.0, the static temperature is $-12^\circ$C, for $M = 1.5$, $T = -57^\circ$C, and will go to a low value of $-220^\circ$C at $M = 5.0$. 
The static temperatures reached during expansion of air at a stagnation temperature of 40°C to Mach number above 1 are considerably below the dew-point temperatures, normally found in the atmosphere. Hence, the static temperature in a supersonic tunnel can easily be low enough to condense out normal atmospheric water vapour. The static pressure in a wind tunnel drops more rapidly than the static temperature, with Mach number increase.
The condensation of moisture in an air stream is the result of molecules colliding and combining and eventually building up into droplet size. The likelihood of condensation in a supersonic wind tunnel with supercooling (cooling of the air below the dew-point) of less than $-12^\circ\text{C}$ is negligible. Since condensation is a result of a gradual buildup from molecular to droplet size due to molecular collisions, it is obviously a time-dependent process.
There are two ways of solving the problem of condensation in supersonic tunnels. The first is to heat the air so that upon expansion to the desired Mach number, its static temperature will be above the temperature corresponding to $-12^\circ C$ of supercooling. But this approach requires heating of air to high temperatures, and hence impractical. For example, with a $3^\circ C$ dew-point, $-12^\circ C$ of supercooling would correspond to a static air temperature of $-9^\circ C$. If this occurs at Mach 2.0, the total temperature required to be $202^\circ C$. The total temperature required would increase rapidly with increasing Mach number.
The second method is to dry the air before storing it in the pressure tanks. Being simpler, this is the commonly used approach. Equipment for drying air to dew-points in the neighborhood of $-20^\circ\text{C}$ is commercially available and not expensive.
Calibration

The calibration of a supersonic wind tunnel includes determining the test-section flow Mach number throughout the range of operating pressure of each nozzle, determining flow angularity, and determining an indication of turbulence level effects.
Mach Number Determination

The following methods may be employed for determining the test-section Mach number of supersonic wind tunnels. Mach numbers from close to the speed of sound to 1.6 are usually obtained by measuring the static pressure ($p$) in the test-section and the total pressure ($p_{01}$) in the settling chamber and using the isentropic relation

$$\frac{p_{01}}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$
For Mach numbers above 1.6, it is more accurate to use the pitot pressure in the test-section ($p_{02}$) with the total head in the settling chamber ($p_{01}$) and the normal shock relation.

\[
\frac{p_{02}}{p_{01}} = \left[ 1 + \frac{2\gamma}{\gamma + 1} \left( M_1^2 - 1 \right) \right]^{-\frac{1}{\gamma - 1}} \left[ \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2} \right] \frac{\gamma}{\gamma - 1}
\]
Measurement of static pressure, $p_1$, using a wall pressure tap in the test-section and measurement of pitot pressure, $p_{02}$, at the test-section axis, above the static tap can be used through the Rayleigh pitot formula,

$$\frac{p_1}{p_{02}} = \left( \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left( \frac{\gamma + 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

for accurate determination of the Mach number.
Measurement of shock wave angle, $\beta$, from schlieren and shadowgraph photograph of flow past a wedge or cone of angle $\theta$ can be used to obtain the Mach number through $\theta - \beta - M$ relation,

$$\tan \theta = 2 \cot \beta \left[ \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 \gamma + \cos 2\beta + 2} \right]$$
The Mach angle, \( \mu \), measured from a schlieren photograph of a clean test-section can also be used for determining the Mach number with the relation

\[
\sin \mu = \frac{1}{M_1}
\]

For this the schlieren system used must be powerful enough to capture the Mach waves in the test-section.

Mach number can also be obtained by measuring pressures on the surface of cones or two-dimensional wedges, although this is rarely done in calibration.
Pitot Pressure Measurement

Pitot pressures are measured by using pitot probe. The pitot probe is simply a tube with a blunt end facing into the air stream. The tube will normally have an inside to outside diameter ratio of 0.5 to 0.75, and a length aligned with the air stream of 15 to 20 times the tube diameter. The inside diameter of the tube forms the pressure orifice. For test-section calibration, a rake consisting of a number of pitot probes are usually employed. The pitot tube is simple to construct and accurate to use. It should always have a squared-off entry and the largest practical ratio of hole (inside) diameter to outside diameter.
At this stage, it is important to note that an open-ended tube facing into the air stream always measures the stagnation pressure (a term identical in meaning to the ‘total head’) it sees. For flows with Mach number greater than 1, a bow shock wave will be formed ahead of the pitot tube nose. Therefore, the flow reaching the probe nose is not the actual freestream flow, but the flow traversed by the bow shock at the nose.
Thus, what the pitot probe measures is not the actual static pressure but the total pressure behind a normal shock (the portion of the bow shock at the nose hole can be approximated to a normal shock). This new value is called *pitot pressure* and in modern terminology refers to pressure measured by a pitot probe in a supersonic stream.
The pressure measured by a pitot probe is significantly influenced by very low Reynolds numbers based on probe diameter. However, this effect is seldom a problem in supersonic tunnels, since a reasonable-size probe will usually have a Reynolds number well above 500, the Reynolds numbers below this is the troublesome range for pitot pressure measurements.
Static Pressure Measurement

