

Nonlinear behavior of an ultrasonic transducer

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Abstract

A transducer model, based on nonlinear piezoelectric constitutive equations, has been developed to describe the harmonic generation that occurs on current or velocity signal analysis when the transducer is driven at high voltage. The symmetrical Langevin transducer studied close to its resonance frequency is composed of Navy III type power ceramics. The nonlinearities create an energy spread over the frequency spectrum and consequently result in an amplitude saturation effect at the driving frequency, which can be interpreted in terms of the loss mechanism. Experiments have been carried out to measure the nonlinear losses and the harmonic amplitude evolutions. Results are given and discussed. Eventually, by experimental means, identification of the influential nonlinear coefficients permits comparison between theoretical predictions and experimental results.

Keywords: Nonlinearities; Transducers; PZT

1. Introduction

For power transducers driven at high electrical levels, the nonlinear domain for piezoelectric ceramics is rapidly reached. Then, instantly, instabilities appear and the performances of the system are strongly limited by the nonlinearities [1-4].

The first - experimental - part of this paper presents different observations made on a Langevin transducer. Besides the resonance frequency shift and the hysteresis phenomenon often described [5], nonlinearities create current and power signal distortions and an energy spread resulting in an amplitude saturation effect.

To describe the transducer nonlinear behavior, a model, based on piezoelectric constitutive equations extended up to the second order, has been proposed [6,7]. This model leads to the resonance curve of the transducer for different driving amplitudes and the quantification of the frequency hysteresis phenomenon.

2. Resonator structure and measurement method

The ceramic behavior under heavy duty working conditions has been studied with a prestressed Langevin structure driven at its resonance frequency.

A small symmetrical transducer, using Navy III type power ceramics, P189 from Quartz & Silice (St Gobain), has been built. It consists of two tail masses in brass, four ceramic rings, four electrodes and a prestress screw; its resonance frequency, under 40 MPa prestress, is around 27 kHz.

Moreover, a thin stress sensor, composed of two disks of P189, can be inserted at the center of the piezostack to monitor the stress level.

A network analyser (HP 3577A), connected to a power amplifier, drives the transducer and allows us to obtain its admittance curve (see Fig. 1). Moreover it permits us to correct the resonance frequency fluctuations.

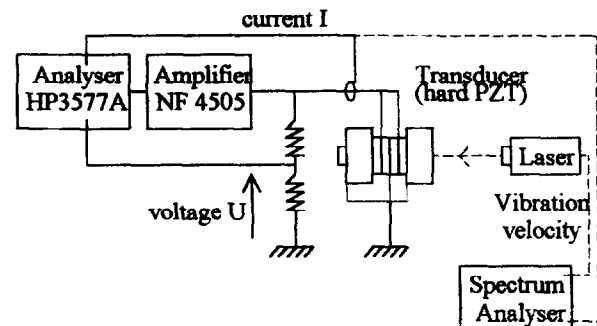


Fig. 1. Experimental set-up.

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The tail mass displacement is measured by laser vibrometry, and a spectrum analyser (HP35665A) gives the signal spectrum. The transducer radiates in air, consequently the coupling with the fluid is minimal.

3. Experimentation

The resonance frequency shift and the hysteresis phenomenon that occur when the transducer excitation level is increased are characteristic of a nonlinear behavior (see Fig. 2). A more detailed explanation of these phenomena will be given in the next section, where a piezoelectric nonlinear model of the transducer is proposed.

When the transducer is driven at a high voltage level, a distinct distortion appears on current, velocity, stress or electrical power signals. Figs. 3, 4 and 5 display these observations. The Fourier transforms of these signals exhibit a spectrum splitting over a fundamental frequency, which corresponds to the external driving frequency, and its overtones (see Figs. 6 and 7). The amplitudes of these harmonics increase with the source level. This results in an amplitude saturation of the fundamental frequency. Figs. 8 and 9 present the evolution of the fundamental, first and second harmonic of the current for different prestresses; it can be noticed that

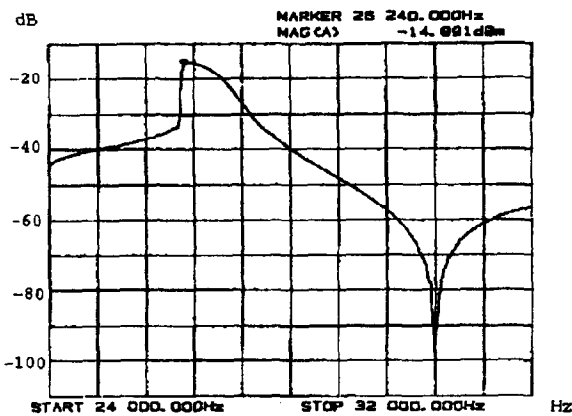


Fig. 2. Hysteresis phenomenon (current curve of the transducer for decreasing frequencies).

