ES427: The Natural Environment and Engineering

## Global warming and renewable energy

# Lecture 4: Wind energy

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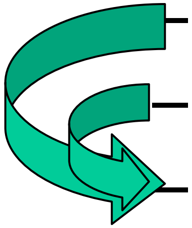
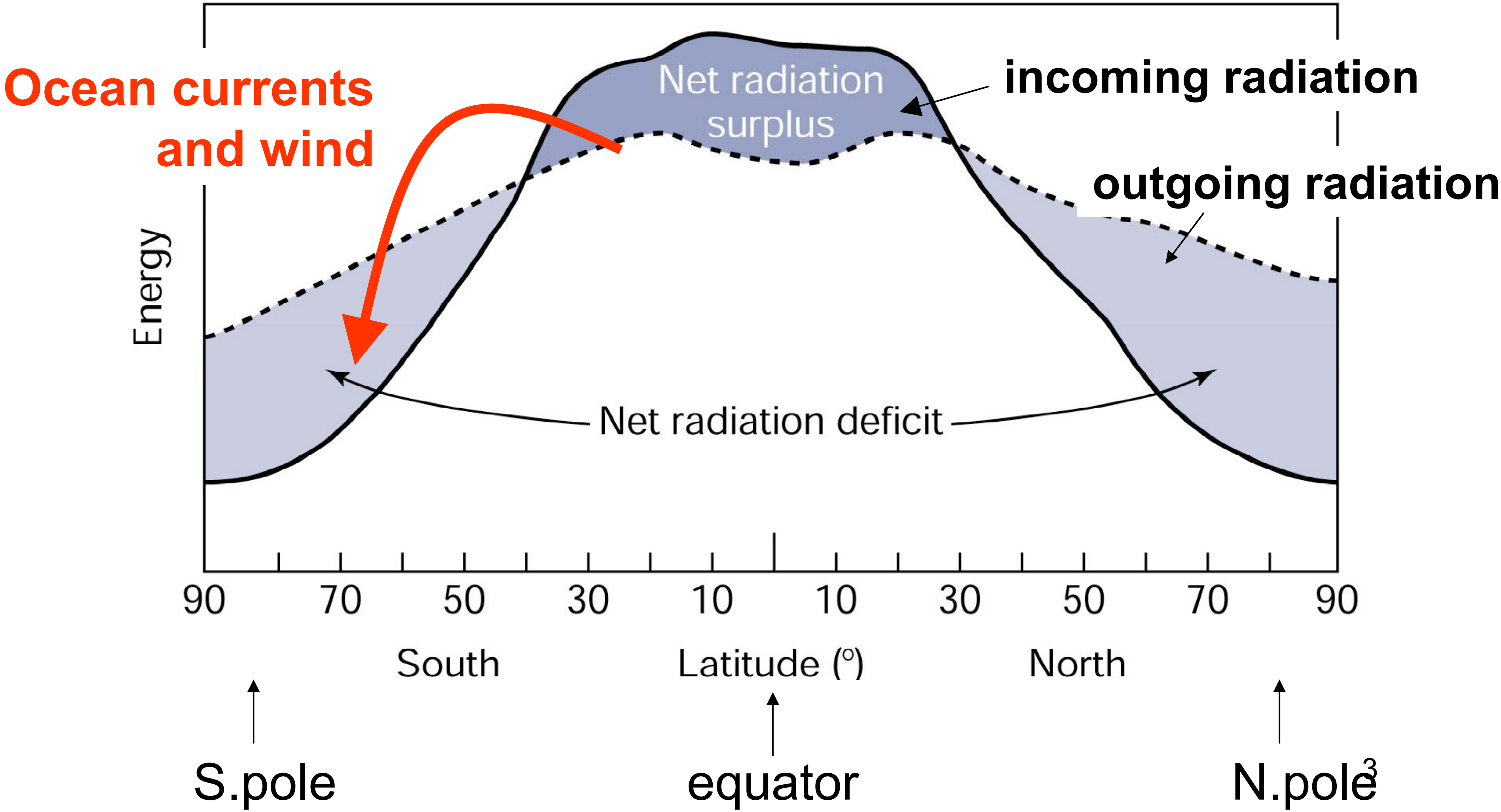
# Overview of topic

- Wind resources
  - Origin of the wind
  - Estimating available wind power
- How a horizontal axis wind turbine (HAWT) works
  - Power coefficient
  - Betz relation
  - Aerofoil concept
  - Blade element–momentum theory
  - Brakes, gearbox, generators
  - Aesthetic considerations