Supersonic flow static pressure measurements are much more difficult than the measurement of pitot and static pressures in a subsonic flow. The primary problem in the use of static pressure probes at supersonic speeds is that the probe will have a shock wave (either attached or detached shock) at its nose, causing a rise in static pressure.
The flow passing through the oblique shock at the nose will be decelerated. However, the flow will continue to be supersonic, since all naturally occurring oblique shocks are weak shocks with supersonic flow on either side of them. The supersonic flow of reduced Mach number will get decelerated further, while passing over the nose-cone of the probe, since decrease of streamtube area would decelerate a supersonic stream.
This progressively decelerating flow over the nose-cone would be expanded by the expansion fan at the nose-cone shoulder junction of the probe. Therefore, the distance over the shoulder be sufficient for the flow to get accelerated to the level of the undisturbed freestream static pressure, in order to measure the correct static pressure of the flow. The static pressure hole should be located at the point where the flow comes to the level of freestream Mach number.
Here, it is essential to note that, the flow deceleration process through the oblique shock at the probe nose, and over the nose-cone portion can be made to be approximately isentropic, if the flow turning angles through these compression waves are kept less than $5^\circ$ (Gas Dynamics, E Rathakrishnan, 2nd edi. 2008). The design and measurement procedure of static pressure probe are discussed in detail in chapter 7.
Static pressures on the walls of supersonic tunnels are often used for rough estimation of test-section Mach numbers. However, it should be noted that the wall pressures will not correspond to the pressures on the tunnel center line if compression or expansion waves are present between the wall and the center line. When Mach number is to be determined from static pressure measurements, the total pressure of the stream is measured in the settling chamber simultaneously with the test-section static pressure. Mach number is then calculated using isentropic relation.
Determinations of Flow Angularity

The flow angularity in a supersonic tunnel is usually determined by using either cone or wedge yaw-meters. Sensitivities of these yaw meters are maximum when the wedge or cone angles are maximum. They work below Mach numbers for which wave detachment occurs, and are so used. The cone yaw meter is more extensively used than the wedge yaw meter, since it is easier to fabricate.
Determination of Turbulence Level

Measurements with a hot-wire anemometer demonstrate that, there are high-frequency fluctuations in the air stream of supersonic tunnels that do not occur in free air. These fluctuations, broadly grouped under the heading of ‘turbulence’, consists of small oscillations in velocity, stream temperature (entropy), and static pressure (sound). Some typical values of these fluctuations are given in Table 2.
Table 2  Turbulence level in the settling chamber and test-section of a supersonic tunnel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Settling chamber</th>
<th>Test-section</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>all</td>
<td>2.2</td>
</tr>
<tr>
<td>Sound, $\frac{\Delta p}{p}$</td>
<td>$&lt; 0.1 %$</td>
<td>0.2%</td>
</tr>
<tr>
<td>Entropy, $\frac{\Delta T}{T}$</td>
<td>$&lt; 0.1 %$</td>
<td>$&lt; 0.1 %$</td>
</tr>
<tr>
<td>Velocity, $\frac{\Delta V}{V}$</td>
<td>0.5 to 1%</td>
<td>$&lt; 0.1 %$</td>
</tr>
</tbody>
</table>
The pressure regulating valve, drive system, after cooler, and test-section boundary layer are the major causes for the fluctuations. Velocity fluctuations due to upstream causes may be reduced at low and moderate Mach numbers by the addition of screens in the settling chambers. At high Mach numbers, upstream pressure and velocity effects are usually less, since the large nozzle contraction ratios damp them out. Temperature fluctuations are unaffected by contraction ratio.
Determination of Test-Section Noise

The test-section noise is defined as pressure fluctuations. Noise may result from unsteady settling chamber pressure fluctuations due to upstream flow conditions. It may also be due to weak unsteady shocks originating in a turbulent boundary layer on the tunnel wall. Noise in the test-section is very likely to influence the point of boundary layer transition on a model. Also, it is probable that the noise will influence the other test results as well.
Test-section noise can be detected by either hot-wire anemometry measurements or by high-response pitot pressure measurements. It is a usual practice to make measurements in both the test-section and the settling chamber of the tunnel to determine whether the noise is coming from the test-section boundary layer. It is then possible to determine whether fluctuations in the two places are related. The test-section noise usually increases with increasing tunnel operating pressure, and, that test-section noise originating in the settling chamber usually decreases as tunnel Mach number increases.
The Use of Calibration Results

The Mach number in the vicinity of a model during a test is assumed to be equal to an average of those obtained in the same portion of the test-section during calibrations. With this Mach number and the total pressure \( p_{01} \) measured in the settling chamber, it is possible to define the dynamic pressure, \( q \), as

\[
\frac{q}{p_{01}} = \frac{\gamma}{2} M^2 \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{-\frac{\gamma}{\gamma - 1}}
\]

for use in data reduction.
If the total temperature is also measured in the settling chamber, all properties of the flow in the test-section can be obtained using isentropic relations. The flow angularities measured during calibration are used to adjust model angles set with respect to the tunnel axis to a mean flow direction reference. The transition point and noise measurements made during the calibration may be used to decrease the tunnel turbulence and noise level.
Starting of Supersonic Tunnels

Supersonic tunnels are usually started by operating a quick-operating valve, which causes air to flow through the tunnel. In continuous operation tunnels, the compressors are normally brought up to the desired operating speed with air passing through a by-pass line. When operating speed is reached, a valve in the by-pass line is closed, which forces the air through the tunnel. In blowdown tunnels a valve between the pressure storage tanks and the tunnel is opened.
Quick starting is **desirable** for supersonic tunnels, since the model is subjected to high loads during the starting process. Also, the quick start of blowdown tunnel conserves air. To determine when the tunnel is started, the pressure at an orifice in the test-section wall near the model nose is usually observed. When this pressure **suddenly drops** to a value close to the static pressure for the design Mach number, the tunnel is started. If the model is blocking the tunnel, the pressure will not drop. We can easily identify the starting of the tunnel **from the sound** it makes.
Some tunnels are provided with variable second-throat diffusers, designed to decrease the pressure ratio required for tunnel operation. These diffusers are designed to allow the setting of a cross-sectional area large enough for starting the tunnel and to allow the setting of a less cross-sectional area for more efficient tunnel operation. When used as designed, the variable diffuser throat area is reduced to a predetermined area as soon as the tunnel starts.
Starting Loads

Whenever a supersonic tunnel is being started or stopped, a normal shock passes through the test-section and large forces are imposed on the model. The model oscillates violently at the natural frequency of the model support system and normal force loads of about 5 times those which the model would experience during steady flow in the same tunnel at an angle of attack of 10 degrees are not uncommon. The magnitudes of starting loads on a given model in a given tunnel are quite random and exactly what causes the large loads is not yet understood.
Starting loads pose a serious problem in the design of balances for wind tunnel models. If the balances are designed to be strong enough to withstand these severe starting loads, it is difficult to obtain sensitivities adequate for resolving the much smaller aerodynamic loads during tests. Number of methods have been used for alleviating this problem. Among them the more commonly used methods are

- Starting at a reduced total pressure in continuous tunnels.
- Changing the model during starting.
- Shielding the model with retractable protective shoes, at start.
- Injecting the model into the air stream after the tunnel is started.
Reynolds Number Effects

The primary effects of Reynolds number in supersonic wind tunnel testing are on drag measurements. The aerodynamic drag of a model is usually made up of the following four parts.