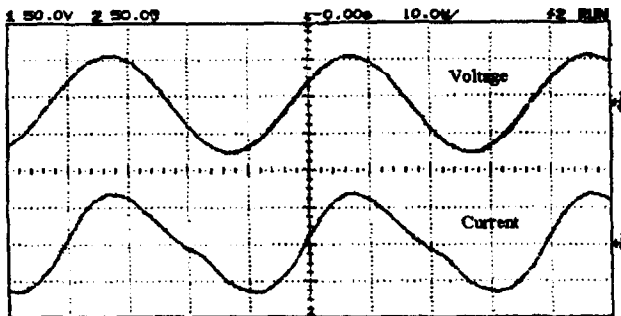


Fig. 3. Voltage and current signals (transducer 400 bars P189 65 V_{peak}).

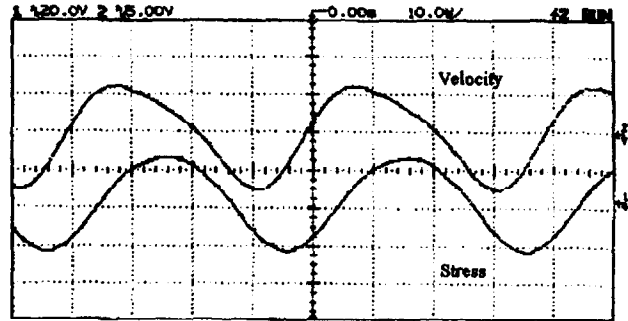


Fig. 4. Stress and velocity signals (transducer 400 bars P189 65 V_{peak}).

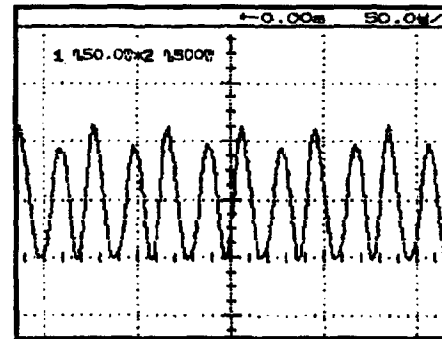


Fig. 5. Electrical power signal (transducer P189 400 bars 65 V_{peak}).

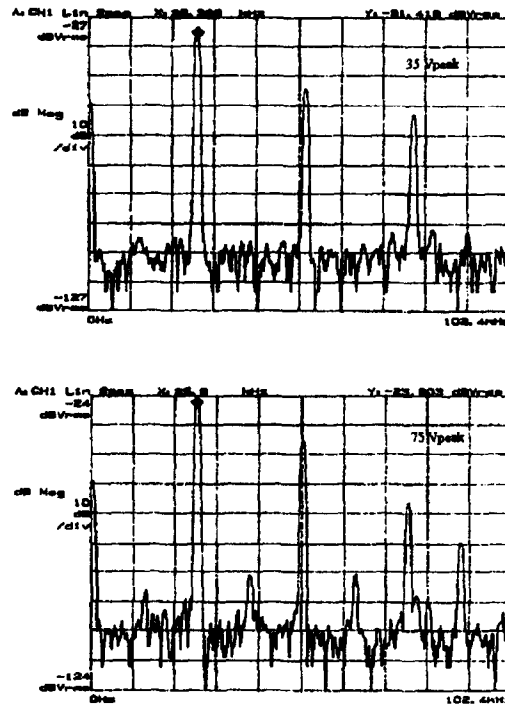


Fig. 6. Current spectra for different excitation levels 35 V_{peak} and 75 V_{peak} (P189 400 bars).

differences lower than 10 dB between the fundamental and the first harmonic amplitudes are rapidly obtained by increasing the driving voltage.