# Why does the wind blow?

- **Differential heating of the earth's surface by the sun.**



**Ocean currents and wind**

**incoming radiation**

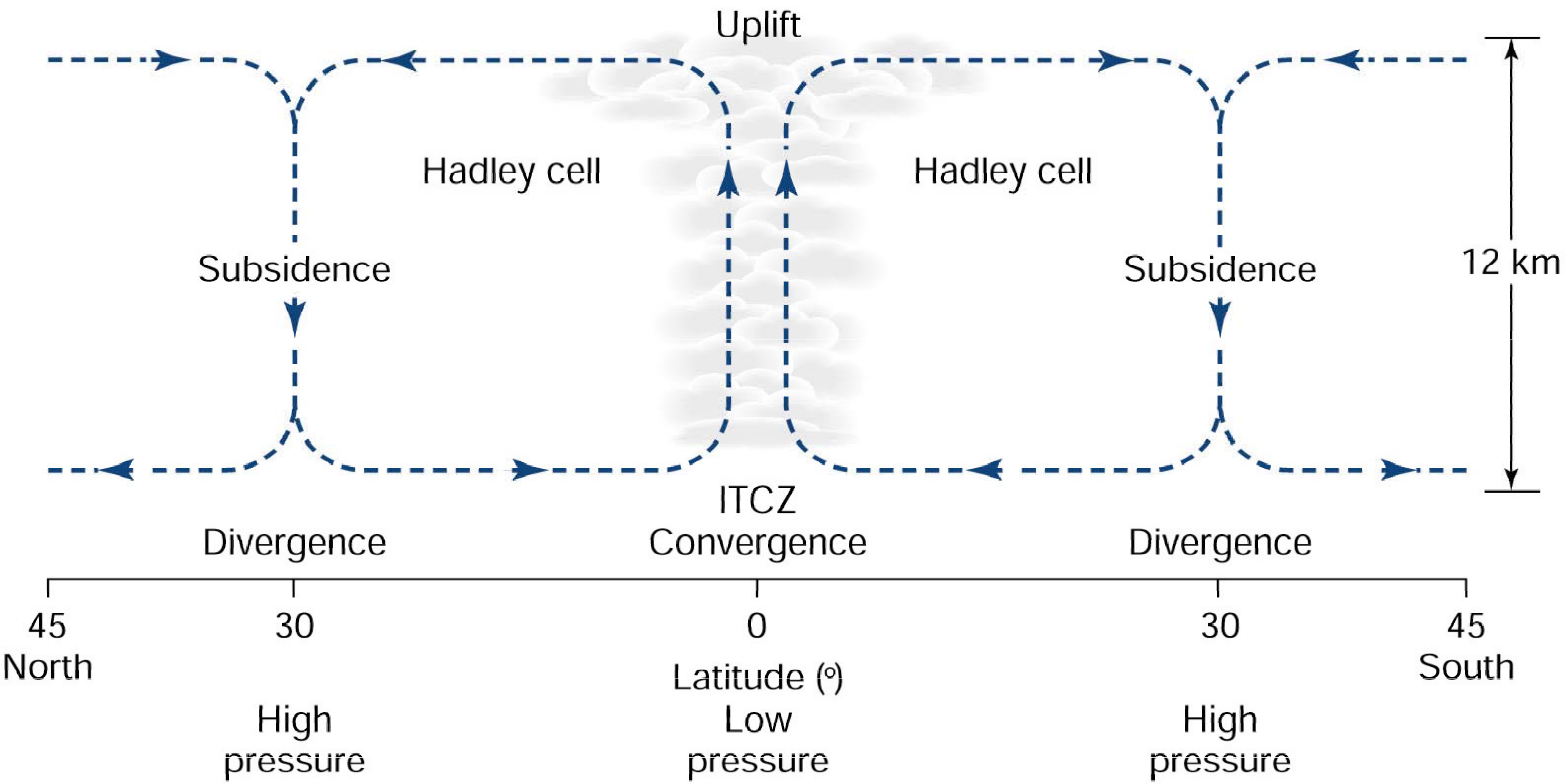
**outgoing radiation**

S.pole

equator

N.pole

# Convection cells

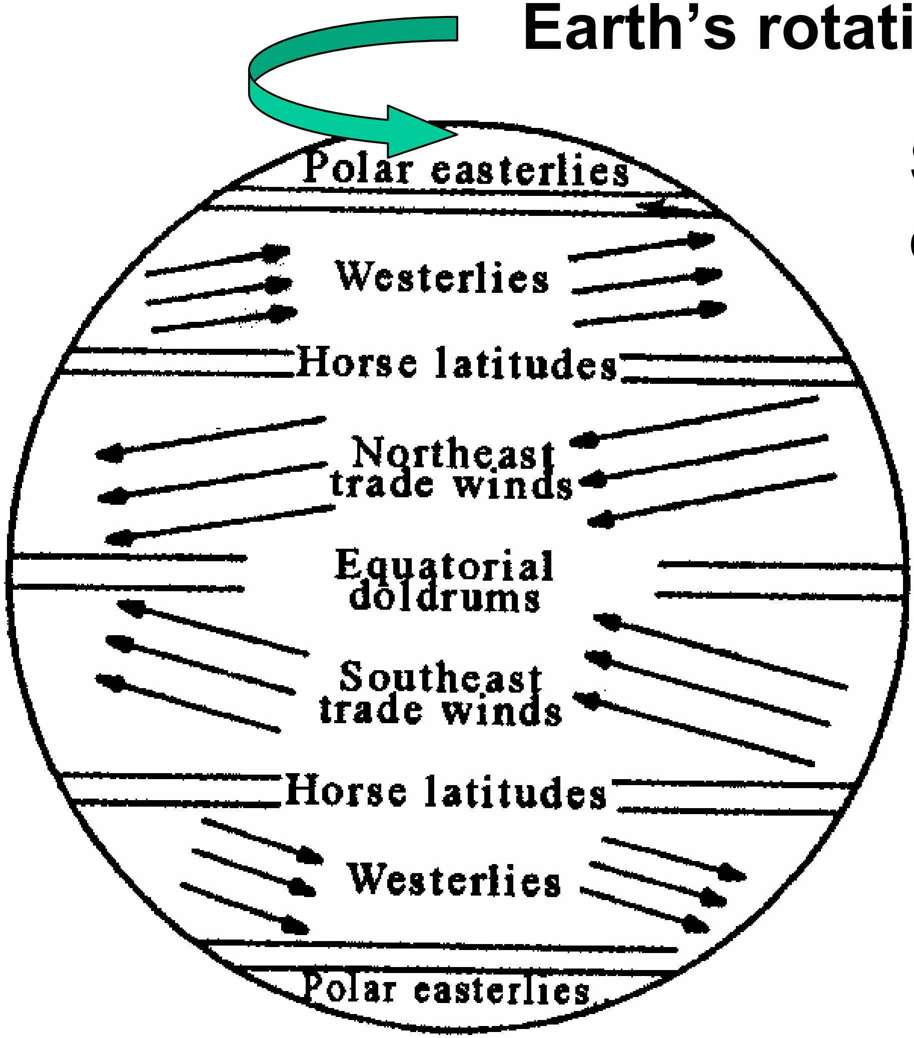


← S.pole

equator ↑

N.pole →

# Coriolis force



**Earth’s rotation**

**Speed at earth’s surface:**

**500 km/h**

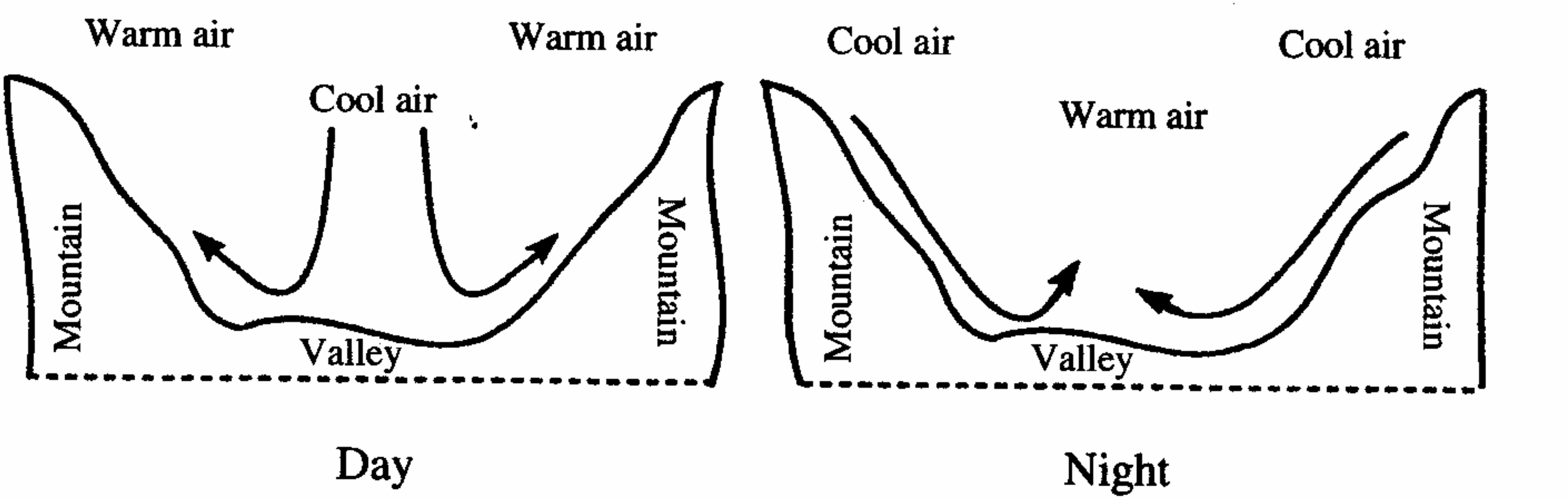
**600 km/h**

# Secondary and tertiary circulations

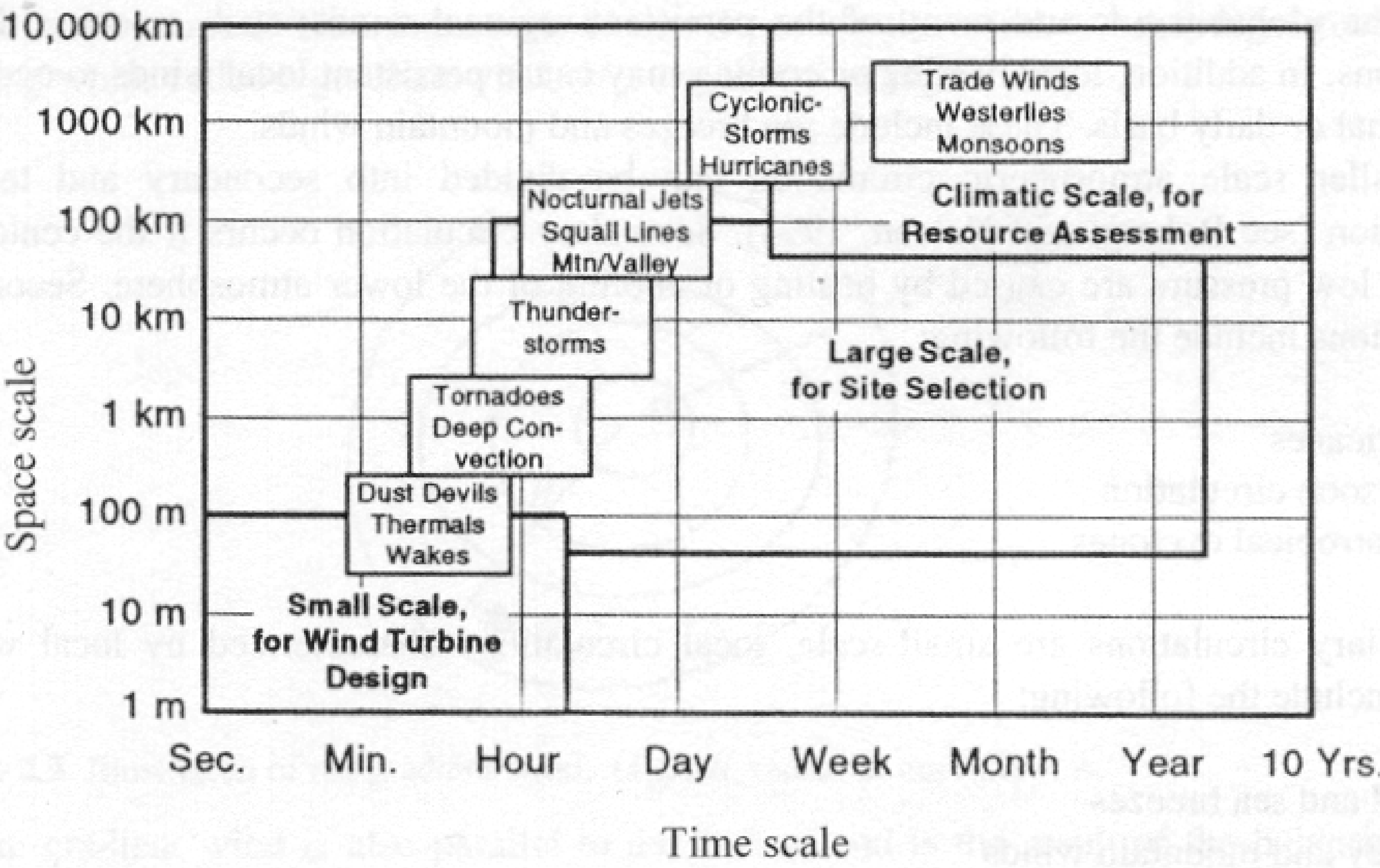
- Secondary
  - Hurricanes (tropical cyclone)
  - Extratropical cyclone
  - Monsoon circulation