1. The skin friction drag, which is equal to the momentum loss of air in the boundary layer.
2. The pressure drag, which is equal to the integration of pressure loads in the axial direction, over all surfaces of the model ahead of the base.
3. The base drag, which is equal to the product of base pressure differential and base area.
4. The drag due to lift, which is equal to the component of normal force in the flight direction.
The pressure drag and drag due to lift are essentially independent of model scale or Reynolds number, and can be evaluated from wind tunnel tests of small models. But the skin friction and base drags are influenced by Reynolds number. In the supersonic regime, the skin friction is only a small portion of the total drag due to the increased pressure drag over the fore body of the model. However, it is still quite significant and need to be accounted for. Although the probability of downstream disturbances affecting the base pressure and hence the base drag is reduced because of the inability of downstream disturbances to move upstream in supersonic flow, enough changes make their way through the subsonic wake to cause significant base interference effects.
Model Mounting-Sting Effects

Any sting extending downstream from the base of a model will have an effect on the flow and therefore, is likely to affect model base pressure. For actual tests the sting must be considerably larger than that to withstand the tunnel starting loads and to allow testing to the maximum steady load condition, with a reasonable model deflection. Sting diameters of 1/4 to 3/4 model base diameters are typical in the wind tunnel tests. The effects on the base pressure of typical sting diameters are significant, but represent less than 1 per cent of the dynamic pressure and therefore, a small amount of the total drag of most of the models.
Calibration and Use of Hypersonic Tunnels

Hypersonic tunnels operate in the Mach number range of 5 to 10 or more. The stagnation pressure varies from 1 MPa to 10 MPa, and the stagnation temperature varies from 60°C to 2000°C. They mostly have solid-walled test-sections and require contoured nozzles which are most frequently axially symmetric instead of two-dimensional. Models which are larger than those that can be tested in supersonic tunnels can be tested in hypersonic tunnels.
Hypersonic tunnel models sometimes have frontal area as high as 10 per cent of the test-section area. Model size will probably be limited by the large model wake, which takes up too much flow area in the diffuser and blocks it during tunnel starting. The tunnel wall effect is unlikely to affect the flow over the model.
The air used in a hypersonic tunnel is **heated to avoid liquefaction** during expansion to the high Mach number and the corresponding low-temperatures, and to facilitate heat transfer studies. In fact, the use of heated air is the major factor that makes the hypersonic tunnel operation to be **more complicated** than supersonic tunnel operation.
The air in hypersonic tunnels must also be **dry to avoid condensation effects** due to the expansion of the air to high Mach numbers and the associated low-temperatures. However, this problem is less serious here than in supersonic tunnels, because in the process of compressing the air to the necessary high pressures for hypersonic flow, most of the natural water will be simply squeezed out.
Calibration of Hypersonic Tunnels

The calibration procedure for hypersonic tunnel test-section is generally the same as that of a supersonic tunnel. However, in hypersonic tunnels it is much more important to calibrate over the complete range of conditions through which the tunnel will operate. The boundary layers at the nozzle wall are much thicker and subject to larger changes in thickness than in supersonic tunnels, due to operating pressure and temperature. Also, the real gas effects make the test-section Mach number very sensitive to total temperature.
Further, a significant axial temperature gradient may exist in the settling chamber, with the temperature decaying as the nozzle throat is approached. In addition to axial gradients, serious lateral temperature gradient is also present in the settling chamber. These must be eliminated before uniform flow can be achieved in the test-section.
Mach Number Determination

As in the supersonic tunnels, Mach number in hypersonic tunnels are usually obtained by using pitot pressure measurements, which differ from those in supersonic tunnels in pressure and Reynolds number range. Pitot pressure in hypersonic tunnels will \textit{usually be lower}. It should be ensured that the Reynolds number based on probe diameter is above 500 (or preferably 1000) since inaccurate measurements are likely if it is lower.
The determination of Mach number from the measured pitot and total pressures become highly complicated if the air temperature is 500°C or above, because of the real gas effects. The procedure for determining the Mach number from the measured pitot and total pressures and a measured total temperature is as follows.

1. From the measured $p_{02}/p_{01}$, determine the corresponding Mach number from the normal shock table for perfect gas.

2. From the chart of $(p_{02}/p_{01})_{\text{thermperf}}/(p_{02}/p_{01})_{\text{perf}}$ versus $M$, determine this ratio of pressure ratios at the above $M$ and measured $T_0$. 

1 Divide the experimental pressure ratio by the ratio determined above to obtain corresponding value of \((p_{02}/p_{01})_{\text{perf}}\).

2 For the new \((p_{02}/p_{01})_{\text{perf}}\), determine the corresponding \(M\) from perfect gas normal shock table.

3 If the Mach number obtained above is not equal to that used in step 2, enter step 2 with the Mach number from step 4 and repeat. When the two Mach numbers agree closely, the interpolation is complete.

Note that, for accurate determination of \(M\), we need pressure-Mach number charts for high temperature (> 500°C) regime. For this we should refer to books specializing on high temperature gases. This method of determining Mach number from measured pressure ratio is cumbersome and inaccurate. A high-speed computer may be used for this purpose.
The following facts about hypersonic tunnel operation will be of high value for experimentation with hypersonic tunnels.

- If the air in the settling chamber is at room temperature, we can achieve a test-section Mach number of 5 without liquefaction of air.