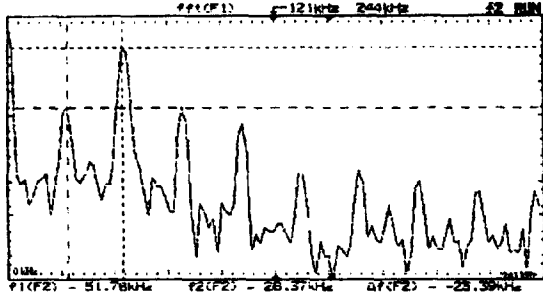


Fig. 7. Spectral distribution of the electrical power (transducer P189 400 bars 65 V_{peak}).

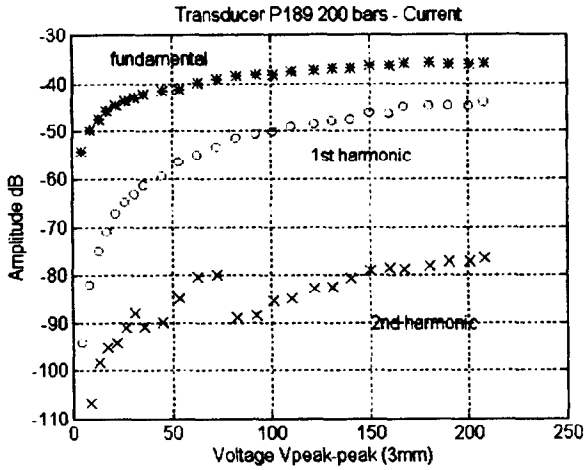


Fig. 8. Evolution of the fundamental, first and second harmonic of the current (in dB) (transducer P189 prestress: 200 bars).

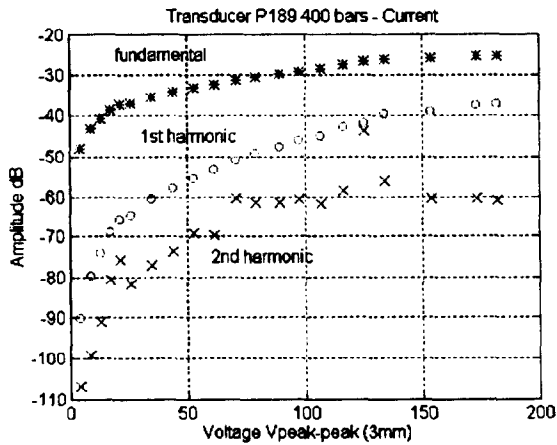


Fig. 9. Evolution of the fundamental, first and second harmonic of the current (in dB) (transducer P189 prestress: 400 bars).

4. Theoretical approach

In the low frequency regime, the transducer can be described as a lumped system. Each tail mass, assumed rigid, is shaken by two ceramic rings. Consider M as the mass of each tail mass, Σ the ceramic surface, l the half length of the ceramic stack, u the tail mass displacement and T the stress, it can then be written:

ment and T the stress, it can then be written:

$$Mu'' = -T\Sigma \quad (1)$$

where the primes specify time differentiations.

The stress T is given by the piezoelectric constitutive equations extended to the nonlinear domain:

$$T = cS + eE + \alpha_1 S^2 + \gamma SE \quad (2)$$

where S is the strain ($S \approx u/l$), E the electrical field, c and e the conventional linear piezoelectric coefficients, and α_1 and γ are the nonlinear ones.

The experimental observations must be taken into account to proceed further. Indeed, the current or velocity spectra display discrete harmonics, so we seek a solution of the form:

$$u(t) = \sum_{n=-\infty}^{n=+\infty} C_n e^{jn\Omega t} \quad (3)$$

with Ω the driving pulsation.

Substituting Eqs. (2) and (3) in Eq. (1) and adding the frictional force, whose damping coefficient is λ , the following equation is obtained:

$$\begin{aligned} & -\Omega^2 \sum_{n=-\infty}^{n=+\infty} n^2 C_n e^{jn\Omega t} + w^2 \sum_{n=-\infty}^{n=+\infty} C_n e^{jn\Omega t} \\ & + \alpha \sum_{n=-\infty}^{n=+\infty} \sum_{m=-\infty}^{m=+\infty} C_n C_m e^{j(n+m)\Omega t} \\ & + \frac{\gamma \Sigma E_0}{2lM} \left(\sum_{n=-\infty}^{n=+\infty} C_n e^{j(n+1)\Omega t} + \sum_{n=-\infty}^{n=+\infty} C_n e^{j(n-1)\Omega t} \right) \\ & + 2\lambda w j \Omega \sum_{n=-\infty}^{n=+\infty} n C_n e^{jn\Omega t} = \frac{-e \Sigma E_0}{2M} (e^{j\Omega t} + e^{-j\Omega t}) \quad (4) \end{aligned}$$