- Tertiary e.g.
  - Land/sea breezes
  - Mountain/valley breezes
  - Thunderstorms
  - Tornadoes

# Example of tertiary circulation: valley and mountain winds

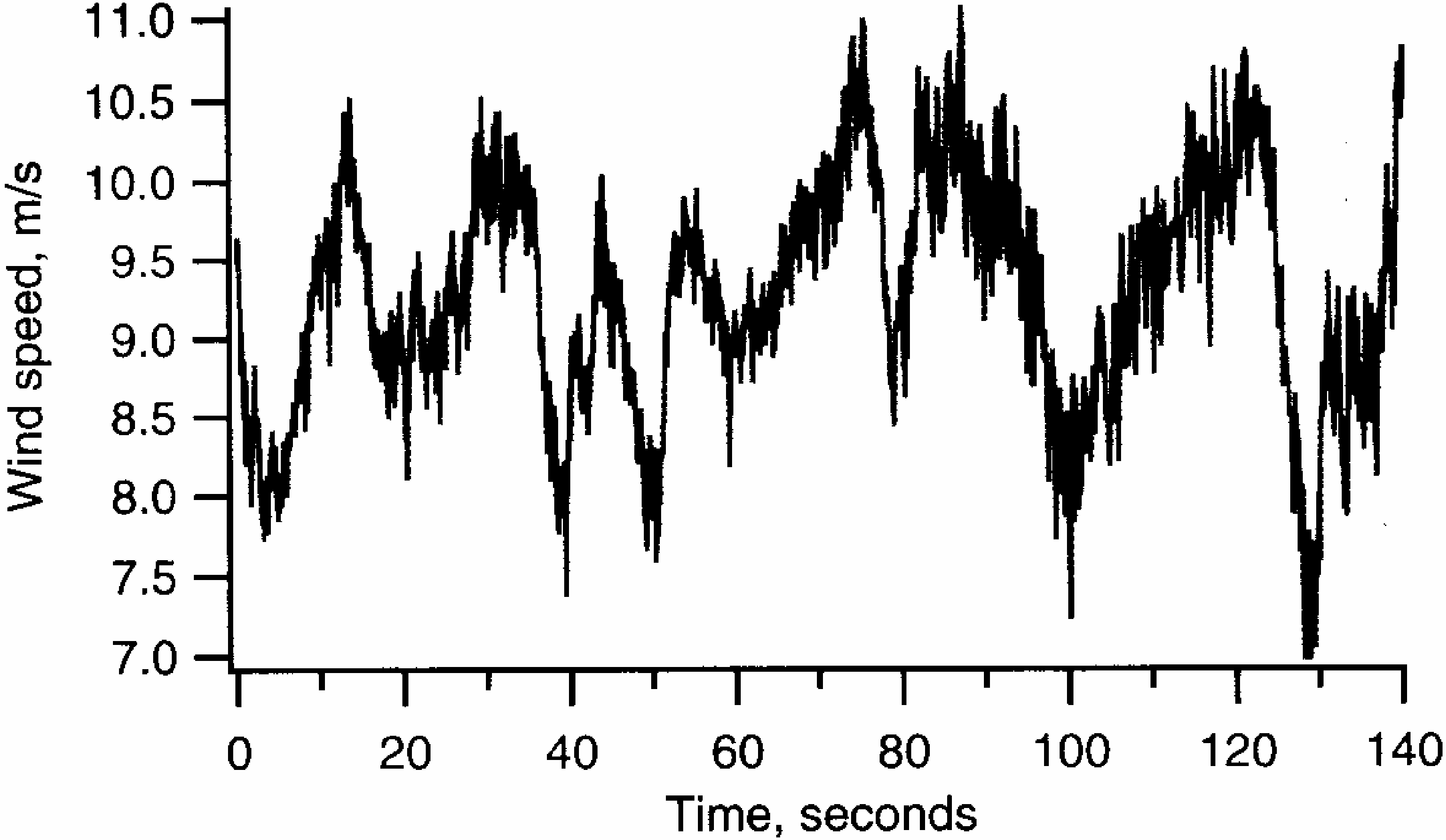


# Types of wind and wind energy



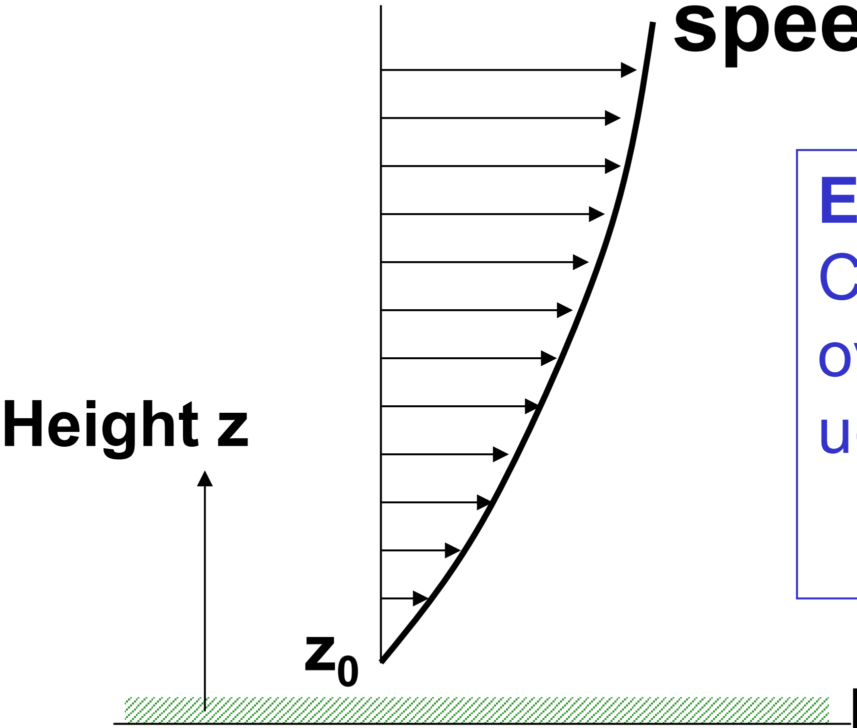
# Turbulence and gusts

## Causes vibrations and fatigue loadings



# Effect of height

**speed $u(z)=U\ln(z/z_0)$**



**Example:**

Compare u at 3 m and 30m over grass

$u(30)/u(3)=\ln(30/0.01)/\ln(3/0.01)$

$=1.40$

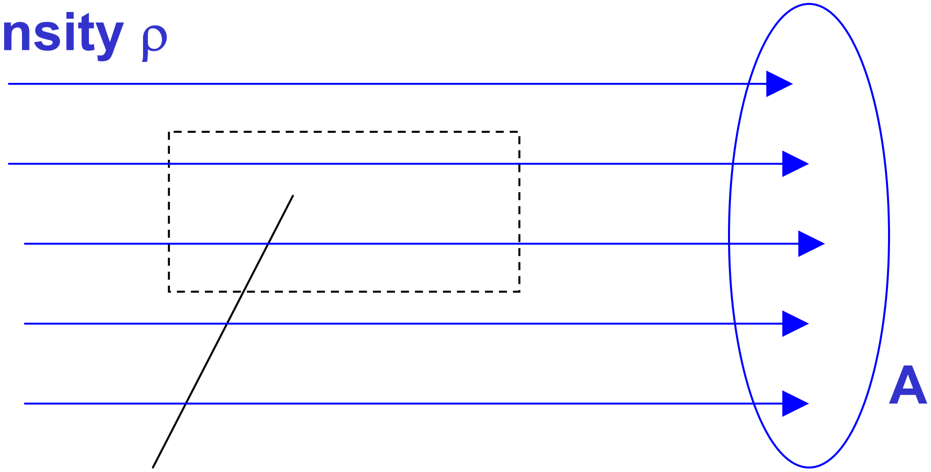
**Rough surface, characterised by parameter $z_0$, e.g.**

| | |
|---|---|
| **Grass** | **$z_0 = 0.01$ m** |
| **Forest** | **$z_0 = 0.5$ m** |
| **City** | **$z_0 = 3$ m** |

# Measure-correlate-predict

- Method for assessing wind resource at a site, based on measurements from a nearby weather station
- Measurements are taken at the site over, say, 6 months
- These are correlated with those taken at the weather station
- The correlation is used to assess long term wind resource based on records from weather station