- But to avoid liquefaction of air as it expands to the test-section conditions where the Mach number is 10, the stagnation temperature (i.e. the temperature in the settling chamber) should be approximately 1060 K.
Because of the limitation of heater capacity, the maximum Mach number for a continuous-flow wind tunnel using air as the test gas is approximately 10.

However, using better insulation and heater buildup methods, there are tunnels operating at Mach numbers of 12 and even 14 with air as the test gas.
One of the major concerns in hypersonic tunnel operations is the combination of maximum Mach number and minimum stagnation temperature for which condensation-free flow can be generated. Daum and Gyarmathy (1968) showed, based on their experimental study of air condensation that, in a rapidly expanding nozzle flow at low stream pressures (less than about 0.05 mm Hg), significant supercooling of the air can be achieved, since the onset of condensation was due to the spontaneous condensation of nitrogen. At these low pressures, an approximately constant experimental supercooling value of about $22 \text{ K}$ was obtained.
To achieve higher Mach number flows, the stagnation temperature must be in **excess of 1060 K**. The high-pressure–high-temperature condition required for hypersonic tunnels can be generated by many ways. For example, use of flow conditions downstream of the reflected shock wave in a shock tube as the reservoir conditions for a shock tunnel is one such means. But the tunnel run time for such facilities is very short. The short run time reduces the energy requirements and alleviates tunnel and model thermal-structural interactions.
Based on run time, hypersonic wind tunnels are classified into the following three categories.

- **Impulse facilities**, which have run times of about 1 second or less.
- **Intermittent tunnels** (blowdown or indraft), which have run times of a few seconds to several minutes.
- **Continuous tunnels**, which can operate for hours (this is only of theoretical interest, since continuous hypersonic tunnels are extremely expensive to build and operate).
The facilities with the shortest run times have the higher stagnation temperatures. Arc discharge or reflected shock waves in a shock tube are used to generate the short-duration, high-temperature stagnation conditions.

The flow quality (including uniformity, noise, cleanliness, and steadiness) in the test-section can affect the results obtained in a ground-test program. Disturbance modes in the hypersonic tunnels include vorticity (turbulence fluctuations), entropy fluctuations (temperature spottiness) which are traceable to the settling chamber, and pressure fluctuations (radiated aerodynamic noise). These disturbances can affect the results of boundary layer transition studies conducted in hypersonic wind tunnels.
It is known that, even the high-enthalpy, short-duration facilities operate on the borderline between perfect gas and real gas flow. Let us look at the reservoir temperature required to maintain perfect air at a test-section temperature of 50 K and Mach number $M_1$. The following table gives these values for different $M_1$. 
Table 3  Stagnation temperature \((T_{01})\) variation with \(M_1\) for \(T_1 = 50\) K, based on perfect gas assumption

<table>
<thead>
<tr>
<th>(M_1)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{01}) (K)</td>
<td>300</td>
<td>410</td>
<td>540</td>
<td>690</td>
<td>860</td>
<td>1050</td>
<td>1260</td>
<td>1490</td>
<td>1740</td>
<td>2010</td>
<td>2300</td>
</tr>
</tbody>
</table>
When the test-section Mach number is 8.5, the stagnation temperature for perfect air must be 772 K. However, it is well established that, the vibrational state is exited beginning approximately at 770 K. Thus, for the high Mach number facilities, vibrational excitation occurs in the settling chamber, followed by vibrational freezing downstream of the throat, and subsequent rapid relaxation in the downstream section of the nozzle. This kind of improper characteristics of hypersonic flow field manifests itself as an error in the Mach number.
Hypersonic tunnels heated by conventional clean air heaters have exhibited a Mach number error of as much as 1.5 per cent compared to that predicted by isentropic flow using the ratio of freestream pitot pressure to reservoir pressure. Even these relatively small errors in Mach number can result in significant errors in the non-dimensionalized data, if they are ignored.
It is important to note that, usually hypersonic wind tunnels operate such that the static temperature in the test-section approaches the liquefaction limit. Thus, hypersonic Mach numbers are achieved with relatively low freestream velocities (which is related to the kinetic energy), because of the speed of sound (which is related to the static temperature) is relatively low.
Determination of Flow Angularity

Flow angularity in hypersonic tunnels is usually determined by cones with included angle in the range of 20 to 90 degrees. The shock waves on cones with higher angles are detached throughout the hypersonic Mach number range and the surface pressure variation with angle of attack cannot be easily calculated.
Determination of Turbulence Level

The large contraction ratios of hypersonic tunnels have a tendency to reduce the turbulence percentage level in the test-section to insignificant values. Therefore, there is no need to determine the turbulence level of hypersonic tunnels.
Blockage Tests

Blockage tests are made during the calibration phase, to determine the size of the models that may be tested in the tunnel, and to find the effect of model size on the starting and operating pressure ratios of the tunnel.
Starting Loads

There is no published data available on starting loads in hypersonic tunnels. But from experience it is found that, at Mach 7 the starting loads are not so severe as indicated for supersonic tunnels.
Reynolds Number Effects

At hypersonic speeds the boundary layers are relatively thicker and prone to separate in the presence of unfavorable (adverse) pressure gradients than at supersonic speeds. Also, there is likely to be an intense interaction between the shock waves and boundary layers. For example, on a wedge or cone leading edge, the shock at hypersonic speeds will be very close to the surface. The boundary layer on the wedge or cone will be an important part of the distance between the surface and the shock.
Under these conditions, loads on the model can no longer be considered simply as those due to an inviscid flow field which exerts pressure through the boundary layer and onto the model surface. Since the boundary layer primarily depends on the Reynolds number, we can say that, the complete flow field around a vehicle at hypersonic speeds is dependent to a significant extent on Reynolds number. Thus, force and moment coefficients in addition to drag are likely to be influenced significantly by Reynolds number.
The boundary layers on models in hypersonic tunnels are mostly, if not entirely, laminar. However, it is not clear that tripping the boundary layer is the answer to the problem of obtaining comparable flow fields over the model in the tunnel and the full-scale vehicle in the flight. In flight at hypersonic speeds, the full-scale vehicle is likely to have long runs of laminar flow if it has a reasonably smooth surfaces.
Reynolds number as high as $7 \times 10^7$ without transition have been reported on rockets. This highlights the difficulty of predicting the location where the transition will occur on the model in flight and consequently where or if a boundary layer trip should be used. The usual practice at present is to test models without transition strips in hypersonic tunnels. If it is found that, the smooth model has extensive boundary layer separation at some point at which it is not expected on the vehicle in flight, then a transition strip may be tried as a means of eliminating separation.
Force Measurements

Force measurements in hypersonic tunnels are similar to those of subsonic and supersonic tunnels. However, there are a few problems in hypersonic tunnel force measurements which do not exist in the lower-speed tunnels. They are the following.