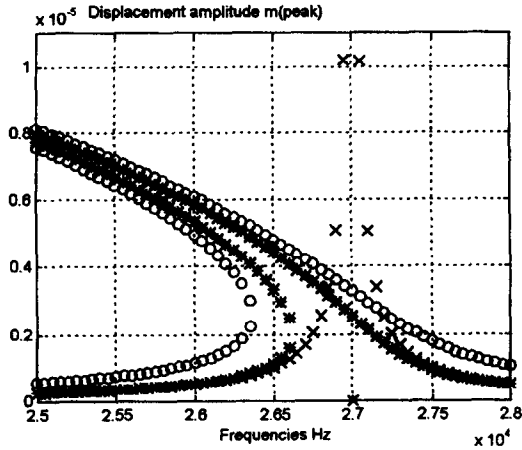
where w is the pulsation associated with the natural resonance frequency of the linear system and $\alpha = \alpha_1 \Sigma / (l^2 M)$.

From this equation, equalities can be written for each frequency; their resolution gives the fundamental or harmonic amplitude evolution for the displacement u . According to previous experiments, a 10% nonlinearity, for strain close to 10^{-3} , has been introduced on the stress function.

The resolution of Eq. (4) leads to a third degree equation in $|C_1|$.

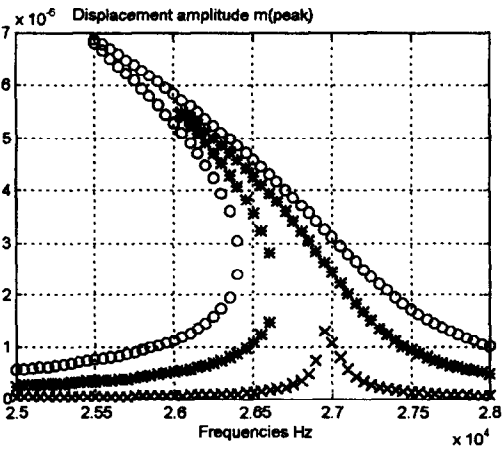
To help understanding, it is useful to consider the undamped system ($\lambda = 0$). The solutions obtained for the fundamental amplitude $|C_1|$ consist of two branches of a parabola meeting at infinity. When the excitation E_0 is low, these two branches are symmetrical and centered on the linear resonance pulsation w , whereas when the excitation level is increased the parabola branches bend to left (see Fig. 10).

If damping is introduced, the solutions do not increase indefinitely and an amplitude maximum is reached: this



$\lambda = \gamma = 0$
 'o': $\alpha = 1.38e15$ and $U=150V_{peak}$
 '*': $\alpha = 1.38e15$ and $U=70V_{peak}$
 'x': $\alpha = 0$ and $U=70V_{peak}$

Fig. 10. Simulation of the undamped system.



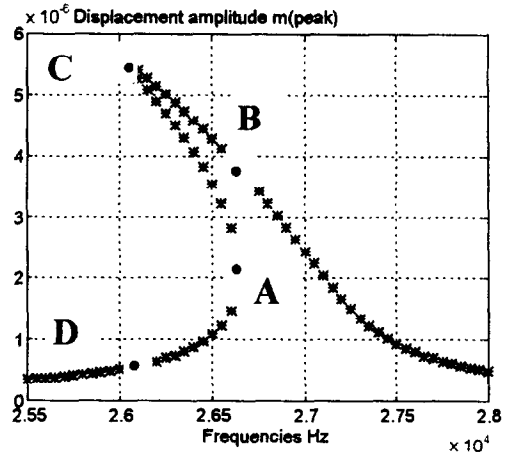
$\lambda = 0.002$
 $\gamma = 5e4$ and $\alpha = 1.38e15$
 'o': $U=150V_{peak}$
 '*': $U=70V_{peak}$
 'x': $U=10V_{peak}$

Fig. 11. Simulation of the damped system.

corresponds to the resonance amplitude (see Fig. 11) [8,9].