# Power in the wind



Air travelling at uniform velocity v
Density ρ

Area A

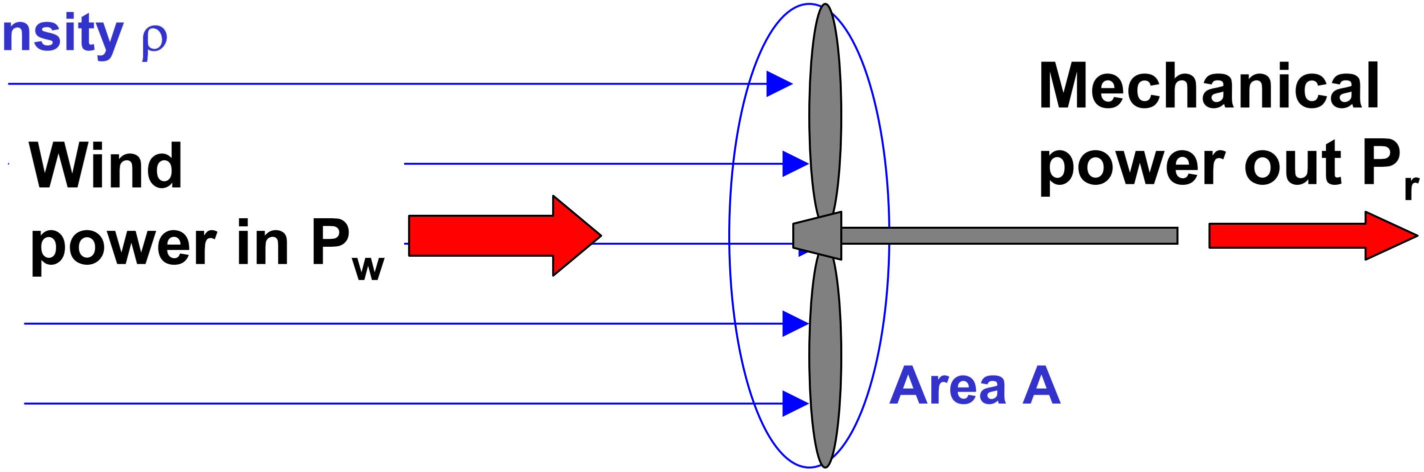
Mass flow $\dot{m} = \rho A v$

Kinetic energy of a portion of the flowing air of mass $\delta m$ is $\frac{1}{2}\,\delta m\, v^2$

The rate of energy flow (ie. power) $P_w$ for the whole flow is

$$\boxed{P_w = \tfrac{1}{2}\,\dot{m}\, v^2 = \tfrac{1}{2}\,\rho A v^3}$$

# Power coefficient



**Air travelling at uniform velocity v**
**Density $\rho$**

**Wind power in $P_w$**

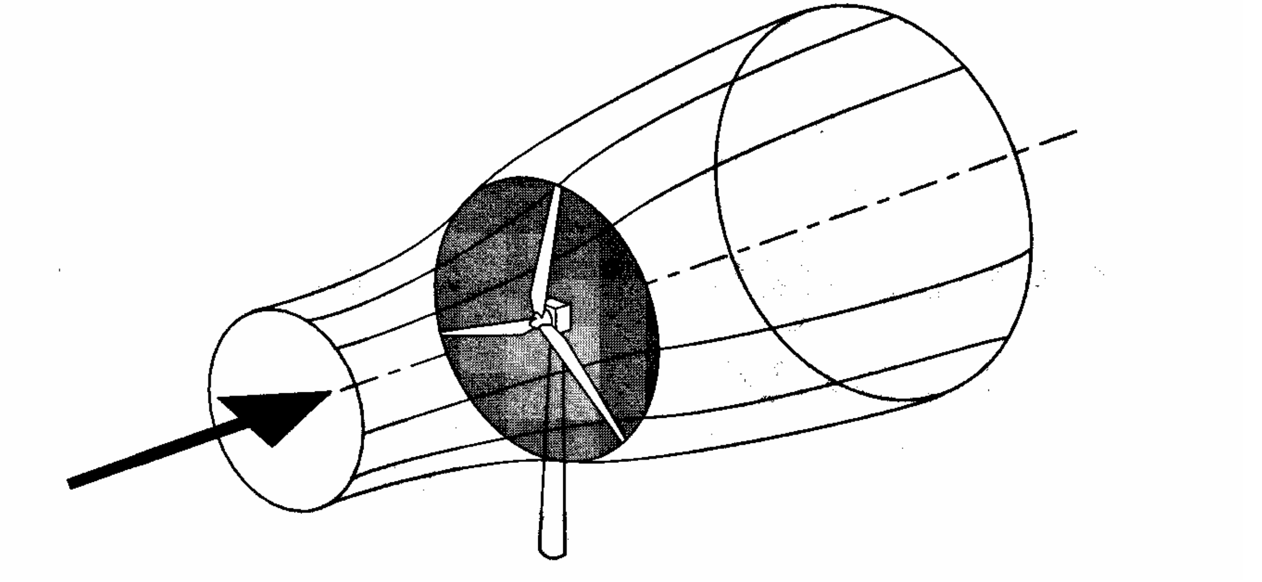
**Mechanical power out $P_r$**

**Area A**

**Define power coefficient $C_p$**

$$\boxed{C_p = P_r / P_w}$$

# Betz limit



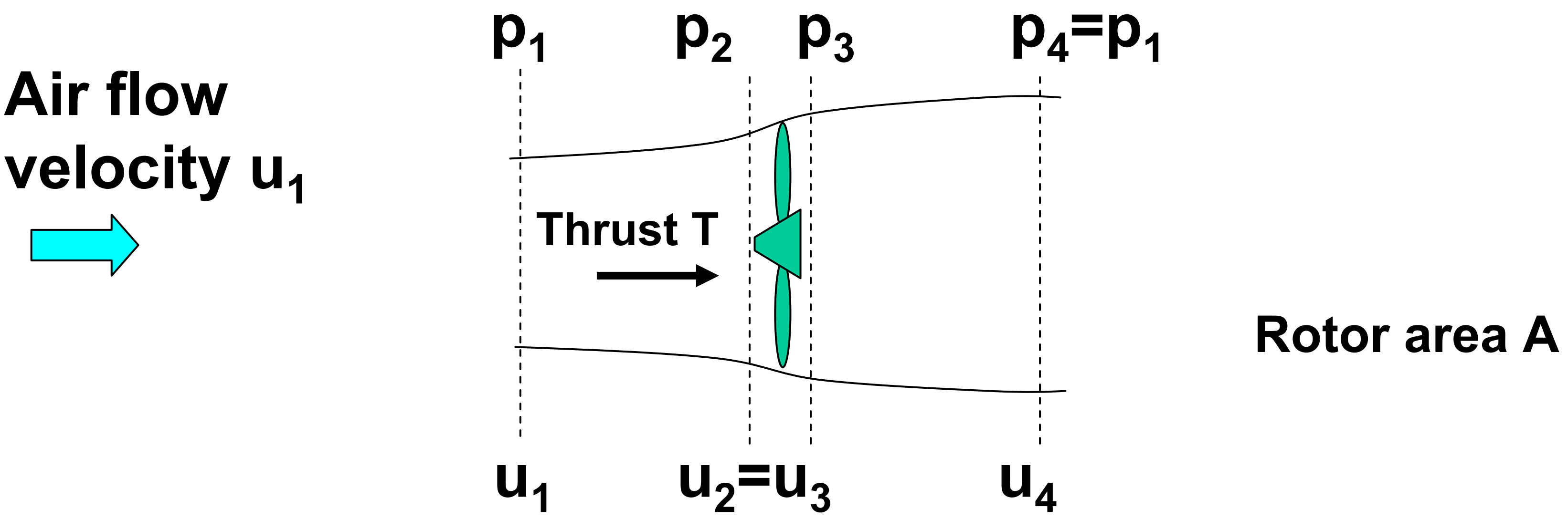
- A wind turbine cannot stop the wind completely (where would the air go?)
- The flow is spread out as it slows down
- Therefore not all of the power can be extracted
- The limit to how much power can be extracted was determined by Betz as $C_p$=0.59

# Betz limit: assumptions

- Frictionless, incompressible and steady flow: use Bernoulli equation $p + \frac{1}{2}\rho\, v^2 = \text{constant}$
- Velocity varies in direction of rotor axis:

  1-D theory
- The work done by the fluid passing through the rotor is all converted into useful work extracted from the turbine

# Betz limit:details



**Bernoulli:**

$$p_1 + \tfrac{1}{2}\rho u_1^2 = p_2 + \tfrac{1}{2}\rho u_2^2 \qquad p_3 + \tfrac{1}{2}\rho u_3^2 = p_4 + \tfrac{1}{2}\rho u_4^2$$

**Momentum balance** $T = \dot{m}(u_1 - u_4) = \rho u_2 A(u_1 - u_4)$

**Force balance** $T = A(p_2 - p_3)$

# Betz: Details (…cont)

$$p_1 + \tfrac{1}{2}\rho u_1^2 = p_2 + \tfrac{1}{2}\rho u_2^2$$

$$p_3 + \tfrac{1}{2}\rho u_3^2 = p_4 + \tfrac{1}{2}\rho u_4^2$$

Subtract using $p_1=p_4$ and $u_2=u_3$

$$p_2 - p_3 = \tfrac{1}{2}\rho(u_1^2 - u_4^2)$$

$$T = A(p_2 - p_3)$$

**Substitute For $p_2$-$p_3$**

$$T = \tfrac{1}{2}\rho A(u_1^2 - u_4^2)$$

$$T = \rho u_2 A(u_1 - u_4)$$

**Eliminate T**

$$\boxed{u_2 = \tfrac{1}{2}(u_1 + u_4)}$$

…showing that **the air velocity $u_2$ at the rotor is the mean of the upstream and downstream velocities $u_1$ and $u_4$**