- Models will get **heated up** during the tests, since hypersonic tunnels use heated air.
- It is essential to ensure that the model heating and the heated air do not affect the electrical signals from the strain gauge balance.
- There are **possibilities of significant temperature effects** on balance readout at temperatures well below those for which the cement holding the gauges to the flexures fails.
With the model at an angle of attack, surface heating rates on the model will be higher on the windward side than on the leeward side. Air circulating from the model base through the balance cavity will also heat the gauges on one side of the balance more than the gauges on the other side.

These cases of uneven heating of balance are not taken care of by temperature compensation of the bridges of the balance. Keeping the balance temperature essentially constant at a near ambient value during the test is the remedy for the variable balance temperature problem.
In addition to effects on balance readings, uneven heating on the windward and leeward model surfaces may cause model distortion of significant magnitude, especially when the length to diameter ratio of the model is large. This effect is usually avoided by cooling the model. In intermittent operating tunnels it may be eliminated by increasing model wall thickness or by using a material such as Invar, which has a low coefficient of thermal expansion.
Low model loads at high Mach numbers is another problem in hypersonic force tests. Aerodynamic loads in some cases may be considerably less than the model weight. This poses a problem in balance design. The balance must be strong enough to hold the model but it must also be weak enough to be sensitive to loads smaller than the model weight. Under these conditions, there is likelihood of continuous low-frequency oscillations of the model during a test. These oscillations may cause inertia loads to become a significant portion of the aerodynamic loads to be measured and a satisfactory data cannot be obtained unless data acquisition system is equipped with suitable electronic filtering.
Flow Visualization

Schlieren system for high-speed tunnels are often designed for passing the light through the test-section two times using a ‘double-pass’ system. This is accomplished by using a circular arc mirror adjacent to one wall of the test-section and a light source and mirror focal point as close together as possible on the opposite side of the test-section, in order to increase the system sensitivity.
However, it is found that obtaining good schlieren pictures of the flow around a model when the test-section pressures are less than about 1 mm mercury (absolute) is difficult, even with double pass system. Pressures below 1 mm mercury are common in wind tunnels operating at Mach 8 and above.
To obtain better flow visualization at low test-section pressures, the air in the flow field the model may be ionized using an electric current (used by Jet Propulsion Laboratory). An electrode is placed a few centimeters upstream and a few centimeters above the model in the test-section. A potential of 5000 volts direct current is established between the electrode and model with a current flow of 0.4 amp.
The flow of current ionizes the flow field, with the result that shock waves are clearly shown in regular photographs and are much more clearly visible in this schlieren photographs than in schlieren photographs taken without ionization. Care must be taken to interlock (possibly with a low pressure switch) the power system to prevent injury to personnel.
Hypervelocity Facilities

These are experimental aerodynamic facilities that allow testing and research at velocities considerably above those achieved in the wind tunnels discussed in the previous sections of this chapter. The high velocities in these facilities are achieved at the expense of other parameters, such as Mach number, pressure, and/or run time.

From the discussions on supersonic and hypersonic tunnels, it is obvious that the aerodynamic problems of high-speed flight are not completely answered by tests in these facilities, where the tunnel operating temperature is only high enough to avoid liquefaction.
Also, we know that, if the static temperatures and pressures in the test-section of a wind tunnel have to be equal to those at some altitude in the atmosphere and at the same time that the velocity in the wind tunnel equals the flight velocity of a vehicle at that altitude, then the total temperatures and pressures in the wind tunnel must be quite high.
It is important to keep the static temperature, static pressure, and velocity in the test-section same as those in the actual flight condition, since only then the temperature and pressure in the vicinity of the model (behind shock waves and in boundary layers) correspond to conditions for the vehicle in flight.
Having the proper temperature and pressure in the vicinity of the model is considered important since at high temperatures, the characteristics of air are completely different from those at low temperatures. Experimental facilities which have been developed to simulate realistic flow conditions at high-speeds and which are used extensively for high-speed testing are the following.
- Hotshot tunnels.
- Plasma jets.
- Shock tubes.
- Shock tunnels.
- Light gas guns.

Though it is not our aim to discuss these facilities in this book, let us briefly see them to have an idea about the facilities which are expected to dominate the experimental study in the high-speed regime in the future.
Hotshot Tunnels

Hotshot tunnels are devices meant for the generation of high-speed flows with high temperatures and pressures for a short duration. The high temperatures and pressures required at the test-section are obtained by rapidly discharging a large amount of electrical energy into an enclosed small volume of air, which then expands through a nozzle and a test-section. The main parts of a hotshot tunnels are shown schematically in Figure 43.
**Figure: 43 Main parts of hotshot tunnel**
The arc chamber is filled with air at pressures up to 270 MPa. The rest of the circuit is evacuated and kept at low pressures at the order of few microns. The high- and low-pressure portions are separated by a thin metallic or plastic diaphragm located slightly upstream of the nozzle throat. Electrical energy from a capacitance or inductance energy storage system is discharged into the arc chamber over a time interval of a few milliseconds. The energy added to the air causes an increase in its temperature and pressure, and this makes the diaphragm to get ruptured. When the diaphragm ruptures, the air at high temperature and pressure in the arc chamber expands through the nozzle and establishes a high velocity flow.
The high velocity flow typically lasts for 10 to 100 milliseconds periods, but varies continuously during the period. The flow variation is due to a decay of a pressure and temperature in the arc chamber with time. The high velocity flow is terminated when the shock that passed through the tunnel in starting the flow is reflected from a downstream end of the vacuum tank and arrives back upstream at the model.
Presently the common operating conditions of hotshot tunnels are about 20 MPa, 4000°C and 20 Mach and above, although there is much variation between facilities. Data collection in hotshot tunnels are much more difficult than the conventional tunnels because of the short run times.
Plasma Arc Tunnels