When the electrical excitation is small, the fundamental amplitude solution is represented by a symmetrical curve whose maximum is located at ω , the natural resonance of the system; the equation solved has one real root over the limited full frequency range.

If the excitation is increased, the curve changes and presents three real roots over a frequency domain that corresponds to an unstable area. Indeed, when the driving frequency increases, the amplitude $|C_1|$ reaches the point 'A' then jumps to point 'B', therefore creating



$\alpha = 1.38e15$ $\gamma = 5e4$ $\lambda = 0.002$ $U=70V_{peak}$

Fig. 12. Jump phenomenon description.

a discontinuity, whereas when the driving frequency decreases, the amplitude gets to point 'C' before jumping to point 'D' (see Fig. 12).

The system never gets into the part 'AC' of the curve since this corresponds to unstable oscillations. The hysteresis phenomenon and the associated instability appear.

It is interesting to note in experiments that, if the transducer is directly driven at a frequency located in the unstable area, the system will always run on the low amplitude branch 'AD' of the curve since, on this branch, the energy is minimal.

The influence of the two main nonlinear coefficients introduced (α and γ) has been tested. The nonlinear parameter γ , affecting the crossed term 'SE', has a major effect on the amplitude of the solution $|C_1|$ (see Fig. 13), while the nonlinear coefficient α , affecting the term S^2 , tends to accentuate the hysteresis phenomenon (see Fig. 14): it modifies the locations of the increasing and decreasing resonant frequencies.

5. Conclusions

Nonlinear piezoelectric constitutive equations lead to a comprehensive interpretation of the experimental observed amplitude saturation and frequency hysteresis effects.

The nonlinear coefficients' influence on the resonance curve has been studied and it shows their relative importance: the term in S^2 mainly changes the folding of the resonance curve, whereas the term in SE mainly reduces the resonant amplitude.

The displacement amplitude saturation with the electric driving field is commonly interpreted in terms of a viscous factor increase with the input voltage. In this

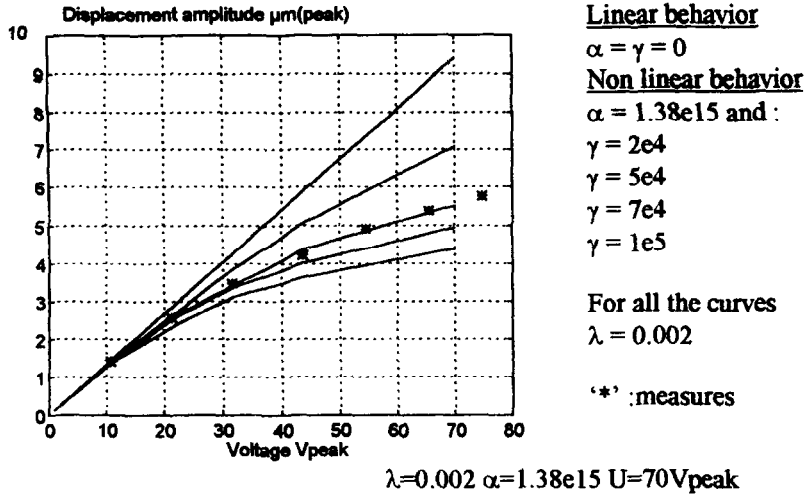


Fig. 13. Influence of the nonlinear parameter γ .

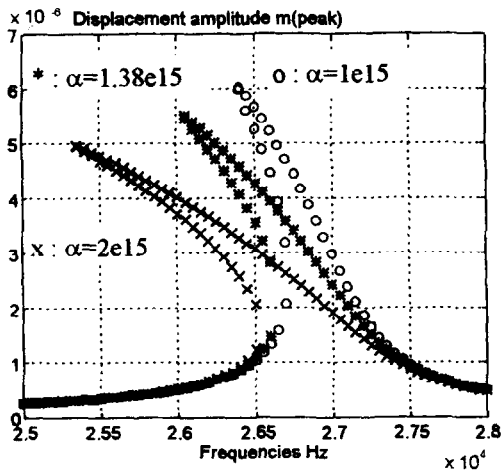


Fig. 14. Influence of the nonlinear parameter α .

paper, an alternative explanation, invoking the nonlinear term SE , is given. A dissipated power analysis will be performed to associate or distinguish between these two possible interpretations.

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