# Betz limit: Axial induction factor

**Introduce the axial induction factor ‘a’ as a measure of the decrease in axial air velocity through the turbine.**

**Define:**

$$\boxed{u_2 = u_1(1 - a)}$$

**The last slide showed that the velocity drops by the same amount before and after the turbine. Therefore:**

$$\boxed{u_4 = u_1(1 - 2a)}$$

**Now we can write the thrust on the rotor in terms of $u_1$ and a:**

$$T = 2\,\rho A u_1^2\, a\,(1-a)$$

# Betz limit: conclusion

**The rate at which work is transferred to the rotor $P_r$ is given by the thrust times the velocity:**

$$P_r = T\,u_2 = \tfrac{1}{2}\,\rho A u_1^3\, 4a\,(1-a)^2$$

**From the definition of power coefficient (slide 13)**

$$\boxed{C_p = 4a(1-a)^2}$$

**At a = 1/3, a maximum value of $C_p$ occurs, known as the Betz limit**

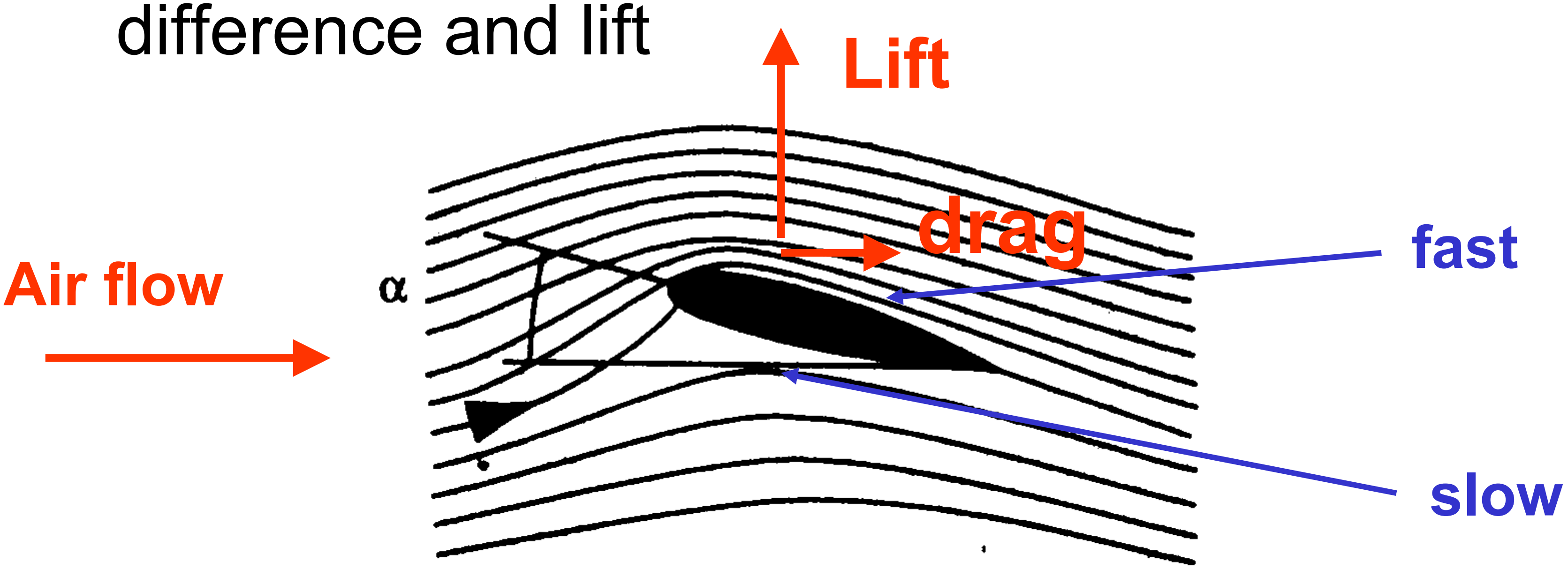
$$\boxed{C_p = 16/27 = 0.593}$$

**The corresponding thrust is**

$$\boxed{T = (4/9)\,\rho A u_1^2}$$

# Aerofoil

- Air flows more quickly over the top surface than the bottom surface, resulting in a pressure difference and lift



Lift

drag

fast

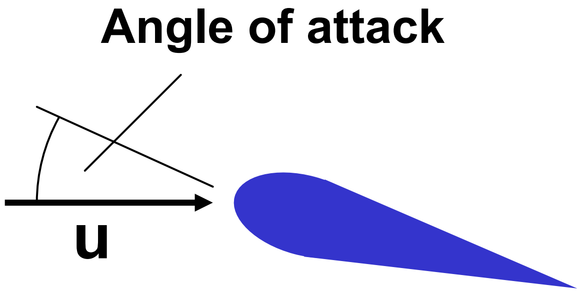
Air flow

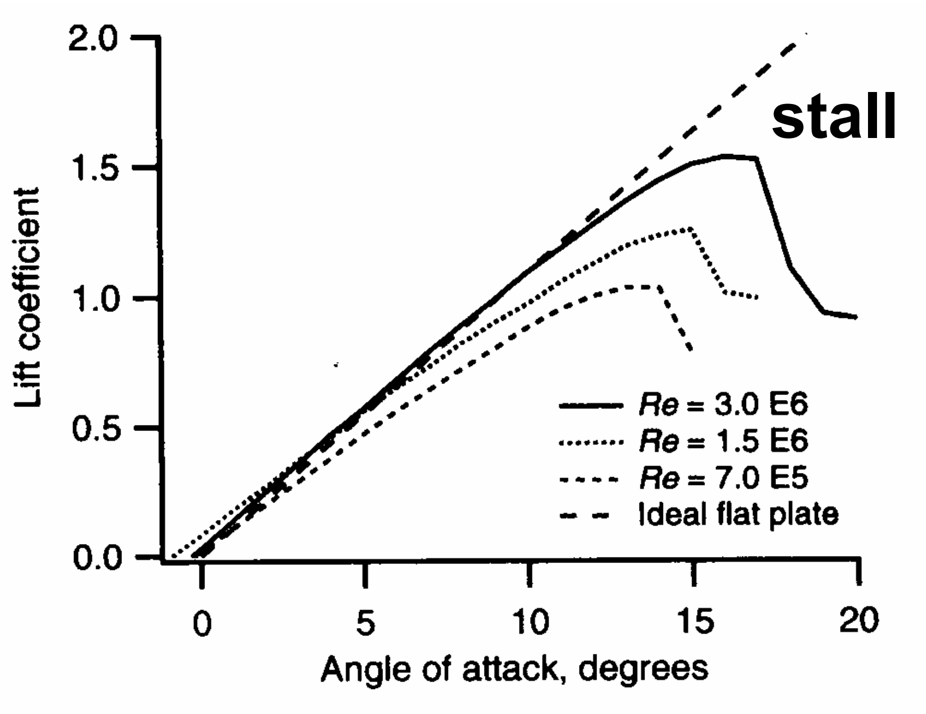
slow

- The lift on an aerofoil is several times the drag

# Aerofoils: Angle of attack and stall

- As the angle of attach increases, lift increases until stall occurs:





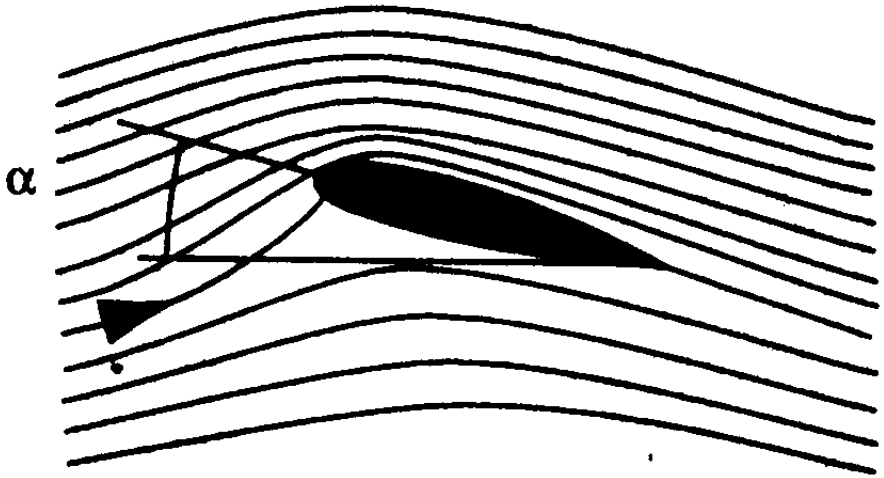
**Lift coefficient**
**$C_L$ defined so that:**