Plasma arc tunnels are devices capable of generating high-speed flows with very high temperature. It uses a high-current electric arc to heat the test gas. Unlike hotshot tunnels, plasma arc tunnels may be operated for periods of the order of many minutes, using direct or alternating current. Temperatures of the order of $13000^\circ C$ or more can be achieved in the test gas.
A typical plasma arc tunnel consists of an arc chamber, a nozzle usually for a Mach number less than 3, an evacuated test-chamber into which the nozzle discharges, and a vacuum system for maintaining the test-chamber at a low pressure, as shown in Figure 44.
Figure: 44 Schematic of plasma arc tunnel
In the plasma arc tunnel, a flow of cold test gas is established through the arc chamber and the nozzle. An electric arc is established through the test gas between an insulated electrode in the arc chamber and some surface of the arc chamber. The electric arc raises the temperature of the test gas to an ionization level, rendering the test gas as a mixture of free electrons, positively charged ions, and neutral atoms. This mixture is called plasma and it is from this that the plasma arc tunnel gets its name.
Plasma tunnels operate with low stagnation pressures of the order of 700 kPa or less, with gases other than air. The enthalpy level of the test gas, and consequently the temperature and velocity in a given nozzle, are higher for a given power input when the pressure is low. Argon is often used as the test gas since high temperature and high degree of ionization can be achieved with a given power input, also the electrode will not get oxidized in argon environment.
Mostly, plasma arc tunnels are used for **studying materials for reentry vehicles**. Surface material ablation tests, which are not possible in low-temperature tunnels or high-temperature short duration tunnels, can be made. These tunnels can also be used for ‘**magneto-aerodynamics**’ and **plasma chemistry fields** to study the electrical and chemical properties of the highly ionized gas in a flow field around a model.
Shock Tubes

The shock tube is a device to produce high-speed flow with high temperatures, by traversing normal shock waves which are generated by the rupture of a diaphragm which separates a high-pressure gas from a low-pressure gas. Shock tube is a very useful research tool for investigating not only the shock phenomena, but also the behavior of the materials and objects when subjected to very high pressures and temperatures. A shock tube and its flow process are shown schematically in Figure 45.
Figure: 45 Pressure and wave diagram for a shock tube
The diaphragm between the high- and low-pressure sections is ruptured and the high-pressure driver gas rushes into the driven section, setting up a shock wave which compresses and heats the driven gas. The pressure variation through the shock tube at the instant of diaphragm rupture and at two short intervals later are shown in Figure 46. The wave diagram simply shows the position of the important waves as a function of time.
When the shock wave reaches the end of the driven (low-pressure) tube, all of the driven gas will have been compressed and will have a velocity in the direction of shock wave travel. Upon striking the end the tube, the shock gets reflected and starts traveling back upstream. As it passes through the driven gas and brings it to rest, additional compression and heating is accomplished.
The heated and compressed gas sample at the end of the shock tube will retain its state except for heat losses until the shock wave reflected from the end of the tube passes through the driver gas-driven interface and sends a reflected wave back through the stagnant gas sample, or the rarefaction wave reflected from the end of the driver (high-pressure) section reaches the gas sample. The high temperature gas samples that are generated make the shock wave useful for studies of the chemical physics problems of high-speed flight, such as dissociation and ionization.
Shock Tunnels

Shock tunnels are wind tunnels that operate at Mach numbers of the order 25 or higher for time intervals up to a few milliseconds by using air heated and compressed in a shock tube. Schematic diagram of a shock tunnel, together with wave diagram, is shown in Figure 46.

Figure: 46 Schematic of shock tunnel and wave diagram
As shown in the figure, a shock tunnel includes a shock tube, a nozzle attached to the end of the driven section of the shock tube, and a diaphragm between the driven tube and the nozzle. When the shock tube is fired and the generated shock reaches the end of the driven tube, the diaphragm at the nozzle entrance is ruptured. The shock is reflected at the end of the driven tube and the heated and compressed air behind the reflected shock is available for operation of the shock tunnel. As the reflected shock travels back through the driven section, it travels only a relatively short distance before striking the contact surface, it will be reflected back towards the end of the driven section.
When the reflected shock reaches the end of the driven section, it will result in a change in pressure and temperature of the gas adjacent to the end of the driven section. If the change in the conditions of the driven gas is significant, the flow in the nozzle will be unsatisfactory and the useful time will be terminated. The stagnation pressure and temperature in shock tunnels are about 200 MPa and 8000 K, respectively, to provide test times of about 6.5 milliseconds.
Gun Tunnels

The gun tunnel is quite similar to the shock tunnel, in operation. It has a high-pressure driver section and a low-pressure driven section with a diaphragm separating the two, as shown in Figure 47.

Figure: 47 A gun tunnel and its wave diagram
A piston is placed in the driven section, adjacent to the diaphragm, so that when the diaphragm ruptures, the piston is propelled through the driven tube, compressing the gas ahead of it. The piston used is so light that it can be accelerated to velocities significantly above the speed of sound in the driven gas. This causes a shock wave to precede the piston through the driven tube and heat the gas.
The shock wave will be reflected from the end of the driven tube to the piston, causing further heating of the gas. The piston comes to rest with equal pressure on its two sides, and the heated and compressed driven gas ruptures a diaphragm and flows through the nozzle.
As can be inferred, gun tunnels are limited in the maximum temperature that can be achieved by the piston design. The maximum temperatures normally achieved are about 2000 K. Run times of an order of magnitude higher than the shock tunnels are possible in gun tunnels. In general, the types of tests that can be carried out in gun tunnels are the same as those in the hotshot tunnels and the shock tunnels.
Ludwig Tube

The Concept of Ludwig Tube was first proposed in 1955 by Hubert Ludwig, a German scientist. The beauty of the Ludwig Tube is that, it provides clean supersonic/hypersonic flow with relative ease and low cost. Some of the well established Ludwig Tube facilities are the following.
Ludwieg Tube Group-GALCIT at Caltech: A Mach 2.3 facility is located at the California Institute of Technology, in Pasadena, CA, USA. The facility has a test time of 80 milliseconds.

