Overview of topic

• Wind resources
  – Origin of the wind
  – Estimating available wind power
• How a horizontal axis wind turbine (HAWT) works
  – Power coefficient
  – Betz relation
  – Aerofoil concept
  – Blade element–momentum theory
  – Brakes, gearbox, generators
  – Aesthetic considerations
Why does the wind blow?

- Differential heating of the earth’s surface by the sun.
Convection cells

- Hadley cells
- Uplift
- Subsidence
- Divergence
- Convergence

45 North
30
High pressure

0 Latitude (°)
Low pressure

30
45 South
High pressure

S.pole
equator
N.pole
Coriolis force

Earth’s rotation

Speed at earth’s surface:
- 500 km/h
- 600 km/h

- Polar easterlies
- Westerlies
- Horse latitudes
- Northeast trade winds
- Equatorial doldrums
- Southeast trade winds
- Horse latitudes
- Westerlies
- Polar easterlies

Earth’s rotation

Coriolis force
Secondary and tertiary circulations

• Secondary
  – Hurricanes (tropical cyclone)
  – Extratropical cyclone
  – Monsoon circulation

• Tertiary e.g.
  – Land/sea breezes
  – Mountain/valley breezes
  – Thunderstorms
  – Tornadoes
Example of tertiary circulation: valley and mountain winds
Types of wind and wind energy
Turbulence and gusts
Causes vibrations and fatigue loadings

![Wind speed graph](image)

- **Wind speed, m/s**
- **Time, seconds**
Effect of height

Rough surface, characterised by parameter $z_0$, e.g.
- Grass $z_0 = 0.01$ m
- Forest $z_0 = 0.5$ m
- City $z_0 = 3$ m

speed $u(z) = U \ln(z/z_0)$

Example:
Compare $u$ at 3 m and 30 m over grass
$u(30)/u(3) = \ln(30/0.01)/\ln(3/0.01) = 1.40$
Measure-correlate-predict

- Method for assessing wind resource at a site, based on measurements from a nearby weather station
- Measurements are taken at the site over, say, 6 months
- These are correlated with those taken at the weather station
- The correlation is used to assess long term wind resource based on records from weather station
Power in the wind

Air travelling at uniform velocity $v$
Density $\rho$

Mass flow $\dot{m} = \rho A v$

Area $A$

Kinetic energy of a portion of the flowing air of mass $\delta m$ is $\frac{1}{2} \delta m v^2$

The rate of energy flow (ie. power) $P_w$ for the whole flow is

$$P_w = \frac{1}{2} \dot{m} v^2 = \frac{1}{2} \rho A v^3$$
Power coefficient

Air travelling at uniform velocity $v$
Density $\rho$

Wind power in $P_w$

Mechanical power out $P_r$

Area $A$

Define power coefficient $C_p$

$$C_p = \frac{P_r}{P_w}$$
Betz limit

- A wind turbine cannot stop the wind completely (where would the air go?)
- The flow is spread out as it slows down
- Therefore not all of the power can be extracted
- The limit to how much power can be extracted was determined by Betz as $C_p = 0.59$
Betz limit: assumptions

• Frictionless, incompressible and steady flow: use Bernoulli equation $p + \frac{1}{2} \rho v^2 = \text{constant}$
• Velocity varies in direction of rotor axis: 1-D theory
• The work done by the fluid passing through the rotor is all converted into useful work extracted from the turbine
Betz limit: details

Air flow velocity \( u_1 \)

\[ \begin{align*}
\text{Bernoulli:} & \quad p_1 + \frac{1}{2} \rho u_1^2 = p_2 + \frac{1}{2} \rho u_2^2 \\
& \quad p_3 + \frac{1}{2} \rho u_3^2 = p_4 + \frac{1}{2} \rho u_4^2 \\
\text{Momentum balance} & \quad T = \dot{m}(u_1 - u_4) = \rho u_2 A(u_1 - u_4) \\
\text{Force balance} & \quad T = A(p_2 - p_3)
\end{align*} \]
Betz: Details (...cont)

\[ p_1 + \frac{1}{2} \rho u_1^2 = p_2 + \frac{1}{2} \rho u_2^2 \]

\[ p_3 + \frac{1}{2} \rho u_3^2 = p_4 + \frac{1}{2} \rho u_4^2 \]