$$\text{Lift} = \tfrac{1}{2} \rho u^2 C_L A$$

**where A is the area of the aerofoil**

# Aerofoils: Stall

**No stall**



**Stall**

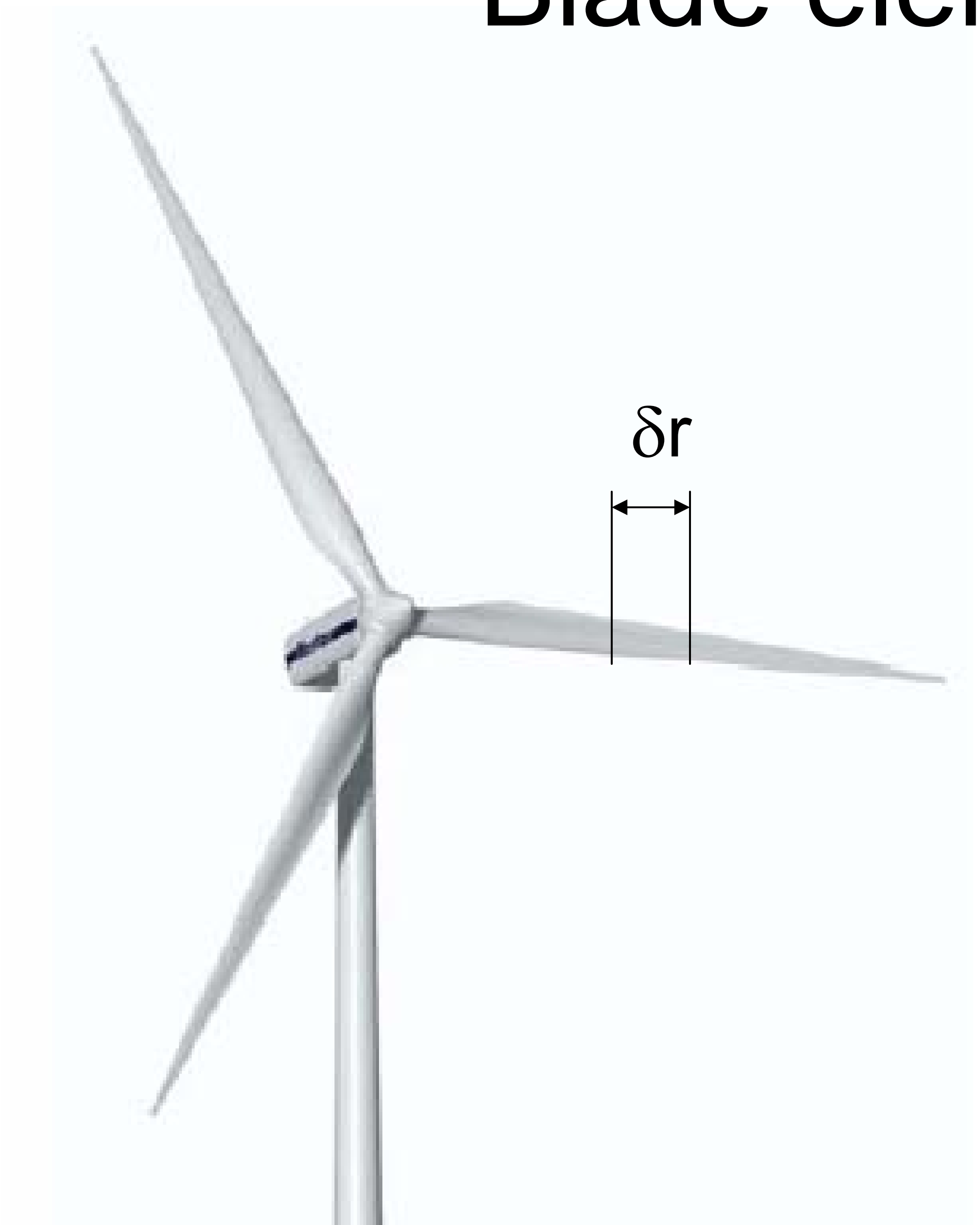
**Separation**

# Blade-element momentum theory Designing the ideal rotor

- Combine the Betz theory and aerofoil theory to determine the shape of the ideal rotor

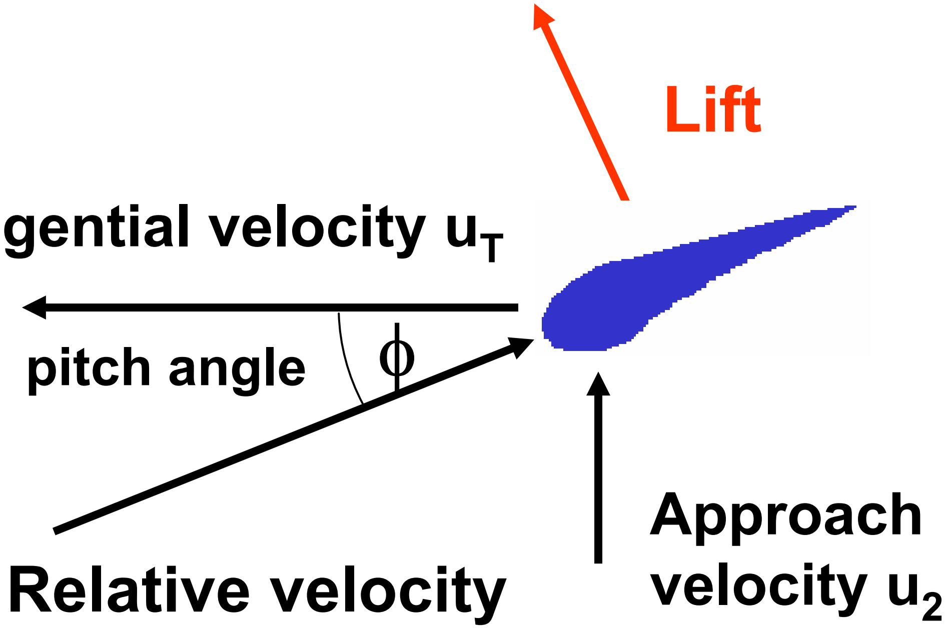
- Conditions will vary from the hub to the outside of the rotor, so we imagine the blade as consisting of small radial element each $\delta r$ long

# Blade element



# Ideal rotor : Relative velocity and lift

**The tangential velocity of the blade is usually many times the wind speed. This gives rise to a large lift force.**



$$u_{rel}=\sqrt{(u_T^2 + u_2^2)}$$

# Tip speed ratio

- Defined as speed of the blade tip divided by wind speed

  $$\lambda = u_T/u_1 \text{ or } u_T = \lambda\, u_1$$

  At a radius r:

  $$u_T = (r/R)\, \lambda\, u_1$$

  where R is the radius of the whole blade

- We will also make use of the Betz relation assuming the ideal situation of $a = 1/3$:

  $$u_2 = (2/3)u_1$$

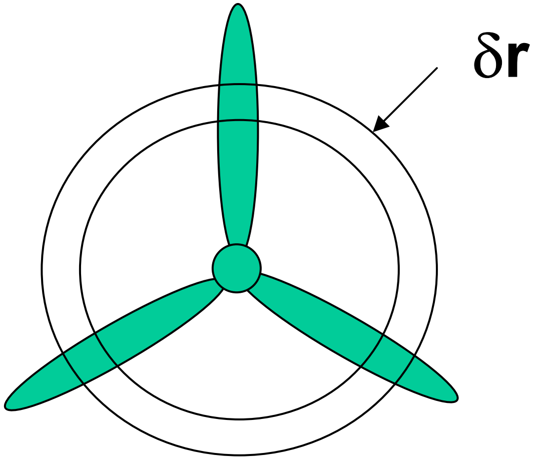
- We can now express the pitch angle as $\tan\phi = u_2/u_T$ and therefore

$$\boxed{\phi = \arctan\left[\frac{2}{3(r/R)\lambda}\right]}$$

# Thrust on the section of the rotor

- We can apply the Betz expression for thrust to just an annular section of the rotor.