The Boeing/AFOSR Mach 6 quiet tunnel at Purdue University. This tunnel has a 9.5 inch exit diameter, runs for about 10 seconds, about once an hour for about US $10/short (Built during 1995-2001).

The Ludwieg Tube tunnel at Marshall Space Flight Center. This 32 inch Ludwieg Tube built at Marshall Space Flight Center is a long tube of constant diameter for storing air at 50 atm. The run times were short, but for a time duration of half a second or less the model is bathed in airflow that was constant in pressure and temperature and displayed very little turbulence.
The Ludwieg Tube tunnel at Hypersonic Technology Gottingen, and the large number of tunnels fabricated and supplied by HTG to different universities in Germany. These tunnels have test-section size varying from 250 mm diameter to 500 mm diameter and Mach number range from 5 to 12.

The significant characteristics of Marshalls’ Ludwieg Tube was the high Reynolds number achieved—roughly three times that in conventional existing wind tunnels. This capability found immediate application in basic fluid dynamic research as well as for the determination of aerodynamic forces acting on launch vehicles. However, the Ludwieg Tube had limited use in testing winged aircraft because of high stresses encountered and the consequent distortion of models.
Operating Principle of Ludwieg Tube\textsuperscript{(23)}

Basically Ludwieg tube tunnel is a blowdown type hypersonic tunnel. Due to its special fluid dynamic features no devices are necessary to control pressure or temperature during the run. It thus can be regarded as an ‘intelligent blowdown facility’. The test gas storage occurs in a long charge tube which is, by a fast acting valve, separated from the nozzle, test-section, and the discharge vacuum tank. Upon opening of the valve an unsteady expansion wave travels with speed of sound $a_T$ into the charge tube. This wave accelerates, the gas along its way, to a tube Mach number $M_T$ given by the area ratio of the tube/nozzle throat.
During the fro-and backward traveling time of the wave a constant steady flow to the expansion nozzle is established with pressure and temperature determined by one-dimensional unsteady expansion process. Upon return of the end wall reflected wave at the nozzle throat the valve is closed and the test is finished. Thus, the length of the tube $L_T$ and the speed of sound $a_T$ of the charge tube gas determines the run time $t_R$ of the tunnel. It is given by

$$t_R = 2 \frac{L_T}{a_T}$$
From opening to closing of the valve only a short column of the charge tube gas has been discharged. To start a new test or shot, the charge tube to be refilled and the dump tank must be re-evacuated.

The optimum run time and interval between runs achieved with optimized valve operated Ludwieg tunnel are

Run time $t_R$: 0.1 to 0.5 second
Interval times $t_I$: 200 to 600 seconds

The operating principle of a typical Ludwieg tube is explained with a wave traveling diagram in the distance-time frame in Figure 48.
Figure: 48 The principle and the elements of a hypersonic Ludwieg tunnel \(^{(23)}\)
The area ratio between sonic throat and Ludwieg charge tube determines the subsonic Ludwieg tube Mach number $M_L$ is

$$\frac{A^*}{A_L} = \left(\frac{\gamma + 1}{2}\right) \left[\frac{\gamma + 1}{2(\gamma - 1)}\right] M_L \left(1 + \frac{\gamma - 1}{2} M_L^2\right) - \left[\frac{\gamma + 1}{2(\gamma - 1)}\right]$$

where $A_L$ is the charge tube cross-sectional area, $A^*$ is nozzle throat area and $\gamma$ is the specific heats ratio of the gas in the charge tube.

Unsteady one-dimensional expansion relates the flow conditions behind the expansion wave to flow conditions of the charged Ludwieg tube.
The following are the working relations.

Pressure ratio is given by

\[
\frac{p_L}{p_{L0}} = \left(1 + \frac{\gamma - 1}{2} M_L\right)^{-2\gamma}
\]

Temperature ratio is given by

\[
\frac{T_L}{T_{L0}} = \left(1 + \frac{\gamma - 1}{2} M_L\right)^{-2}
\]

where subscript ‘L’ refers to the state of the gas behind the expansion wave and ‘L0’ refers to the stagnation state of the gas in the charge tube.
Speed of sound is given by

\[
\frac{a_L}{a_{L0}} = \left(1 + \frac{\gamma - 1}{2} M_L\right)^{-1}, \quad \text{with} \quad a_{L0} = \sqrt{\gamma RT_{L0}}
\]

The measurement time \( t_m \) available is given by

\[
t_m = \frac{2L_L}{a_L} = \frac{2L_L}{a_{L0}} \frac{1}{1 + M_L} \left(1 + \frac{\gamma - 1}{2} M_L\right) \frac{\gamma + 1}{2(\gamma - 1)}
\]

During the test time \( t_M \) a gas column length \( L_D \) is discharged with velocity \( V_L \) from the tube of length \( L_L \). The subscript ‘0’ refers to properties corresponding to stagnation condition. In a first approximation this discharged gas column length \( L_G \) is given by

\[
L_G = V_L \times t_m = M_L \times a_L \times t_m = 2 \times L_L M_L a_L / a_{L0}
\]
The discharged gas column length decreases with tube Mach number, which usually is kept between 0.05 and 0.15\(^{(23)}\). Thus, approximately 10–30% of the charge tube gas is discharged during one shot.