Subtract using \( p_1 = p_4 \) and \( u_2 = u_3 \)

\[ p_2 - p_3 = \frac{1}{2} \rho (u_1^2 - u_4^2) \]

Substitute For \( p_2 - p_3 \)

\[ T = \frac{1}{2} \rho A (u_1^2 - u_4^2) \]

Eliminate \( T \)

\[ u_2 = \frac{1}{2} (u_1 + u_4) \]

...showing that the air velocity \( u_2 \) at the rotor is the mean of the upstream and downstream velocities \( u_1 \) and \( u_4 \)
Betz limit: Axial induction factor

Introduce the axial induction factor ‘a’ as a measure of the decrease in axial air velocity through the turbine.

Define:

\[ u_2 = u_1(1 - a) \]

The last slide showed that the velocity drops by the same amount before and after the turbine. Therefore:

\[ u_4 = u_1(1 - 2a) \]

Now we can write the thrust on the rotor in terms of \( u_1 \) and \( a \):

\[ T = 2 \rho Au_1^2 a (1-a) \]
Betz limit: conclusion

The rate at which work is transferred to the rotor $P_r$ is given by the thrust times the velocity:

$$P_r = T \ u_2 = \frac{1}{2} \ \rho A u_1^3 \ 4a \ (1-a)^2$$

From the definition of power coefficient (slide 13)

$$C_p = 4a(1-a)^2$$

At $a = 1/3$, a maximum value of $C_p$ occurs, known as the Betz limit

$$C_p = \frac{16}{27}=0.593$$

The corresponding thrust is

$$T= \ (4/9) \ \rho A u_1^2$$
Aerofoil

- Air flows more quickly over the top surface than the bottom surface, resulting in a pressure difference and lift.

- The lift on an aerofoil is several times the drag.
Aerofoils: Angle of attack and stall

• As the angle of attack increases, lift increases until stall occurs:

Lift coefficient $C_L$ defined so that:

$$\text{Lift} = \frac{1}{2} \rho u^2 C_L A$$

where $A$ is the area of the aerofoil
Aerofoils: Stall

No stall

Stall

Separation
Blade-element momentum theory
Designing the ideal rotor

- Combine the Betz theory and aerofoil theory to determine the shape of the ideal rotor

- Conditions will vary from the hub to the outside of the rotor, so we imagine the blade as consisting of small radial element each $\delta r$ long
Blade element
Ideal rotor: Relative velocity and lift

The tangential velocity of the blade is usually many times the wind speed. This gives rise to a large lift force.

\[ u_{\text{rel}} = \sqrt{u_T^2 + u_2^2} \]
Tip speed ratio

- Defined as speed of the blade tip divided by wind speed
  \[ \lambda = \frac{u_T}{u_1} \] or \[ u_T = \lambda u_1 \]

At a radius \( r \):
\[ u_T = (r/R) \lambda u_1 \]

where \( R \) is the radius of the whole blade.

- We will also make use of the Betz relation assuming the ideal situation of \( a = 1/3 \):
  \[ u_2 = (2/3)u_1 \]

- We can now express the pitch angle as \( \tan \phi = \frac{u_2}{u_T} \) and therefore
  \[ \phi = \arctan \left[ \frac{2}{3(r/R)\lambda} \right] \]
Thrust on the section of the rotor

- We can apply the Betz expression for thrust to just an annular section of the rotor.

\[ \delta T = \frac{4}{9} \rho \pi r \delta r u_1^2 \]

\[ T = \frac{4}{9} \rho A u_1^2 \]

\[ \delta T = \frac{4}{9} \rho 2\pi r \delta r u_1^2 \]

see slide 19
Ideal rotor: Relative velocity and lift

Lift force
\[ \delta F_L = \frac{1}{2} C_L \rho u_{rel}^2 \delta A \]

Tangential force
\[ \delta F_T = \delta F_L \sin \phi \]

Relative velocity
\[ u_{rel} = \sqrt{(u_T^2 + u_2^2)} = u_2 / \sin \phi \]