*see slide 19*

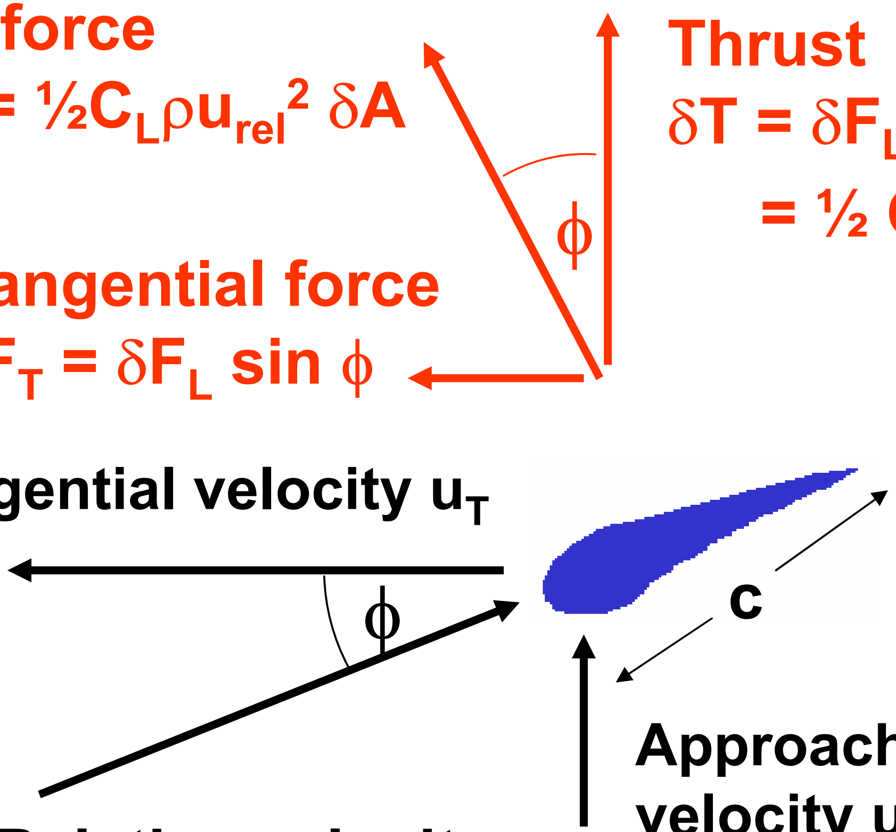


$$\boxed{\mathbf{T = (4/9)\,\rho A u_1^2}}$$

$$\Downarrow$$

$$\boxed{\mathbf{\delta T = (4/9)\,\rho\, 2\pi r\, \delta r\, u_1^2}}$$

# Ideal rotor : Relative velocity and lift



**Lift force**

$\delta F_L = \frac{1}{2} C_L \rho u_{rel}^2 \, \delta A$

**Thrust**

$\delta T = \delta F_L \cos\phi$

$= \frac{1}{2} C_L \rho u_{rel}^2 \, \delta A \cos\phi$

**Tangential force**

$\delta F_T = \delta F_L \sin\phi$

**Tangential velocity $u_T$**

**Approach velocity $u_2$**

**Relative velocity**

$u_{rel} = \sqrt{(u_T^2 + u_2^2)} = u_2 / \sin\phi$

**Equate with expression for $\delta T$ from last slide, setting $\delta A = B \, \delta r \, c$ (B is number of blades) and use $u_2 = (2/3) u_1$**

$$\boxed{\tan\phi \sin\phi = C_L Bc/4\pi r}$$

# Designing the ideal rotor: Results

- Combining this with the expression for tan ϕ from slide 26 gives the following expression for aerofoil chord length in the direction of the pitch angle:

$$\boxed{c = \frac{8\pi \sin\phi}{3BC_L\lambda_r}}$$

And from slide 27 we have the corresponding ideal pitch angle:

$$\boxed{\phi = \arctan\left[\frac{2}{3(r/R)\lambda}\right]}$$

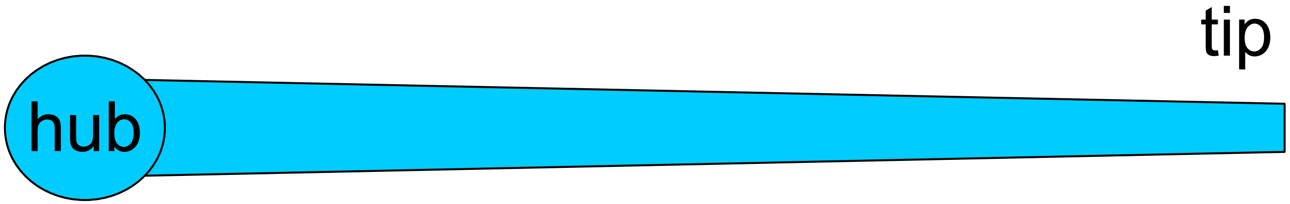
# Ideal rotor design: example

Tip speed ratio $\lambda = 7$

Radius R = 5 m

Number of blades B = 3

Lift coefficient $C_L=1$



| r/R | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|
| Chord c (m) | 0.86 | 0.46 | 0.31 | 0.24 | 0.19 |
| Pitch $\phi$ (°) | 27 | 15 | 11 | 8 | 7 |

# Causes of non-ideal performance

- Friction on blades (especially if dirty) decreases drag and increases lift
- Rotation, vortices and turbulence in wake correspond to wasted energy
- Tip losses: air takes a short cut around tip of blade
- Tower shadow
- Fixed speed operation (or limited speed range): mismatch with wind speed

# Putting on the brakes

- A wind turbine can only absorb so much power. It has to cope with very high wind speeds occasionally.
- Therefore brakes are needed (also for safety)
- Stall braking
  - Passive stall (no pitch control)
  - Pitch controlled stall or feathering
- Mechanical brake on shaft: hydraulic disc brake

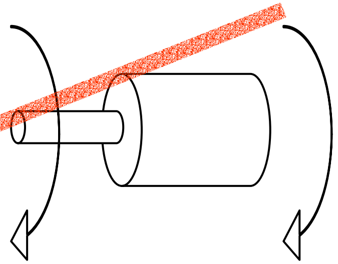
# Generators

~~1. Synchronous generator:~~

~~Fixed speed determined by grid~~

~~Torque proportional to lag $\theta_r - \theta_f$~~

~~Acts like a spring → resonances~~



~~Rotor angle $\theta_r$~~

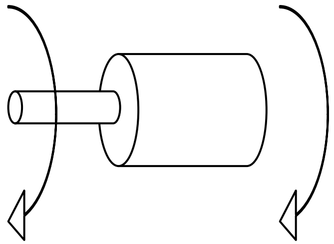
~~Magnetic field angle $\theta_f$~~

2. Induction generator:

Almost fixed speed determined by grid

Torque proportional to slip speed = $\omega_r - \omega_f$

Acts like a damper → no resonances



Rotor speed $\omega_r$

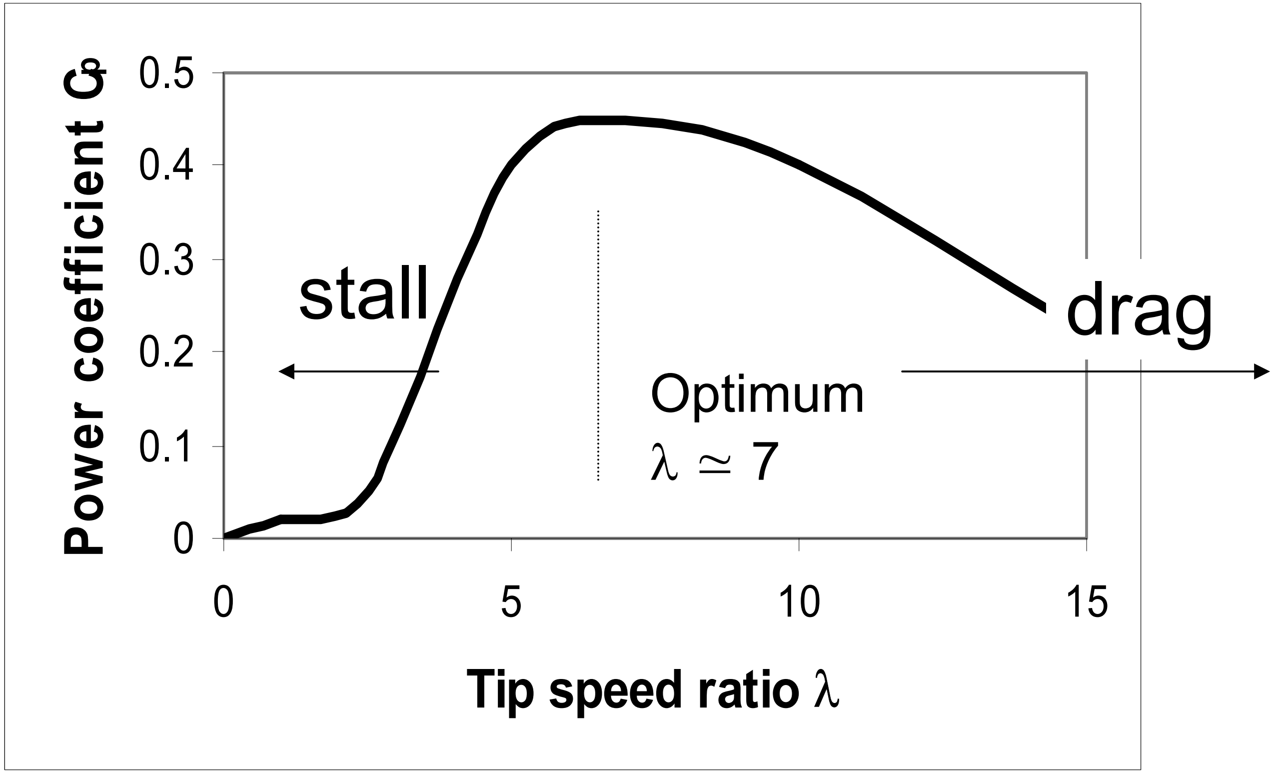
Magnetic fiel speed $\omega_f$

# Gearing requirements: generator

- The speed of the induction generator only varies over a restricted range

- The approximate speed is determined by the mains frequency and number of poles e.g. for 50 Hz
  - 2 poles → 3000 rpm (50Hz)
  - 4 poles → 1500 rpm
  - 8 poles → 750 rpm