The Ludwieg tube principle can also be used for high subsonic or transonic wind tunnels. As the principle requires a discharge through a critical throat area \(A^*\) this throat is now placed behind the test-section. The throat can be combined with the fast acting valve and the ratio of valve opening area \(A^*\) to Ludwieg tube cross-sectional area \(A_L\), determines the tube \(M_L\) and finally the test-section Mach number \(M\).
Some Specific Advantages and Disadvantages of Ludwieg Tube

We have seen that, the Ludwieg tube tunnel is basically a short duration blowdown type tunnel. The idealized discharge process from a Ludwieg tube and the discharge from a pressurized sphere of equal volume are compared in Figure 49. Each pressure step in the discharge from Ludwieg tube results from a forward and backward running expansion wave.
Figure: 49 Discharge from a Ludwieg tube and from an equivalent spherical pressure vessel\(^{(23)}\)
Advantages of Ludwieg tube tunnel compared to standard blowdown tunnels are the following.

1. The Ludwieg tube tunnel requires extremely short start and shut-off time.
2. No regulation of temperature and pressure during run is necessary. No throttle valve upstream of the nozzle is necessary. The gas dynamic principle regulates the temperature and pressure.
3. From points 1 and 2 above follows an impressive economy, since there exists no waste of mass and energy of flow during tunnel start and shut-off.
4. Due to elimination of pressure regulation valve, the entrance to the nozzle can be kept clean. This ensures low turbulence level in the test-section.
5. Ludwieg tube is well suited for transient heat transfer tests.
6. There is no unit Reynolds number effect in the Ludwieg tube, as in the case of many facilities.
Disadvantages of Ludwieg tube tunnel

Like any facility, Ludwieg tube tunnel also has some disadvantages. They are

1. **Short run time**—of the order of 0.1 to 0.5 second.
2. **Limitation of maximum stagnation temperature** due to heater and charge tube material.
Hypersonic Simulation Requirements

Wind tunnel tests are usually meant for simulating the aerothermodynamic phenomena of free flight atmosphere in a scale test. A true simulation of the free flight conditions can in principle only be achieved if the dynamic similarity between the free flight and scale tests are established. This requires that the scale test must be based on the similarity laws for the problems to be investigated. In principle, the following flow phenomena need to be investigated at hypersonic conditions.

- **Compressibility** of the air.
- **Viscous effects** like friction and heat conduction.
- **Chemistry** due to dissociation of air molecules at high temperatures for flight velocities of the order of Mach 8.
It is now accepted to divide the simulation into the following regimes:\(^{(1)}\).

(a) Mach–Reynolds–Simulation for Compressibility and Viscous Effects

Geometric similarity: \textit{Model scale} \quad S = \frac{L_{WT}}{L_{FL}}

Compressibility: \textit{Mach number} \quad M_{WT} = M_{FL}; \quad \left(\frac{V}{a}\right)_{WT} = \left(\frac{V}{a}\right)_{FL}

Viscous effects: \textit{Reynolds number} \quad Re_{WT} = Re_{FL}; \quad \left(\frac{\rho VL}{\mu}\right)_{WT} = \left(\frac{\rho VL}{\mu}\right)_{FL}

Wall stagnation temperature ratio: \quad \left(\frac{T_w}{T_0}\right)_{WT} = \left(\frac{T_w}{T_0}\right)_{FL}
where the subscripts $WT$ and $FL$ refer to wind tunnel test model and actual body in flight, respectively. If we keep $V_{WT} = V_{FL}$ and $\mu_{WT} = \mu_{FL}$, then viscous similarity requires a scale test at increased density $\rho_{WT} = \rho_{FL}$. Thus, the model loads will increase with $1$/scale factor. If $V_{WT} = V_{FL}$ is not the case, the viscosity shall have the same and identical power law in both wind tunnel test and actual flight

$$\frac{\mu(T_0)}{\mu(T_\infty)} = \left(\frac{T_0}{T_\infty}\right)^\omega$$

where $\omega_{WT} = \omega_{FL}$. 
(b) Simulation of Real Gas Effects

This requires the simulation of the dissociation and recombination process of molecules behind the shock wave. In order to reach the high temperature behind shock waves in the scale test also, the first requirement is to duplicate the flight velocity.

- Duplication of velocities: \( V_{WT} = V_{FL} \)

- For chemical similarity the Damkohler numbers for forward (dissociation) and backward (recombination) have to be simulated.

  Forward reaction: \( (Da, f)_{WT} = (Da, f)_{FL} \left( \frac{V}{\rho L} \frac{1}{k_f} \right)_{WT} = \left( \frac{V}{\rho L} \frac{1}{k_f} \right)_{FL} \)

  Backward reaction: \( (Da, b)_{WT} = (Da, b)_{FL} \left( \frac{V}{\rho^2 L} \frac{1}{k_b} \right)_{WT} = \left( \frac{V}{\rho^2 L} \frac{1}{k_b} \right)_{FL} \)
As the backward reaction rate contains $\rho^2L$ this is not compatible with viscous similarity which requires $\rho L$ to be simulated. Thus, a complete chemistry simulation can not be combined with viscous flow simulation. This is the high enthalpy simulation ‘dilemma’\(^{(1)}\).

**Industrial tunnels**

Depending on the simulation task, i.e. small missiles or re-entry vehicles, the test-section diameter should range between 500 and 1000 mm.
Research tunnels

These tunnels are meant for studying and exploring basic phenomena and validating theoretical methods and codes. Therefore, a full Reynolds number simulation on large models of flight vehicles will not be necessary. The tunnels shall however have the capability to reproduce all important local phenomena on a smaller scale. This can be achieved with small facilities having the capability to simulate local phenomena at various Reynolds numbers. Adequate test-section for these tunnels range from 150 to 500 mm in diameter.
Pressure Measuring System

The pressures involved in the hypersonic flows generally are from orders of mega pascals to fraction of torr. Therefore, the pressure measuring device preferably should have a wide range.

One of the important temperatures to be measured is the charge tube temperature. We encounter a maximum of about 1400 K. For this any standard thermocouple made up of suitable ‘dissimilar’ metals will be sufficient. In addition to this if the temperature on the model surface is of interest then one has to select thermocouple foils.