Tangential velocity \( u_T \)

Approach velocity \( u_2 \)

Thrust
\[ \delta T = \delta F_L \cos \phi \]
\[ = \frac{1}{2} C_L \rho u_{rel}^2 \delta A \cos \phi \]

Equate with expression for \( \delta T \) from last slide, setting \( \delta A = B \ \delta r \ c \) (\( B \) is number of blades) and use \( u_2 = (2/3)u_1 \)

\[ \tan \phi \sin \phi = C_L Bc / 4\pi r \]
Designing the ideal rotor: Results

- Combining this with the expression for $\tan \phi$ from slide 26 gives the following expression for aerofoil chord length in the direction of the pitch angle:

$$c = \frac{8\pi \sin \phi}{3BC_L \lambda_r}$$

And from slide 27 we have the corresponding ideal pitch angle:

$$\phi = \arctan \left[ \frac{2}{3(r/R)\lambda} \right]$$
Ideal rotor design: example

Tip speed ratio $\lambda = 7$
Radius $R = 5$ m
Number of blades $B = 3$
Lift coefficient $C_L = 1$

<table>
<thead>
<tr>
<th>$r/R$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord $c$ (m)</td>
<td>0.86</td>
<td>0.46</td>
<td>0.31</td>
<td>0.24</td>
<td>0.19</td>
</tr>
<tr>
<td>Pitch $\phi$ ($^\circ$)</td>
<td>27</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>
Causes of non-ideal performance

- Friction on blades (especially if dirty) decreases drag and increases lift
- Rotation, vortices and turbulence in wake correspond to wasted energy
- Tip losses: air takes a short cut around tip of blade
- Tower shadow
- Fixed speed operation (or limited speed range): mismatch with wind speed
Putting on the brakes

- A wind turbine can only absorb so much power. It has to cope with very high wind speeds occasionally.
- Therefore brakes are needed (also for safety)
- Stall braking
  - Passive stall (no pitch control)
  - Pitch controlled stall or feathering
- Mechanical brake on shaft: hydraulic disc brake
Generators

1. Synchronous generator:
   Fixed speed determined by grid
   Torque proportional to lag $\theta_r - \theta_f$
   Acts like a spring $\rightarrow$ resonances

2. Induction generator:
   Almost fixed speed determined by grid
   Torque proportional to slip speed $= \omega_r - \omega_f$
   Acts like a damper $\rightarrow$ no resonances
Gearing requirements: generator

• The speed of the induction generator only varies over a restricted range

• The approximate speed is determined by the mains frequency and number of poles e.g. for 50 Hz
  – 2 poles → 3000 rpm (50Hz)
  – 4 poles → 1500 rpm
  – 8 poles → 750 rpm
Gearing requirements: rotor

Example: wind speed 10 m/s, radius = 26 m
Tip speed = 7 x 10 = 70 m/s, giving 25 rpm
• *Therefore substantial gearing is needed e.g. 1:30*
Wind turbine nacelle

- Mechanical Brake
- Gearbox
- Generator
- Rotor
- Pitching mechanism
- Tower
Variable speed machines

- Ring generator
- AC
- Rectifier
- DC
- Inverter
- Grid
Aesthetic considerations

Generally favour:

- Slowly rotating blades
- 3 blades better than 2
- Smooth round towers preferred to lattices
- Not too much noise!
Conclusions

• Wind is primarily generated by equator to pole energy gradients combined with the earth’s rotation
• Wind speed increases significantly with height
• The power in the wind varies with the cube of wind speed
• The Betz theory establishes a limit of 16/27=0.59 to power coefficient
• The wind turbine blade is an aerofoil
• The tip speed is greater than the wind speed
• Lift is created as the blade slices through the wind – the tangential component of the lift is what drives the turbine
• You also need a gearbox, a generator and a tower to put it all on
References

www.windpower.org Danish site, contains lots of background including nice animations

Manwell J.F. et al “Wind Energy Explained”
ISBN 0 471 49972 2 (covers everything in lecture and lots more)

ISBN 0 471 48997 2, more advanced (expensive)

web sites of manufacturers e.g. www.vestas.com
www.enercon.de