# Gearing requirements: rotor

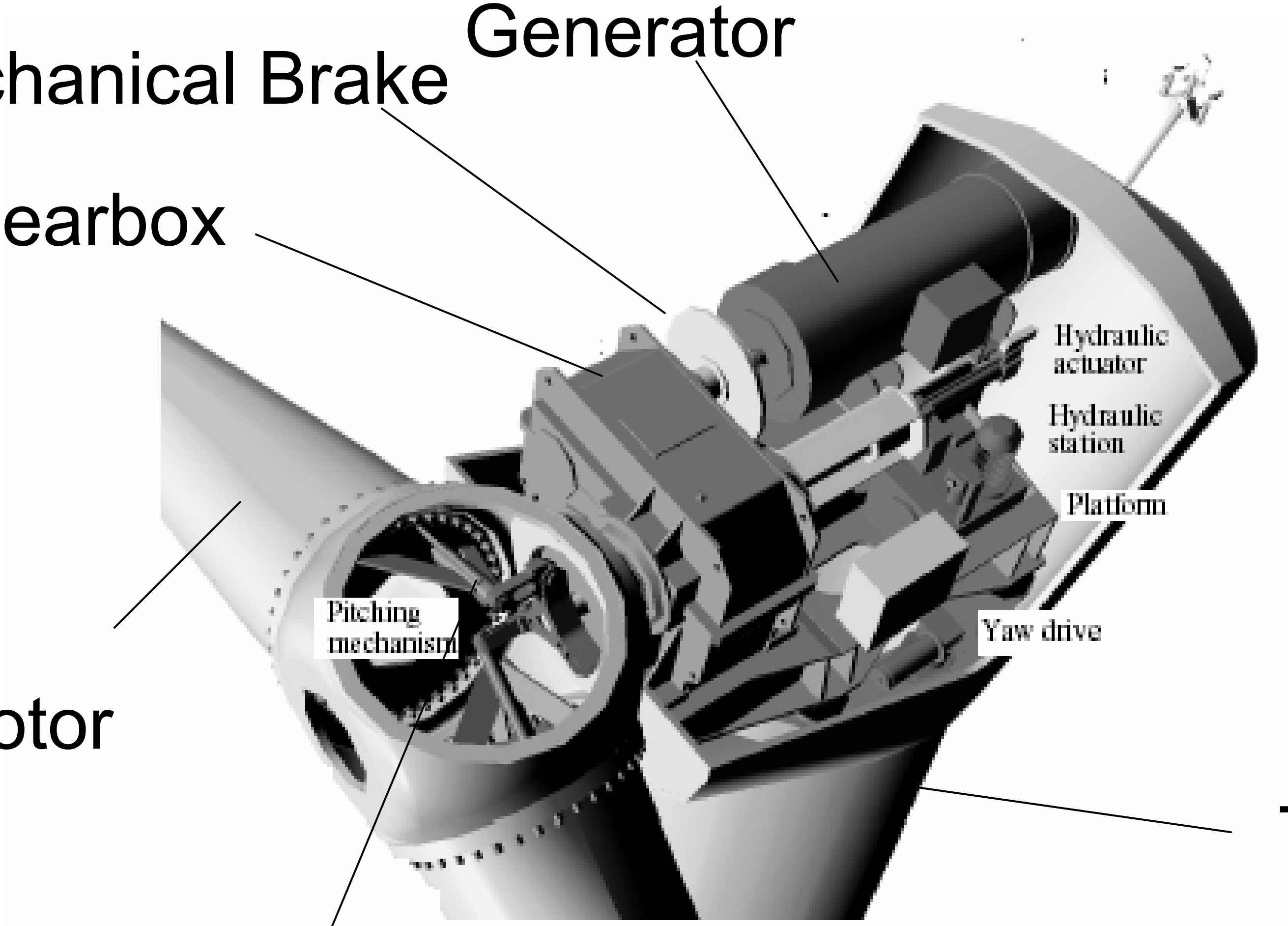


Example: wind speed 10 m/s, radius = 26 m

Tip speed = 7 x 10 = 70 m/s, giving 25 rpm

- *Therefore substantial gearing is needed e.g. 1:30*

# Wind turbine nacelle



Generator

Mechanical Brake

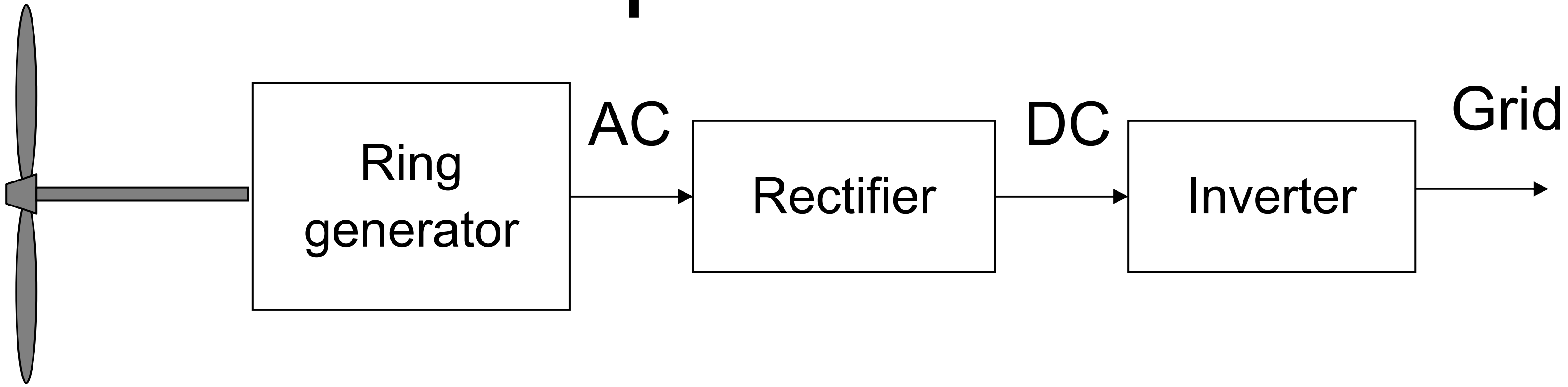
Gearbox

Rotor

Tower

Pitching mechanism

# Variable speed machines



Ring generator

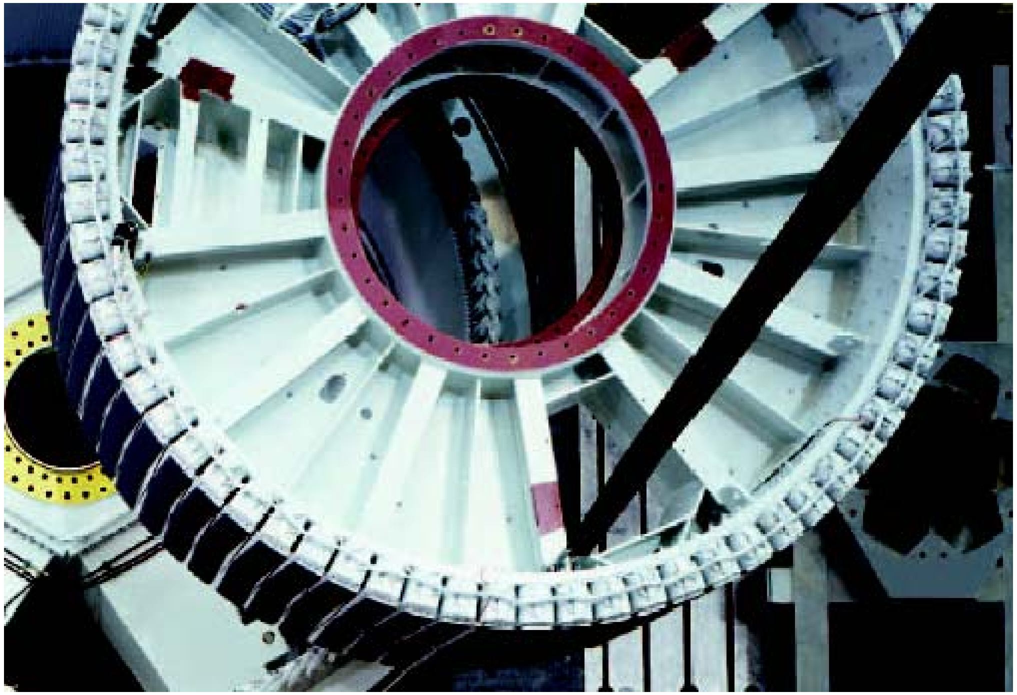
AC

Rectifier

DC

Inverter

Grid



# Aesthetic considerations

Generally favour:

- Slowly rotating blades
- 3 blades better than 2
- Smooth round towers preferred to lattices
- Not too much noise!

# Conclusions

- Wind is primarily generated by equator to pole energy gradients combined with the earth's rotation
- Wind speed increases significantly with height
- The power in the wind varies with the cube of wind speed
- The Betz theory establishes a limit of 16/27=0.59 to power coefficient
- The wind turbine blade is an aerofoil
- The tip speed is greater than the wind speed
- Lift is created as the blade slices through the wind – the tangential component of the lift is what drives the turbine
- You also need a gearbox, a generator and a tower to put it all on

# References

www.windpower.org Danish site, contains lots of background including nice animations

Manwell J.F. et al “Wind Energy Explained”

ISBN 0 471 49972 2 (covers everything in lecture and lots more)

Burton T. et al: “Wind Energy Handbook”,

ISBN 0 471 48997 2, more advanced (expensive)

web sites of manufacturers e.g. www.vestas.com www.enercon.de